Sushi or Fish Fingers?

Preferences for diversity and the sustainability of fisheries

Martin F. Quaas* and Till Requate

Department of Economics, University of Kiel, Germany

draft version, May 2008

Abstract. We consider consumers' preferences for food fish diversity in a multi-species fishery model. Studying both the long-run equilibria and the dynamics of an open-access fishery we conclude that the outcome is generally less sustainable the stronger preferences for diversity are. We show that even without biological interactions the optimal landing fees for the different species are dynamically interdependent and have to be adjusted in a non-monotonic way. One policy implication is that substantial landing fees should be levied also on a fish species with a healthy stock if it is a substitute for an endangered species.

JEL-Classification: Q22, Q57

Keywords: marine biodiversity, sustainability, fisheries, fishery economics, multispecies fishery

*Address: Department of Economics, University of Kiel, Olshausenstr. 20, 24118 Kiel, Germany. Email: quaas@economics.uni-kiel.de.

1 Introduction

The overexploitation of marine fish stocks is a major problem of world-wide importance. Many fish species, particularly those which are on the top of the food chain, are under serious threat of extinction (Pauly et al. 1998). Overfishing thus is also a threat for biodiversity conservation (Millennium Ecosystem Assessment 2005).

There are numerous contributions to the literature in environmental economics that try to estimate economic values for the usually non-marketed public good biodiversity (e.g., Ninan 2007, Markussen et al. 2005). The purpose of these efforts is to attach an economic value to biodiversity in order to generate incentives for biodiversity conservation.

In the case of fish, biodiversity has a clear and direct value: many consumers like to consume food fish of different species. A prominent example for this type of preferences is the Japanese sushi: it is traditionally prepared with a clearly defined set of different species of fish. The aim of the present paper is to include these preferences for diversity in a multi-species fishery model and to study how preferences for diversity affect (i) the outcome of the open access fishery, (ii) the Pareto-optimal dynamic fishing strategy and (iii) the policy implications.

For this sake, we model the consumers' preferences for food fish in form of a 'love-of variety' effect (Dixit and Stiglitz 1977): the utility derived from consuming fish is higher, the more species of fish are consumed in a more equal quantity. In particular we compare the cases that the different species are close, but imperfect, substitutes, and that the different species are complements. We describe the dynamics of the fish stocks in a very simple way by abstracting from any ecological interactions between the different stocks. This allows us to focus on the economic interactions leading to interdependent harvest quantities of the different species even without of ecological complexity. The recent literature on fishery ecology has developed several multispecies models of fisheries that include such biological interactions between different species of fish (e.g., Lande et al. 2003, Quinn and Deriso 1999). However, there is no thorough analysis of how preferences for diversity affect the fishery in the growing literature on multispecies fisheries.

Studying both the long-run equilibria and the transitional dynamics of an open-access fishery we conclude that better substitutes generally lead to a more sustainable outcome. In other words, these results imply that stronger preferences for diversity lead to a less sustainable outcome in the open access regime. Under the optimal fishing regime it may in the contrary be the case that the more endangered species should be harvested less. Thus, the need for regulation may be higher the more complementary the different species of fish are in order to attain a sustainable outcome.

We show that even without biological interactions the optimal landing fees for the different stocks of fish are interdependent. This interaction results in the need to levy substantial landing fees also on fish species with a stock that is still relatively high if this species is a substitute for an endangered species. If, on the other hand, the different species are complements, the fees should be targeted in a more direct way to the endangered species.

The paper is structured as follows: Section 2 describes the model including an arbitrary number of species of edible fish. In the analysis we concentrate on the case of just two species, because this facilitates the derivation and presentation of results. It is clear how the results generalize to a larger number of species. In Section 3 we study the equilibrium and the dynamics of the open access fishery. Section 4 presents the conditions for the Pareto optimal fishing policy, both in the steady state and the transition dynamics. The final section concludes.

2 The Model

The model is set up for an arbitrary number s of fish species, labeled $i = 1, \ldots, n$. To keep the analysis tractable, we will confine ourselves to the case of just two species. An extension of most results to the case of more than two species is straightforward.

The stock of species i is described by a simple equation of motion for its biomass x_i

$$\dot{x}_i = g_i(x_i) - n_i\, h_i = \rho_i\, x_i \left[1 - \frac{x_i}{\kappa_i}\right] - n_i\, h_i, \tag{1}$$

where ρ_i is the intrinsic growth rate and κ_i is the carrying capacity of species i. The strong simplifying assumption here is that there is no ecological interaction between species: The growth rate of species i depends only on its own stock and on the harvest of species i. Total harvest of species i is $n_i\, h_i$, where n_i is the number of vessels and h_i is the harvest per vessel.

Harvest per vessel is determined by fishing effort e_i directed at catching species i and the stock x_i. It is described by the generalized Gordon-Schaefer production function (cf. e.g. Clark 1991)

$$h_i = h_i(x_i, e_i) = x_i\, e_i^{\eta}, \tag{2}$$

where we assume positive but decreasing returns to effort e_i, i.e. $0 < \eta < 1$. Effort is measured in units of labor. In addition to the variable costs due to fishing effort, fishing also has fixed costs for operating a vessel. We assume that operating a vessel needs ϕ units of a numeraire commodity in each period of time, which are the same for all types of vessels, independently of which species is caught.

There is a mass of households equal to one. All households are identical. They inelastically supply one unit of labor each so that total labor force is unity. Rather than being employed in the fishery sector, workers can produce a numeraire commodity with a constant returns to scale technology where each unit of labor produces $\zeta > 0$ units of output. This implies that the wage rate equals ζ. Given the effort levels to catch each species of fish, output in the numeraire sector which is remaining for consumption is

$$y = \zeta \left(1 - \sum_i n_i\, e_i\right) - \sum_i n_i\, \phi. \tag{3}$$

Consumer preferences are described by the utility function

$$u(v, y) = y + \alpha \ln v, \tag{4}$$

where y is the consumption of a numeraire commodity and v is the consumption of fish. Consumption of fish is composed of different species, i.e.

$$v = v(q_1, \ldots, q_n) = \left[\sum_i q_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

where q_i is the quantity of species i consumed. The parameter σ measures the elasticity of substitution in consumption of two different species. Different species are considered to be substitutes if the elasticity of substitution exceeds unity, $\sigma > 1$, and complements if the elasticity of substitution is smaller than one, $\sigma < 1$. Hence, $\sigma > 1$ corresponds to a consumer of fish fingers, while $\sigma < 1$ describes a consumer of sushi which has to consist of certain different varieties of fish. The limiting case $\sigma = 1$, corresponds to a Cobb-Douglas utility function $v(q_1, \ldots, q_n) = \prod_i q_i^{1/s}$.

As a slightly different interpretation, a higher σ may be regarded as a stronger preference for (bio-)diversity, since it is more important for the consumer that all species are equally represented in the consumption bundle.

3 Open access

The question we pose in this section is how consumers' preferences for diversity, measured by the elasticity of substitution σ, affect the outcome of a competitive fishery under the institutional setting of open access.

Given the quasi-linear utility function (4), the representative household's utility maximization problem can be formulated as follows.

$$\max_{\{q_i\}} \left\{ \frac{\alpha\,\sigma}{\sigma - 1} \ln \left[\sum_i q_i^{\frac{\sigma-1}{\sigma}}\right] + \zeta - \sum_i p_i\, q_i \right\}, \tag{6}$$

where ζ equals the wage rate, and p_i is the price for one kilo of fish of species i. The first-order conditions for this problem lead to the inverse demand functions

$$p_i = \alpha\, v^{-\frac{\sigma-1}{\sigma}}\, q_i^{-\frac{1}{\sigma}} \qquad \text{for all } i. \tag{7}$$

The inverse demand function is declining in the consumption of species i. It is also declining in the consumption of all other species of fish, if these are substitutes ($\sigma > 1$). If, on the other hand, the different species are complements ($\sigma < 1$), the inverse demand increases in the consumption of other species.

With little rearrangement (see Appendix A), we derive the demand function

$$q_i = \alpha\, p_i^{-\sigma} \left[\sum_i p_i^{1-\sigma}\right]^{-1}, \tag{8}$$

which is decreasing in the price p_i of the species under consideration and increasing in the prices of the other species if they are substitutes, $\sigma > 1$, or decreasing if they are complements, $\sigma < 1$.

A fisherman who fishes for species i maximizes profit by choosing the effort level e_i, taking the wage rate ζ and output price p_i as given,

$$\max_{e_i} p_i \, x_i \, e_i^\eta - \zeta \, e_i - \phi. \tag{9}$$

The first-order condition determines the optimal effort level

$$e_i = \left[\frac{p_i \, \eta \, x_i}{\zeta} \right]^{\frac{1}{1-\eta}} \tag{10}$$

and the harvest of each vessel, which is

$$h_i = x_i \, e_i^\eta = \left[\frac{p_i \, \eta}{\zeta} \right]^{\frac{\eta}{1-\eta}} x_i^{\frac{1}{1-\eta}}. \tag{11}$$

The harvest per vessel increases with the output price, p_i, and decreases with the wage rate ζ, as this increases the costs of fishing effort. The harvest increases also with the stock x_i of fish, because this decreases fishing costs.

When there are n_i vessels fishing for species i, total harvest is $n_i \, h_i$. Equating supply and demand (8), we get that total harvest of each species is equal to demand

$$n_i \, h_i = \alpha \, p_i^{-\sigma} \left[\sum_i p_i^{1-\sigma} \right]^{-1}. \tag{12}$$

As long as profits are positive new vessels will enter the business. In equilibrium, profits are vanish, i.e., for each vessel we have $p_i \, h_i - w \, e_i - \phi = 0$. Using the profit-maximizing effort level (10) and the profit-maximizing harvest per vessel, (11) in this condition, we derive the equilibrium price of fish,

$$p_i = \frac{\phi^{1-\eta} \, \zeta^\eta}{(1-\eta)^{1-\eta} \, \eta^\eta} \, x_i^{-1} = \Phi^{-1} \, x_i^{-1}, \tag{13}$$

where we defined the parameter cluster $\Phi = \frac{(1-\eta)^{1-\eta} \, \eta^\eta}{\phi^{1-\eta} \, \zeta^\eta}$ to simplify notation. Plugging this result into (10), we find that effort per vessel is independent of the stock of fish,

$$e_i = \left[\frac{\phi^{1-\eta} \, \zeta^\eta}{(1-\eta)^{1-\eta} \, \eta^\eta} \, x_i^{-1} \frac{\eta \, x_i}{\zeta} \right]^{\frac{1}{1-\eta}} = \frac{\eta \, \phi}{(1-\eta) \, \zeta}. \tag{14}$$

It increases with the fixed costs ϕ and decreases with the wage rate ζ. It also increases with the output elasticity of fishing effort, η.

Using (13) in (12) we obtain the total catch as a function of the stocks

$$n_i\,h_i = \alpha\,\left[\sum_j p_j^{1-\sigma}\right]^{-1}\;p_i^{-\sigma} = \alpha\,\Phi\,\frac{x_i^\sigma}{\sum\limits_j x_j^{\sigma-1}}. \tag{15}$$

This equation relates the total catch of species i to the stocks of the different species. It is only in the case of a Cobb-Douglas utility function, i.e. $\sigma = 1$ that harvest of one species is independent of the stock of all other species.

If the species are perfect substitutes ($\sigma \to \infty$), only the species with the highest stock will be harvested. Total harvest of this species is $\alpha\,\Phi$. It can be easily verified that total harvest of all other species is zero.

If the species are perfect complements ($\sigma \to 0$), an equal quantity of all species will be harvested: $n_i\,h_i = \alpha\,\Phi\,\left[\sum_j x_j^{-1}\right]^{-1}$ for all i.

To simplify the analysis, we will consider just two species of fish in the following. In this case, we can easily derive the comparative statics of total harvest $n_i\,h_i$ with respect to the elasticity of substitution σ:

$$\frac{d\,n_1\,h_1}{d\sigma} = \alpha\,\Phi\,x_1^\sigma\,x_2^{\sigma-1}\,\frac{\ln x_1 - \ln x_2}{\left[x_1^{\sigma-1} + x_2^{\sigma-1}\right]^2} \tag{16}$$

$$\frac{d\,n_2\,h_2}{d\sigma} = -\,\alpha\,\Phi\,x_1^{\sigma-1}\,x_2^\sigma\,\frac{\ln x_1 - \ln x_2}{\left[x_1^{\sigma-1} + x_2^{\sigma-1}\right]^2} \tag{17}$$

Let, without loss of generality, the stock of species 1 be larger than the stock of species 2, i.e. $x_1 > x_2$. Then, the harvest of species 1 will increase and the harvest of species 2 will decrease with the elasticity of substitution σ:

Proposition 1 (better substitutes are more sustainable)
The better substitutes in consumption the species are,

1. *the more will be harvested of the species with the largest stock.*

2. *the less will be harvested of the species with the smallest stock.*

The intuition for this result is as follows: The better substitutes the species are, the more elastic is the demand with respect to a price difference. Thus, the better substitutes the species are the more will be consumed of the cheaper fish and the less will be consumed of the more expensive fish. Because the price at which fish is supplied on the market is lower for the species with the larger stock, more will be consumed of the fish with the larger stock and less of the fish with the smaller stock.

If we assume that a smaller stock means that the species is more endangered, Proposition 1 implies that the harvest of the most endangered species is smaller the better substitutes the species are. In this sense, a higher elasticity of substitution between different species of fish leads to a more sustainable fishery management in the open access regime. That is, weaker preferences for the diversity of food fish contribute to the protection of biodiversity.

Total catch, as determined by (15), together with the equations of motion for the fish stocks (1) determine the dynamics of the open access fishery. The equilibrium is reached when total catch of each species equals the net reproduction of this species, i.e. $n_i\, h_i = g_i(x_i)$.

Using expression (15) for the total harvest of species i in the equation of motion for the fish stocks (1), we obtain the following system of equations for the equilibrium fish stocks in the two-species case

$$\rho_1\, x_1 \left[1 - \frac{x_1}{\kappa_1}\right] = \alpha\, \Phi\, \frac{x_1^\sigma}{x_1^{\sigma-1} + x_2^{\sigma-1}} \tag{18}$$

$$\rho_2\, x_2 \left[1 - \frac{x_2}{\kappa_2}\right] = \alpha\, \Phi\, \frac{x_2^\sigma}{x_1^{\sigma-1} + x_2^{\sigma-1}}. \tag{19}$$

Rearranging these equations, we obtain

$$x_2^\star(x_1) = x_1 \left[\frac{\alpha\, \Phi\, \kappa_1}{\rho_1\, [\kappa_1 - x_1]} - 1\right]^{\frac{1}{\sigma-1}} \tag{20}$$

$$x_1^\star(x_2) = x_2 \left[\frac{\alpha\, \Phi\, \kappa_2}{\rho_2\, [\kappa_2 - x_2]} - 1\right]^{\frac{1}{\sigma-1}}. \tag{21}$$

Plugging the first equation into the second equation, we find that the open access equilibrium is determined by the fixed point equation $x_1 = x_1^\star(x_2^\star(x_1))$.

Proposition 2

In the case of substitutes, $\sigma > 1$, but independently of the exact value of σ, a globally stable open access equilibrium exists with strictly positive stocks of both species, $x_1^\star > 0$ and $x_2^\star > 0$, if $\rho_1 + \rho_2 > \alpha\, \Phi$.

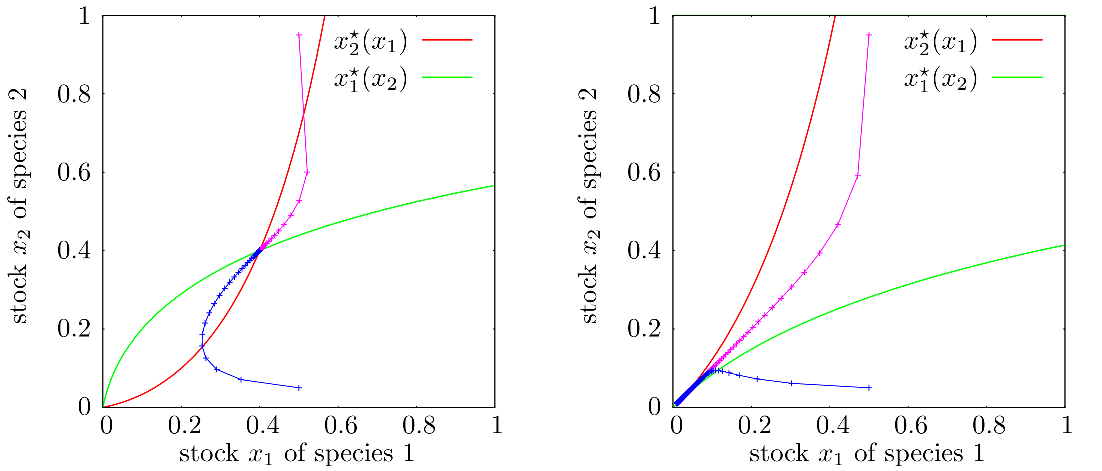
In the case of complements, $\sigma < 1$, but independently of the exact value of σ, a globally stable equilibrium with strictly positive stocks of both species, $x_1^\star > 0$ and $x_2^\star > 0$, exists only if $\min\{\rho_1, \rho_2\} > \alpha\, \Phi$.

Proposition 2 shows that there is a fundamental difference between substitutes and complements with regard to the dynamic behavior of the open access fishery.

In the case of substitutes, if the intrinsic growth rates of the two fish species are very low, $\rho_1 + \rho_2 < \alpha\, \Phi$, the open access outcome is extinction of both species. For substitutes already moderately large intrinsic growth rates suffice to reach a stable open access equilibrium where no species goes extinct. Moreover, it is sufficient that the aggregate growth rate of both species is large enough. In other words, if there is just one species with a sufficiently large intrinsic growth rate, neither of the species will be fished to extinction.

The case of complements is different. Both species need to have a sufficiently large intrinsic growth rate in order to guarantee that neither of the species will be fished to extinction.

To investigate numerically the transition dynamics to the open access equilibrium, we started a numerical computation for a given set of parameters for given



$\rho_1 = \rho_2 = 0.5$ $\rho_1 = \rho_2 = 0.3$

Figure 1: The phase diagrams for the case of substitutes ($\sigma = 2$) and two different intrinsic growth rates: $\rho_1 = \rho_2 = 0.5$ on the left hand side and $\rho_1 = \rho_2 = 0.25$ on the right hand side. The other parameters are $\alpha = 0.6$, $\Phi = 1$ and $\kappa_1 = \kappa_2 = 1$. The initial values are $(x_1, x_2) = (0.05, 0.5)$ and $(x_1, x_2) = (0.95, 0.5)$ in both pictures.

initial stocks of both species and computed the equations of motion (1) recursively using the expression (15) for the aggregate harvest in the open access regime.

In Figure 1, the two equilibrium curves $x_2^\star(x_1)$ and $x_1^\star(x_2)$ are plotted for substitutes ($\sigma = 2$) and for two different values of the intrinsic growth rates. For the sake of clarity, we have assumed that both species have the an identical ecological dynamics, i.e. $\rho_1 = \rho_2$ and $\kappa_1 = \kappa_2$. The figure shows the stock of species 1 in equilibrium as a function of the stock of species 2, i.e. $x_1^\star(x_2)$ (green curve) and the stock of species 2 in equilibrium as a function of the stock of species 1, i.e. $x_2^\star(x_1)$ (red curve). At the intersection between these two curves there is an open access equilibrium. In the picture on the left hand side of Figure 1 such an equilibrium exists at strictly positive values of x_1 and x_2, because the intrinsic growth rates of both species are sufficiently high. This is different in the picture on the right hand side of Figure 1. Here, we assumed low intrinsic growth rates, leading to the extinction of both species under open access fishery (cf. Proposition 2).

Each of the two pictures in Figure 1 shows two dynamic paths, one starting with a low initial stock of species 2 (blue line) and the other one starting with a high initial stock (magenta line), while the stock of species 1 is the same in both cases.

For comparatively high intrinsic growth rates of the fish stock (Figure 1, left) a unique and stable equilibrium exists in which both species have a strictly positive stock (cf. Proposition 2). Interestingly, the stock of fish species 1 changes in a non-monotonic way for the two time paths shown. With a low initial stock of species 2, species 1 which has a higher initial stock is first put under strong fishing pressure and later on the harvest is decreased until the stock has recovered to the equilibrium value. If the initial stock of species 2 is substantially larger than the initial stock of species 1, species 2 is fished more strongly initially and the stock of species 1 can recover. Finally, both stocks decline towards their equilibrium levels.

For very low intrinsic growth rates (Figure 1, right), extinction of both species is the only stable equilibrium. independently of whether the stock of species 2 is large or small initially, both trajectories end up at zero stocks of both species. An interesting feature of the time path with a low initial value of species 2 is that its stock will increase for a while, until it is fished to extinction. The reason is that with substitutes the species with the higher stock is under strong fishing pressure while the species with a small stock is too costly to be harvested.

In Figure 2, the corresponding results for the case of complements ($\sigma = 0.5$) are shown. As in the case of complements it is also possible that instable equilibria occur, we show four graphs here. For very high intrinsic growth rate of the two fish species, a globally stable equilibrium exists with strictly positive stocks of both species (cf. Proposition 2). In this case the growth rates are so high that even species 2 with a very small initial stock recovers, although it does not increase as quickly as species 1 with the higher initial stock (blue curve in the top left graph of Figure 2).

If the intrinsic growth rates are a bit lower, but still high, a *locally* stable equilibrium exists with positive stocks of both species (the graph on the top right of Figure 2). In addition to this stable equilibrium, two saddle point-stable equilibria exist. Whether one of the species is fished to extinction crucially depends on the initial conditions. If they lie inside the domain of attraction of the stable equilibrium both species will survive in the long run (magenta and cyan curve). If the initial conditions lie outside the domain of attraction, one species will go extinct while the other reaches its natural carrying capacity: because both species are complements, fishing will discontinue once one species is extinct (blue curve). Note that the ecological parameters in this figure are identical to those in the left graph of Figure 1: with the same intrinsic growth rates, both species will survive in an open access fishery if the species are substitutes, but one species may be fished to extinction in the case of complements.

For still lower intrinsic growth rates, a saddle point-stable equilibrium exists (graph on the bottom left of Figure 2. But for almost all combinations of initial stocks, ultimately one species will be fished to extinction. The only exception are those initial stocks that exactly lie on the saddle-path.

If the intrinsic growth rates are very low, no equilibrium with positive stocks of the species exists. For any initial stocks, one of the species will be fished to extinction under the open access regime. For the case of substitutes lower growth rates enable the survival of both fish species than in the case of complements. However, if the growth rates are very low, both species will be fished to extinction, while in the case of substitutes one species will survive: it is of no economic use if the complement has been extincted. Thus, it is not fished anymore.

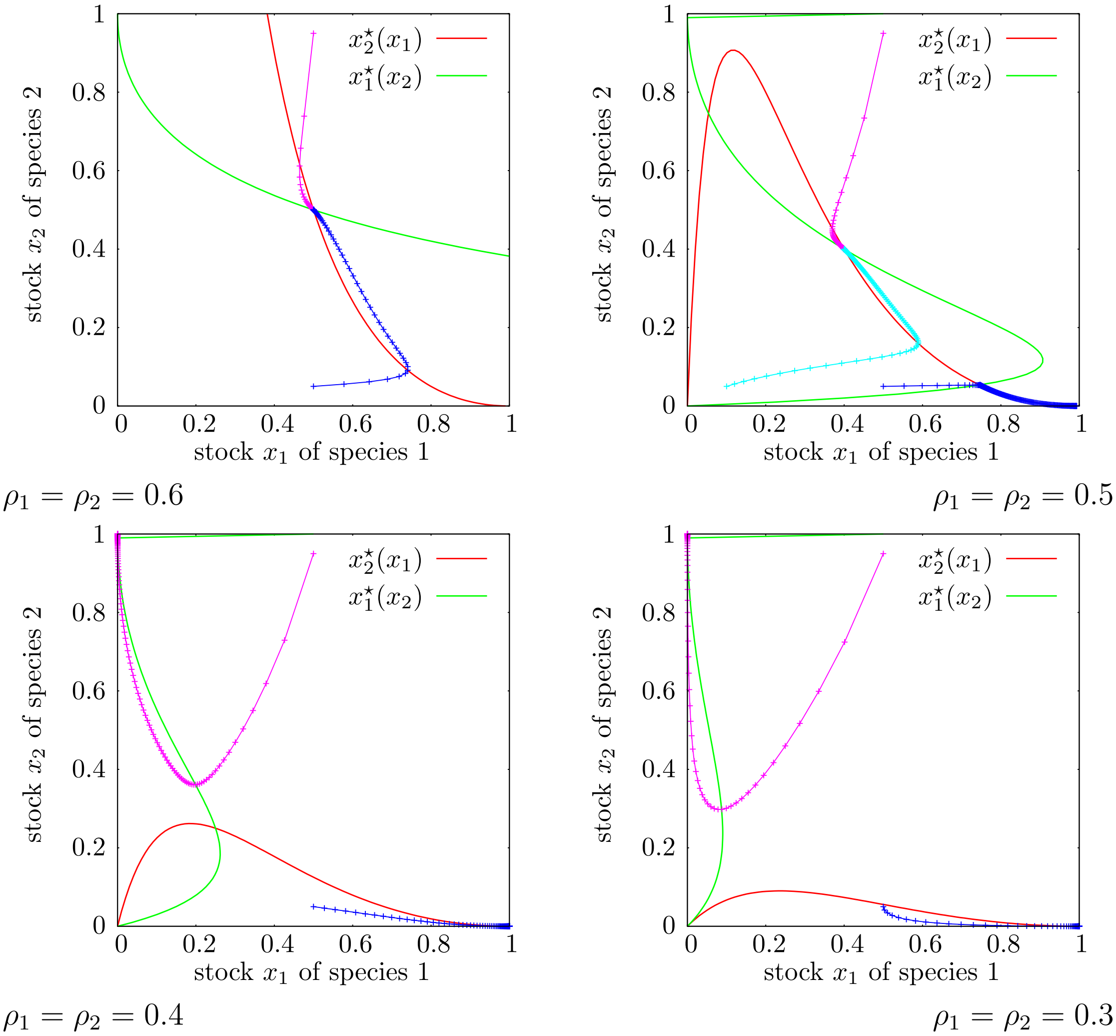


Figure 2: The phase diagrams for the case of complements ($\sigma = 0.5$) and four different intrinsic growth rates: $\rho_1 = \rho_2 = 0.6$ (top left), $\rho_1 = \rho_2 = 0.5$ (top right), $\rho_1 = \rho_2 = 0.4$ (bottom left) and $\rho_1 = \rho_2 = 0.3$ (bottom right). The other parameters are $\alpha = 0.6$, $\Phi = 1$ and $\kappa_1 = \kappa_2 = 1$. The initial values are $(x_1, x_2) = (0.05, 0.5)$ and $(x_1, x_2) = (0.95, 0.5)$ in both pictures.

4 Pareto-optimum

The outcome of the open-access-fishery is certainly not efficient. In order to determine the Pareto-optimal fishery regime we consider a social planner. The planner's optimization problem is to maximize a representative household's utility by choosing effort over time

$$\max_{y,\{q_i,e_i,n_i\}} \int_0^\infty \left[y + \frac{\alpha\,\sigma}{\sigma-1} \ln\left[\sum_i q_i^{\frac{\sigma-1}{\sigma}}\right]\right] \exp(-\delta\, t)\, dt$$

$$\text{subject to } q_i = n_i\, x_i\, e_i^\eta,\ (1), \text{ and } (3). \quad (22)$$

To determine the general properties of the optimal fishery we derive the conditions for the optimal fishing strategy using the Hamilton formalism. The current-value Hamiltonian with co-state variables π_i for $q_i = n_i\, x_i\, e_i^\eta$ and μ_i for the equations of motion (1) of the fish stocks reads

$$\mathcal{H} = \frac{\alpha\,\sigma}{\sigma-1} \ln\left[\sum_i q_i^{\frac{\sigma-1}{\sigma}}\right] + \zeta \left(1 - \sum_i n_i\, e_i\right) - y - \sum_i n_i\, \phi$$
$$+ \sum_i \pi_i \left[n_i\, x_i\, e_i^\eta - q_i\right] + \sum_i \mu_i \left[g_i(x_i) - n_i\, x_i\, e_i^\eta\right] \quad (23)$$

According to the formalism, the necessary conditions for the optimal fishing strategy are

$$\frac{\partial \mathcal{H}}{\partial q_i} = 0 \qquad \alpha \frac{q_i^{-\frac{1}{\sigma}}}{\sum_j q_j^{\frac{\sigma-1}{\sigma}}} = \pi_i \quad (24)$$

$$\frac{\partial \mathcal{H}}{\partial e_i} = 0 \qquad [\pi_i - \mu_i]\ \eta\, n_i\, x_i\, e_i^{\eta-1} = \zeta\, n_i \quad (25)$$

$$\frac{\partial \mathcal{H}}{\partial n_i} = 0 \qquad [\pi_i - \mu_i]\ x_i\, e_i^\eta = \zeta\, e_i + \phi \quad (26)$$

$$\frac{\partial \mathcal{H}}{\partial x_i} = \delta\, \mu_i - \dot{\mu}_i \qquad \left[\pi_i - \mu_i\right] n_i\, e_i^\eta + \mu_i\, g_i'(x_i) = \delta\, \mu_i - \dot{\mu}_i \quad (27)$$

Conditions (24), (26), (25) and (27) hold for all species i.

From (24) we derive the total harvest of species i as a function of the shadowprice of harvest (π_i)

$$n_i\, h_i = q_i = \alpha \frac{\pi_i^{-\sigma}}{\sum_j \pi_j^{1-\sigma}}. \quad (28)$$

This expression for the optimal harvest of species i is similar to the harvest in the open access setting (equation 12) except that the equilibrium price p_i of fish in Equation (12) is replaced by the shadowprice π_i.

From Conditions (26) and (25) we derive the optimal effort levels:

$$e_i = \frac{\eta\,\phi}{(1-\eta)\,\zeta}. \tag{29}$$

Evidently, the effort per vessel in the open access setting is equal to the optimal effort level. What is causing the inefficiency in the open access fishery is the free entry that leads to an excessively high number of vessels fishing for each species.

From the condition for the optimal number n_i of vessels, using (29), we derive the shadowprice of harvested fish

$$\pi_i = \frac{\phi}{1-\eta}\,h_i^{-1} = \Phi^{-1}\,x_i^{-1} + \mu_i. \tag{30}$$

The shadowprice of harvest in the optimum exceeds the price of harvest in the open-access equilibrium by the shadowprice μ_i of the stock of fish species i. This implies that the optimal landing fee for fish species i in the open access regime just equals the shadowprice of the fish stock.

Our next step is to determine the optimal harvest in a steady-state and the optimal equilibrium fish stocks. In a steady state, neither the stocks of fish nor the shadowprices change, i.e. $\dot{\mu}_i = 0$. Hence, the optimal steady-state number of vessels is determined by Condition (27) together with (30). Given the number of vessels, the harvest of species i is determined by (see Appendix B)

$$n_i\,h_i = \alpha\,\Phi\,\frac{\left[x_i - \frac{\rho_i\,x_i\left[1-\frac{x_i}{\kappa_i}\right]}{\delta+\rho_i\,\frac{x_i}{\kappa_i}}\right]^{\sigma}}{\sum\limits_j\left[x_j - \frac{\rho_j\,x_j\left[1-\frac{x_j}{\kappa_j}\right]}{\delta+\rho_j\,\frac{x_j}{\kappa_j}}\right]^{\sigma-1}} \tag{31}$$

The optimal steady-state harvest of species i differs from the harvest in the open access equilibrium, as correcting factors appear both on the numerator and denominator of the right hand side of equation (31).

Again, as in the open-access case, the harvest of species i depends not only on the own stock, but also on the stocks of all other species. Only in the case of Cobb-Douglas preferences, $\sigma = 1$, harvests are independent of each other.

In the case of perfect complements ($\sigma = 0$), the harvests of all species is the same, $n_i\,h_i = \alpha\,\Phi\left[\sum\limits_j x_j^{-1}\left[1-\frac{\kappa_j - x_j}{\delta/\rho_j\,\kappa_j + x_j}\right]^{-1}\right]^{-1}$. It is clearly smaller than in the open access regime with perfect complements.

In the case of perfect substitutes, ($\sigma \to \infty$) it is not necessarily optimal to harvest only the species with the largest stock. Rather, only the species should be harvested for which

$$\tilde{x}_i \equiv x_i - \frac{\rho_i\,x_i\left[1-\frac{x_i}{\kappa_i}\right]}{\delta+\rho_i\,\frac{x_i}{\kappa_i}} \tag{32}$$

is maximal. Equation (32) defines the adjusted steady-state stock of species i. That is the equilibrium stock of species i minus the sustainable yield, i.e. the natural growth at the equilibrium stock, discounted at a rate that is composed of the rate of time preference δ and the natural mortality rate of the fish at the equilibrium stock, $\rho_i\, x_i/\kappa_i$. Of course it may well be that the adjusted steady-state stock of species i is larger than the adjusted steady-state stock of another species j, even if the actual steady-state stock is smaller. In particular this would very likely be the case if species i had a vastly larger intrinsic growth rate than species j, $\rho_i \gg \rho_j$.

Consider again the case of just two species of fish, we can easily derive the comparative statics of total steady-state catch $n_i\, h_i$ with respect to the elasticity of substitution σ:

$$\frac{d\, n_1\, h_1}{d\sigma} = \alpha\, \Phi\, \tilde{x}_1^{\sigma}\, \tilde{x}_2^{\sigma-1}\, \frac{\ln \tilde{x}_1 - \ln \tilde{x}_2}{\left[\tilde{x}_1^{\sigma-1} + \tilde{x}_2^{\sigma-1}\right]^2} \tag{33}$$

$$\frac{d\, n_2\, h_2}{d\sigma} = -\, \alpha\, \Phi\, \tilde{x}_1^{\sigma-1}\, \tilde{x}_2^{\sigma}\, \frac{\ln \tilde{x}_1 - \ln \tilde{x}_2}{\left[\tilde{x}_1^{\sigma-1} + \tilde{x}_2^{\sigma-1}\right]^2} \tag{34}$$

Let, without loss of generality, the adjusted steady-state stock of species 1 be larger than the adjusted steady-state stock of species 2, i.e. $\tilde{x}_1 > \tilde{x}_2$. Then, the harvest of species 1 will increase and the harvest of species 2 will decrease with the elasticity of substitution σ:

Proposition 3

The better substitutes in consumption the species are,

1. *the more will be harvested of the species with the largest* adjusted *steady-state stock.*

2. *the less will be harvested of the species with the smallest* adjusted *steady-state stock.*

While in an open access regime a higher elasticity of substitution between the different species of fish leads to a more sustainable outcome in the sense that the species with the lower stock is harvested less, Proposition 3 shows that this may not be optimal. If, e.g., the intrinsic growth rate of the species with the smaller stock is substantially larger, it may well be optimal to harvest more of the species with the smaller stock if σ increases.

The problem to determine the optimal steady-state stocks in a fishery with two species can be formulated as a fixed-point problem in a similar way as in the open access equilibrium. The exact formulas are somewhat more complicated however. We have

$$x_1^{\text{stst}}(x_2) = \frac{1}{4}\left[Y_2 + \kappa_1\left[1 - \frac{\delta}{\rho_1}\right]\right] + \sqrt{\frac{\delta\,\kappa_1}{2\,\rho_1}\,Y_2 + \frac{1}{16}\left[Y_2 + \kappa_1\left[1 - \frac{\delta}{\rho_1}\right]\right]^2} \tag{35}$$

$$x_2^{\text{stst}}(x_1) = \frac{1}{4}\left[Y_1 + \kappa_2\left[1 - \frac{\delta}{\rho_2}\right]\right] + \sqrt{\frac{\delta\,\kappa_2}{2\,\rho_2}\,Y_1 + \frac{1}{16}\left[Y_1 + \kappa_2\left[1 - \frac{\delta}{\rho_2}\right]\right]^2} \tag{36}$$

where

$$Y_1 = \tilde{x}_1 \left[\alpha\, \Phi \left[\frac{1}{\rho_1 - \rho_1 \frac{x_1}{\kappa_1}} - \frac{1}{\delta + \rho_1 \frac{x_1}{\kappa_1}} \right] - 1 \right]^{\frac{1}{\sigma-1}} \tag{37}$$

$$Y_2 = \tilde{x}_2 \left[\alpha\, \Phi \left[\frac{1}{\rho_2 - \rho_2 \frac{x_2}{\kappa_2}} - \frac{1}{\delta + \rho_2 \frac{x_2}{\kappa_2}} \right] - 1 \right]^{\frac{1}{\sigma-1}}. \tag{38}$$

The steady state stocks of the two fish species are determined by the fixed point equation $x_1 = x_1^{\text{stst}}(x_2^{\text{stst}}(x_1))$.

To determine the optimal transition dynamics towards the steady state, we reformulate the social planner's optimization problem by considering total harvest $H_i = n_i\, x_i^{\chi}\, e_i^{\eta}$ as the decision variable. Using (29), we have

$$n_i = H_i\, x_i^{-\chi}\, e_i^{-\eta} = H_i\, x_i^{-\chi} \left[\frac{\eta\, \phi}{(1-\eta)\,\zeta} \right]^{-\eta} \tag{39}$$

Thus, the optimization problem simplifies to

$$\max_{\{H_i\}} \int_0^{\infty} \left[\frac{\alpha\, \sigma}{\sigma - 1} \ln \left[\sum_i H_i^{\frac{\sigma-1}{\sigma}} \right] - \frac{1}{\Phi} \sum_i H_i\, x_i^{-\chi} \right] \exp(-\delta\, t)\, dt$$

$$\text{subject to } \dot{x}_i = \rho_i\, x_i \left[1 - \frac{x_i}{\kappa_i} \right] - H_i, \tag{40}$$

We solved this optimization problem numerically in a discrete time setting, employing a dynamic program. (The code of the program which is written in **c** is available from the authors upon request.)

The simulation results for the same ecological parameter setting as used to compute the graphs in Figure 1 and for the same initial stocks are shown in Figures 3 and 4. The intrinsic growth rates of both species of fish on the left hand side are $\rho_1 = \rho_2 = 0.5$ while on the right hand side they are $\rho_1 = \rho_2 = 0.5$. The two graphs at the top show the phase diagrams in the state space. The green and red line show the curves of $x_1^{\text{stst}}(x_2)$ and $x_2^{\text{stst}}(x_1)$. Their intersection depicts the steady state. The two graphs at the bottom show the optimal landing fees for both species, i.e. the shadowprices of the two fish stocks (cf. Equation (30)).

In Figure 3 the case of substitutes is shown. The blue curve depicts the optimal path for a low initial stock of species 2, while the magenta curve depicts the optimal path for a high initial stock of species 2. The initial stock of species 1 is the same in both cases. Even with very low growth rates of both species (right hand side), a steady-state equilibrium with large stocks of both species is optimal, very much in contrast to the open-access case where both species are fished to extinction. If the initial stock of species 2 is very low it is optimal to put more fishing effort on species 1 initially and only switch to more harvesting of species 2 when its stock has recovered. The comparison of blue curves in the graphs on the left hand side

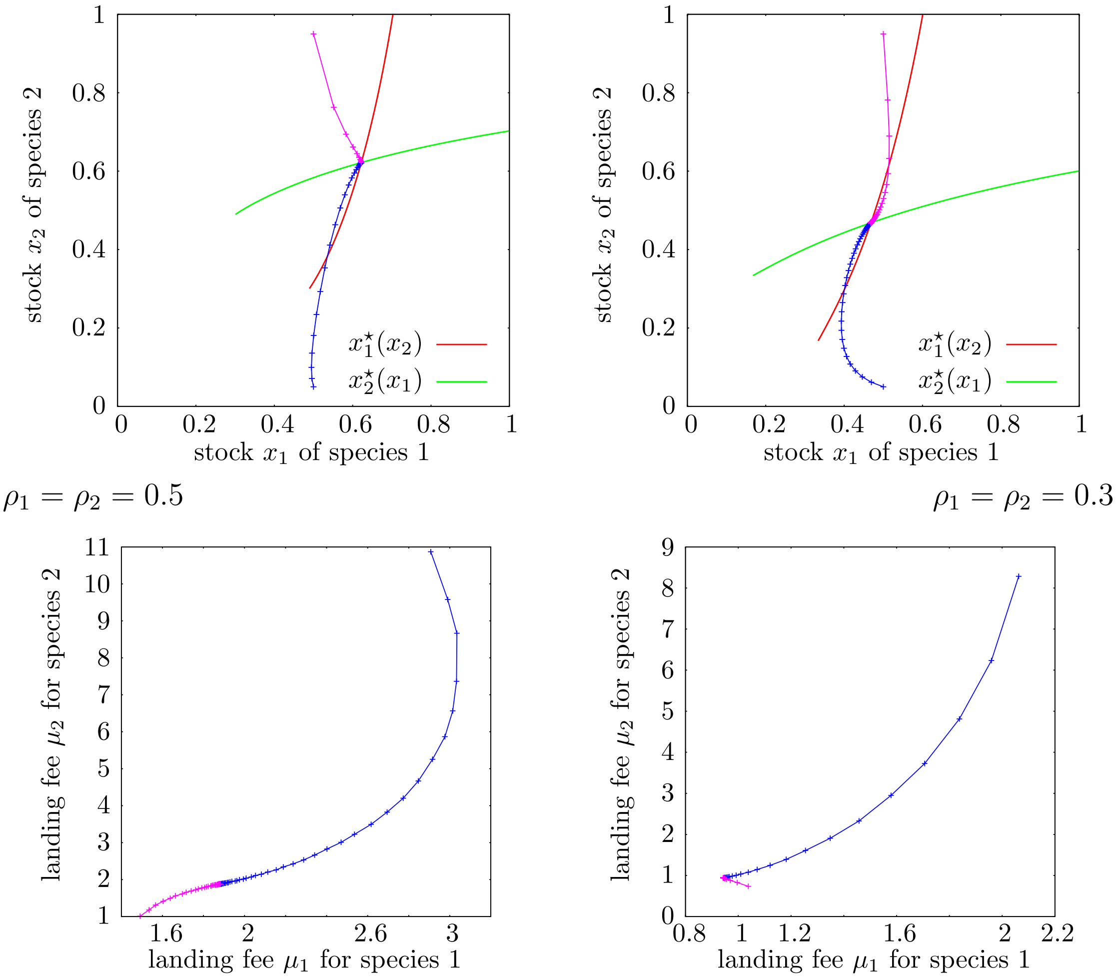


Figure 3: The phase diagrams for the case of substitutes ($\sigma = 2$) and two different intrinsic growth rates: $\rho_1 = \rho_2 = 0.5$ on the left hand side and $\rho_1 = \rho_2 = 0.25$ on the right hand side. The other parameters are $\alpha = 0.6$, $\Phi = 1$, $\delta = 0.2$ and $\kappa_1 = \kappa_2 = 1$. The initial values are $(x_1, x_2) = (0.05, 0.5)$ and $(x_1, x_2) = (0.95, 0.5)$ in both pictures.

of Figures 1 and 3 show the difference in harvesting in the open access and the optimum. In the open access fishery, much more is harvested of the larger stock. Thus, the stock of species 1 declines strongly in the beginning. In the optimum, the species with the larger *adjusted* stock is harvested to a greater extent. This is not necessarily the species with the larger stock: The stock of species 1 declines less in the beginning than the stock of species 2: The reason is that the growth rate of species 1, i.e. the adjustment factor, is much higher.

If the stock of one species is very low initially, the initial optimal landing fee for this species is very high (note that the scales on the x and y axes are different). But also the landing fee on species 1 is high initially, although the initial stock of this species is not much below the steady state value. The reason is that the stock of species 2 is low. Because both species are substitutes, there will be more over-fishing of species 1 than if the stock of species 2 was high.

Figure 4 shows the optimal paths for the case of complements. For the optimal dynamics it is sufficient to show two graphs, as for all the different growth rates of Figure 2 just one steady state exists.[1] The main difference to the case of substitutes is *how* the steady state is approached. Considering the case of a small initial stock of species 2 (i.e. the blue curves) again, in the case of substitutes the stock of species 1 initially declines while in the case of complements it initially increases. The reason is that in the case of complements both species are harvested more equally. Thus, the species with the higher stock, which also exhibits the higher natural growth rate (cf. the equations of motion (1)), increases more strongly.

Since in the case of complements the species with the lower stock is much more under fishing pressure than the species with the larger stock, the optimal landing fee on the species with the small stock is very high, while the landing fee on the species with the large stock is very low (the blue curves in the graphs at the bottom of Figure 4). The landing fee on the species with the larger stock may even decrease in the beginning, because by strong natural reproduction the stock increases, i.e. over-fishing becomes less of an issue.

5 Conclusion

In this paper we have studied the effects of consumers' preferences for diversity of food fish on (i) the outcome of an open access fishery, (ii) the Pareto-optimal dynamic fishing strategy and (iii) policy implications.

We have shown that the better substitutes the species are, the more sustainable is the open access fishery: more will be harvested of the species with the larger stock, but endangered species with a smaller stock will be fished to a lesser extent. The reason is that it is more costly to catch the fish with the lower stock and that this additional effort pays off less the better substitutes the species are. Analyzing the fishing dynamics we have shown that strong preferences for diversity, i.e. when

[1] When the rate of time preference δ is substantially smaller than the growth rates, this may be different. In this paper, we refrain from studying this this case in more detail.

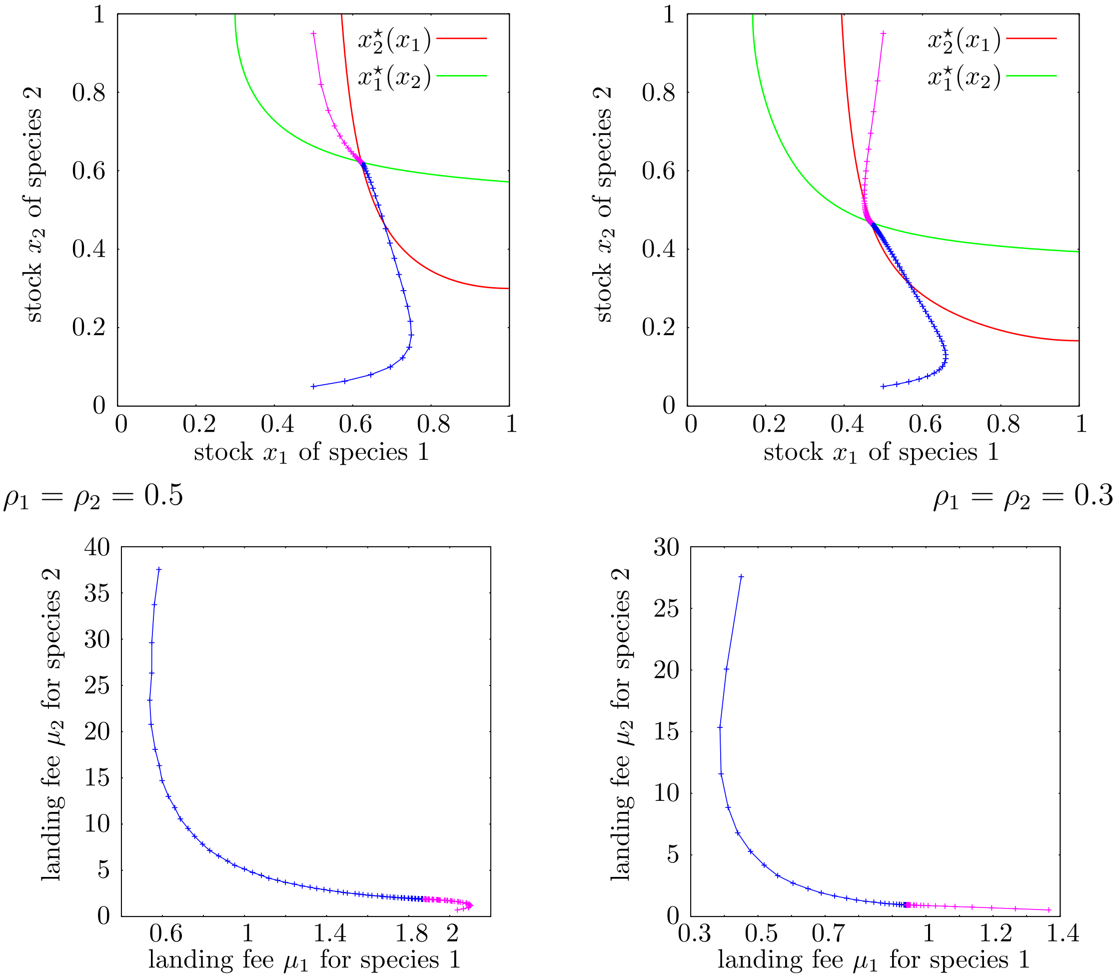


Figure 4: The phase diagrams for the case of complements ($\sigma = 0.5$) and two different intrinsic growth rates: $\rho_1 = \rho_2 = 0.5$ on the left hand side and $\rho_1 = \rho_2 = 0.25$ on the right hand side. The other parameters are $\alpha = 0.6$, $\Phi = 1$, $\delta = 0.2$ and $\kappa_1 = \kappa_2 = 1$. The initial values are $(x_1, x_2) = (0.05, 0.5)$ and $(x_1, x_2) = (0.95, 0.5)$ in both pictures.

the different species are complements lead to extinction of one species for large ranges of ecological parameters for which an open access fishery is sustainable if the different species are substitutes. In other words these results imply that stronger preferences for diversity lead to a less sustainable outcome in the open access regime. In this sense, this is the 'wrong' type of preferences for diversity from the conservationist's point of view.

In contrast to the open access regime, however, in the Pareto optimum it may well happen that the species with the smaller stock is fished less when the elasticity of substitution decreases. This is the case if the growth rate of the species with the smaller stock exhibits the larger growth rate. Also, under reasonable assumptions on the parameters it is optimal to approach a steady-state with positive stocks of all species, i.e. it is not optimal to fish any species to extinction.

Although we did not consider any biological interactions between the different species of fish our analysis has shown that the optimal dynamics are relatively complex: The optimal landing fees for the different stocks of fish are interdependent. Our analysis has shown that this interaction results in the need to levy substantial landing fees also on fish species with a stock that is still relatively high if this species is a substitute for an endangered species. If the different species are complements, the fees should be targeted in a more direct way to the endangered species.

References

Clark, C. W. (1991). *Mathematical Bioeconomics*. Wiley, New York, 2nd edition.

Dixit, A. K. and J. E. Stiglitz (1977). Monopolistic Competition and Optimum Product Diversity. *American Economic Review*, **67**(3): 297–308.

Lande, R., S. Engen, and B. Saeter (2003). *Stochastic Population Dynamics in Ecology and Conservation*. Oxford University Press, Oxford.

Markussen, M., R. Buse, H. Garrelts, M. A. M. Costa, S. Menzel, and R. Marggraf (eds.) (2005). *Valuation and Conservation of Biodiversity: Interdisciplinary Perspectives on the Convention on Biological Diversity*. Springer.

Millennium Ecosystem Assessment (2005). *Ecosystems and Human Well-Being: Synthesis Report*. Island Press, Washington DC.

Ninan, K. N. (2007). *The Economics of Biodiversity Conservation: Valuation in Tropical Forest Ecosystems*. Earthscan Publications.

Pauly, D., V. Christensen, J. Dalsgaard, R. Froese, and F. Torres (1998). Fishing Down Marine Food Webs. *Science*, **279**(5352): 860–863.

Quinn, T. and R. Deriso (1999). *Quantitative fishery dynamics*. Oxford University Press, Oxford.