Optimum Population and Long-run Conservation of Natural Capital Stock

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Abstract

This paper aims at showing how the conservation of a natural capital stock (NCS) differs in the long run, depending on the objective of a local community or economy that owns and utilizes the NCS, by controlling the size of population. We suppose that the economy has the objective to maximize the individual well-being or the total well-being. We show that the NCS will be higher in the long-run under maximizing individual well-being than the total well-being. We also compare them with the states under profit maximization and open access. It is demonstrated that the highest NCS will be attained under maximizing individual well-being, followed by that under profit maximization, which is also higher than the other two states. We also give the necessary and sufficient condition to ensure that the state under the utilitarianism is lower than that under open access.

Key words: Natural Capital Stock, Population, Welfare Criterion, Long-run Equilibrium, Conservation

1 Introduction

Population continues to grow rapidly, and it is expected that the number will reach nine billion in a half century. This might bring a severe pressure on the environment and the well-being of people through the excessive exploitation of natural capital stocks (NCSs). For example, Ehrlich and Holdren (1971) examine the environmental problem in terms of population growth. Dasgupta (2000, 2003) also stresses the importance of some findings in the literature that the population growth as well as the magnitude of poverty have a relation with the degradation of NCSs [1]. In this sense, size of population is a key to determine how the economy conserves NCSs in the long-run.

The purpose of this paper is to see how the size of population in an economy affects the long-run conservation of NCSs that are exploited by the economy. In the analysis, we suppose that the size of population is determined to maximize social welfare. That is, "optimum population" is adopted. This sort of analysis in a standard economy without the NCS is studied by, for example, Dasgupta (1969). Dasgupta supposes a community in which the size of population is determined based on the utilitarian social welfare function, by assuming that the economy equally divides the total production among all the citizens, who are identical. The economy then controls the number of population to maximize the utilitarian welfare function. In this paper, we also determine the size of population in terms of optimality and adopt the utilitarian social welfare function. This might be justified because the total well-being reflects what the economy might focus on when it cares people's welfare as a whole. In addition, we also apply another

[1] He argues that what he calls the "population-poverty-resource nexus" should be the focus of necessity and value in economics.

type of objective or social welfare criterion, which focuses on the well-being of individual in the economy. This is also justified because individual well-being might be the most fundamental for the authority to pay an attention [2].

Depending on these welfare criteria, two long-run optima are obtained [3]. We compare these equilibria with the states under profit maximization and open access. Here profit maximization and open access are linked with the size of population by supposing that NCS is located in a place without community and that people immigrates from outside. First of all, the number of immigrants are determined by a firm who maximizes profit, which is the case of profit maximization. On the other hand, we also suppose the case that there is no one to exclude immigrants. Thus, the number of immigrants are determined for the rent from exploiting NCS to diminish, which is the case of open access [4].

We show that the NCS will be higher under maximizing individual well-being than the total well-being. It is also demonstrated that the highest NCS will be attained under maximizing individual well-being, followed by that under profit maximization, which is also higher than the other two states. We also give the necessary and sufficient condition to ensure that the state under the utilitarianism is higher than that under open access. With a sufficiently small elasticity of marginal utility, utilitarian welfare criterion may lead the NCS to be smaller than open access situation.

[2]Since all the people are equally distributed in our model, this criterion is equivalent to the Rawlsian one.

[3]In the case with constant population and without the harvesting cost, the long run equilibria under some different objectives are sometimes compared. For example, Heal (1998) describes the equilibria under the (discounted) utilitarianism, maximin, and Chilniskyfs criteria. With the harvesting cost, note that the utilitarianism results in essentially the same equilibrium as (discounted) profit maximization.

[4]Brander and Taylor (1998) construct a dynamic model in which the harvest of a natural resource (forest) is determined by the size of population under open-access system. Based on the model, they derive a time path that explains the rise and fall of Easter Island.

Section 2 introduces the model in the paper. In section 3, long-run optima under the two kinds of social welfare function are investigated. Section 4 compares these states with the equilibria under profit maximization and open access. In section 5, some remarks are stated.

2 Model

We consider an economy which owns a renewable natural capital stock. The economy utilizes the natural capital stock, S, by harvesting it every year. The exploited resources are sold at a market price P. Let us assume that the growth of the capital stock is expressed by the function below:

$$G(S) = rS(1 - \frac{S}{K}), \tag{1}$$

where K expresses the carrying capacity. Let S_{msy} stand for the level of NCS at which the yield is maximized. Thus, $G' > 0$ for $S < S_{msy} \equiv K/2$ and $G' < 0$ for $S > S_{msy}$, with $G'(S_{msy}) = 0$.

The size of population is denoted by N. Each individual in the economy supplies a fixed amount of labor, which is expressed by $\bar{l}$. Thus, the total amount of labor, E, is given by $N\bar{l}$, which determines the harvest level, given S, as

$$H = H(E, S), \quad E = N\bar{l}. \tag{2}$$

Here $H(0, S) = H(E, 0) = 0, H_E(\equiv \partial H/\partial E) > 0, H_S(\equiv \partial H/\partial S) > 0 \quad \forall(E, S) > (0, 0)$. For simplicty, we assume $\bar{l} = 1$, so that $E = N$.

Given N, the steady state or equilibrium natural capital stock satisfies

$$G(S) = H(E, S) \tag{3}$$

The steady state is expressed by $S(E)$. We denote the production at the steady state $G(S(E))$

with $Y(E)$. For the analysis, we suppose

$$H(E,S) = hSf(E), h > 0. \tag{4}$$

where f is a continuously twice differentiable function with $f' > 0$.

Under (1) and (4), we obtain

$$S(E) = K(1 - \frac{h}{r}f(E)). \tag{5}$$

and

$$Y(E) = hKf(E)(1 - \frac{h}{r}f(E)) \tag{6}$$

Since $S(E) \geq 0$ and $Y(E) \geq 0$, we assume $E \leq \bar{E} \equiv f^{-1}(\frac{r}{h})$. From this, we have

$$S'(E) = -\frac{h^2}{r}f'(E) < 0, S''(E) = -\frac{h^2}{r}f''(E) \tag{7}$$

Moreover,

$$\frac{dY}{dE} = hKf'(E)(1 - \frac{2h}{r}f(E)) \tag{8}$$

Thus, S_{msy} occurs at $f(E) = r/2h$. We denote such E with E_{msy}. Note that $S(E) > S_{msy}$ for $E < E_{msy}$ and $S(E) < S_{msy}$ for $E > E_{msy}$. Therefore, Y is strictly increasing with Y up to E_{msy} and decreasing beyond the point.

In this paper we have an additional assumption on f as follows.

Assumption 1 *f is convex-concave. That is, there exists some $E_0 \in (0, \bar{E})$ such that $f'' > 0$ for $E < E_0$, $f''(E_0) = 0$, and $f''(E) < 0$ for $E > E_0$. Moreover, f'' is non-increasing. Furthermore, $f'(E)$ is small enough if E is small.*

Figure 1 depicts the shape of f.

[Insert Figure 1 here]

Therefore, given S, $H = hSf(E)$ takes an S-shaped. Under the assumption, Appendix A demonstrates that $Y(E)$ takes S-shaped in $[0, E_{msy}]$. That is, $Y(E)$ is convex-concave in the interval.

The economy harvests the yield from the NCS each year, which is sold at a market price P. The revenue from the sale is assumed to be distributed equally among all the members. Thus, the living standard of each member, C, is equivalent to PY/E. The individual well-being is expressed by $U(C, l)$, where $C = PY(E)/E$ and l is the labor supply. Here $U_C(\equiv \partial U/\partial C) > 0$ and $U_l(\equiv \partial U/\partial l) < 0$, but the labor supply is fixed at $\bar{l}$, so C can be a proxy for the well-being in this paper.

3 Welfare criteria and the natural capital stock conservation

In this section, we examine how the objective the economy has affects the natural capital stock management in the long run. The objective is defined as a social welfare function that represents the norm in the economy. The economy controls the size of population in the long run to maximize its social welfare function. We assume there are two types of objective; first of all, the economy is interested in the individual well-being and tries to maximize it in the long run.

3.1 Maximizing the individual well-being

The individual well-being is expressed by $U(C, \bar{l})$, so the objective is equivalent to maximizing $PY(E)/E$.

By lemma 1 in Appendix A, $Y(E)$ is convex-concave and there exists a maximum at some

$E > 0$. Thus, to maximize this leads to, at the optimum E^e,

$$Y'(E^e) = \frac{Y(E^e)}{E^e} \tag{9}$$

That is, the average productivity of labor must be equal to the marginal productivity of it. So, it must be $Y' > 0$. That is, it holds $E^e < E_{msy}$ so that we obtain

$$S(E^e) > S_{msy}. \tag{10}$$

Note that this property holds independent of the level of P, so it is unchanged even if the price of the goods harvested changes. From this result, we may say that the objective of maximizing the individual well-being will well conserve the natural capital stock, in the sense that the NCS is managed at a level greater than S_{msy} in the long-run.

3.2 Maximizing the total well-being

Next, we deal with the objective that maximizes the sum of utility of all people. This objective is very common in economics and, in a population growing economy, Dasgupta (1969) analyzed under the objective. To formalize it, the social welfare function $W = EU(PY(E)/E, \bar{l})$ is maximized [5] . The maximazation leads to

$$U = P(\frac{Y(E^u)}{E^u} - Y')U_C \tag{11}$$

which is referred to as the Meade Rule by Dasgupta. The second order condition is assumed to be satisfied. [6] Let us assume that the utility function is not always negative, i.e., $U(C, l) > 0$ is

[5] Dasgupta (1969) supposes an intertemporal social welfare function as $\int_0^\infty L(t)U(Y(t)/L(t))e^{-\rho t}dt$, where $L(t)$ is the size of population at time t.

[6] The second order condition is:

$$W'' = \frac{PU_C}{E}(Y' - \frac{Y(E)}{E}) - \frac{P^2U_{CC}}{E}(Y' - \frac{Y(E)}{E})^2 + \frac{PU_C}{E}(Y'' - (Y' - \frac{Y(E)}{E}))$$

possible. Note that without this assumption, the optimum population must be zero; we avoid this optimum by assuming that the utility function can be positive. Under the assumption, it is obvious that $E^e < E^u$ by comparing (11) with (9), since (11) requires in this case

$$Y'(E^u) < \frac{Y(E^u)}{E^u}. \tag{12}$$

Let us compare (12) with (9). Since $Y(E)$ is convex-concave in $[0, E_{msy}]$ (see Appendix A), $Y' > Y/E$ if $E < E^e$. Therefore, the objective of maximizing the individual well-being must generate a smaller population than under the utilitarianism, so that we obtain

$$S^e > S^u. \tag{13}$$

To see the level of S^u more closely, we specify the utility function as

$$U(C, l) = C^\theta + v(l), 0 < \theta < 1, v(l) > 0, v' < 0. \tag{14}$$

This is a sort of utility function in which the elasticity of marginal utility of consumption is constant at $1 - \theta$ [7]. Under (14), (11) shows that E^u is determined to satisfy

$$\theta = \frac{Y(E^u)/E^u}{Y(E^u)/E^u - Y'(E^u)} = \frac{1}{1 + \delta(E^u)} \tag{15}$$

where $\delta(E) = -Y'(E)E/Y$. Since $\theta < 1$, it must hold $Y'(E^u) < 0$. That is, $E^u > E_{msy}$ so that we obtain

$$S(E^u) < S_{msy}. \tag{16}$$

This property also holds independent of the level of P. Thus, we have shown $S^e > S_{msy} > S^u$ in our model. The results obtained in this section are summarized in the following proposition.

$$= P(\frac{PU_{CC}}{E}(Y' - \frac{Y(E)}{E})^2 + \frac{U_C}{E}Y'')$$

$W'' < 0$ is satisfied if it holds $Y'' < 0$, which is supposed in assumption 3.

[7]The case with $\theta < 0$ needs that $U(C, l) < 0$ if we require U is increasing with C. But this is excluded by our supposition that $U(C, l)$ can be positive.

Proposition 1 *Under assumption 1, it holds $S^e > S_{msy}$ and $S^e > S^u$. In addition, if we suppose (14), then we have $S^u < S_{msy}$.*

4 Comparison with the states under profit maximization and open access

This section compares the states S^e and S^u with those under profit maximization and open access. These states are denoted by S^* and S_A respectively.

Profit maximization Profit maximization is formalized in this paper as that the owner of the NCS chooses the population size to maximize

$$PY(E) - wE \tag{17}$$

Here w is the opportunity cost of workers for harvesting the yield in the economy, which is the wage rate prevailing outside of the local economy. The optimal E^* satisfies

$$PY'(E^*) = w \tag{18}$$

That means that $Y'(E^*)$ must be positive. Thus, it is obvious that $E^* < E_{msy}$ so that it holds that $S^* > S_{msy}$, which is a common knowledge in the resource economics. Now we know $S^* > S_{msy} > S^u$, but how about the relationship with S^e? Let us compare S^e with S^* under the following assumption.

Assumption 2 *Profit under E^* is positive. That is, $PY(E^*) - wE^* > 0$.*

In Appendix B, provided that the profit maximized is positive, we demonstrate $E^* > E^e$ so that

$$S^* < S^e \tag{19}$$

That is, maximizing the individual well-being better conserves the NCS than standard profit maximization.

From this, we conclude

$$S^e > S^* > S_{msy} > S^u \tag{20}$$

Open access Now we consider the case in which the economy has no control over its population and immigration from the outside of the economy takes place as long as the exploitation of the NCS generates profit. This is characterized as the open access equilibrium, which is denoted by E_A and S_A. Provided that the NCS is of open access, the economy cannot control the population and the equilibrium with zero profit will finally be attained. We compare the equilibrium (E_A, S_A) with (E^u, S^u). Since Y is convex-concave in $(0, E_{msy})$ by lemma 1, there can be two open access equilibria, one of which is unstable. We refer to only the stable equilibrium here.

Under open access equilibrium, it holds

$$PY(E_A) - wE_A = 0 \tag{21}$$

It is obvious that $E_A > E^*$ so that $S_A < S^*$. So we examine the relationship of S_A with S^u. Although we know from (25) that $Y'' < 0$ for $S \in (E_{msy}, E_{msy} + e)$ for some $e > 0$, but it is not ensured $Y'' < 0$ in $(E_{msy}, \bar{E})$, which we assume the following property.

Assumption 3 *$Y'' < 0$ in $(E_{msy}, \bar{E})$.*

The shape of $Y(E)$ is drawn in Figure 2, in which $Y'' < 0$ in $(E_{msy}, \bar{E})$ is due to the above assumption.

[Insert Figure 2 here]

In Appendix C with this assumption, we derive the following inequality as the necessary and sufficient condition for $S_A > S^u$ as:

$$-Y'(E_A) < \frac{w}{P}(\frac{1}{\theta} - 1) \tag{22}$$

These results are stated in the next proposition.

Proposition 2 *Under assumptions 1 and 2, it holds $S^e > S^* > S_A$. Moreover, under (14) and assumption 3, it holds that* sgn $(S_A - S^u) =$ sgn $(\frac{w}{P}(\frac{1}{\theta} - 1) + Y'(E_A))$.

(22) does not hold if θ is close to 1. Therefore, $S_A > S^u$ holds if and only if θ is sufficiently small, when it holds

$$S^e > S^* > S_A > S^u \tag{23}$$

Otherwise, we have

$$S^e > S^* > S_{msy} > S^u \geq S_A \tag{24}$$

where $S^u > S_A$ holds with a sufficiently large θ. Our result on the levels of S^e, S^*, S_A and S^u are depicted in Figure 3.

[Insert Figure 3 here]

5 Concluding Remarks

In this paper, we investigate how distinct welfare criteria affect the conservation of natural capital stock when population is controlled in the long run. The objectives to maximize the individual well-being and the total well-being are examined. We also compare them with the states under profit maximization and open access.

We show that the criterion of maximizing individual well-being conserves a higher NCS in the long run than the utilitarianism and the other two states. On the other hand, we demonstrate that the utilitarian criterion may require the long-run equilibrium to be even smaller than that under open access, provided that some conditions are met under (14).

It is possible that a economy has its own specific objective when utilizing the NCS. Therefore, the conservation of the NCS may drastically vary, depending on the objective it has. This paper shows such an aspect, using the two criteria. Even if the well-being of member of the economy is focused, the result is shown to be very different depending on whether the focus is on the total well-being of all the people or on the well-being of each and every person in the economy.

This paper stresses the impact of controlling population on the conservation of the NCS, so we assume that desirable population can be attained instantaneously. Since this is an strong assumption, it will be an interesting further study to develop our model based on dynamics of population, which is, for example, introduced by Brander and Taylor (1998).

Appendix

A Convex-Concavity of the function $Y(E)$

From the definition of $Y(E)$, we have

$$\frac{d^2Y}{dE^2} = hKf''(E)(1 - \frac{2h}{r}f(E)) - \frac{2h^2K}{r}(f'(E))^2 \tag{25}$$

Under assumption 1, $\frac{d^2Y}{dE^2} > 0$, for any small enough E. Since $\frac{d^2Y}{dE^2}$ is continuous and it is negative at E_{msy}, $\frac{d^2Y}{dE^2} = 0$ at $E < E_{msy}$, with $f''(E) > 0$. We denote such E by E_v. Note also E_v is unique at least in $(0, E_{msy})$, because f'' is non-increasing and $(1 - \frac{2h}{r} f(E))$ is positive. Thus, it holds $d^2Y/dE^2 > 0$ up to some point E_v and $d^2Y/dE^2 < 0$ in (E_v, E_{msy}). That is,

Lemma 1 *Under the assumption 1, the steady-state yield* $Y(E)$ *is convex-concave in* $[0, E_{msy}]$.

B Profit Maximization

Let us compare S^e and S^u with S^*. Now we prove $E^* > E^e$. Suppose that $E^* \leq E^e$. Note that, for $E \leq E^e$, it must hold $Y'(E) \geq \frac{Y(E)}{E}$ from (9). Then

$$\frac{Y(E^*)}{E^*} \leq Y'(E^*) = \frac{w}{P} \tag{26}$$

However, this implies that

$$PY(E^*) - wE^* \leq 0 \tag{27}$$

which contradicts the assumption 2 that the profit is positive at the optimum. Thus, we obtain $E^* > E^e$ so that $S^* < S^e$.

C Open Access

Let us compare S^u with S_A, i.e., open access equilibrium natural capital stock. Note from (11) that $\frac{Y(E^u)}{E^u} - Y' > 0$. Let us differentiate $\frac{Y(E)/E}{Y(E)/E - Y'}$ with E. Under the assumption 3, we obtain, if $E > E_{msy}$,

$$\left(\frac{Y(E)/E}{Y(E)/E - Y'}\right)' = \frac{-E^{-1}(Y' - (Y/E))^2 + E^{-2}Y(Y' - (Y/E)) + Y''E^{-1}Y}{((Y/E) - Y')^2} < 0 \tag{28}$$

Thus, we have the following lemma.

Lemma 2 $\frac{Y(E)/E}{Y(E)/E - Y'}$ *is decreasing with* E.

Due to this lemma and (15), if $\theta > \frac{Y(E)/E}{Y(E)/E - Y'}$ for a given E, then $E > E^u$. Since $PY(E_A) = wE_A$ at the open access equilibrium, this inequality leads to that open access equilibrium S_A is strictly smaller than S^u if and only if $-Y'(E_A)w^{-1}P < (\frac{1}{\theta} - 1)$. This means that given w and P which determine E_A, there always exists $\theta^* < 1$ such that $E_A = E^u$ and $E_A < E^u$ for $\theta < \theta^*$.

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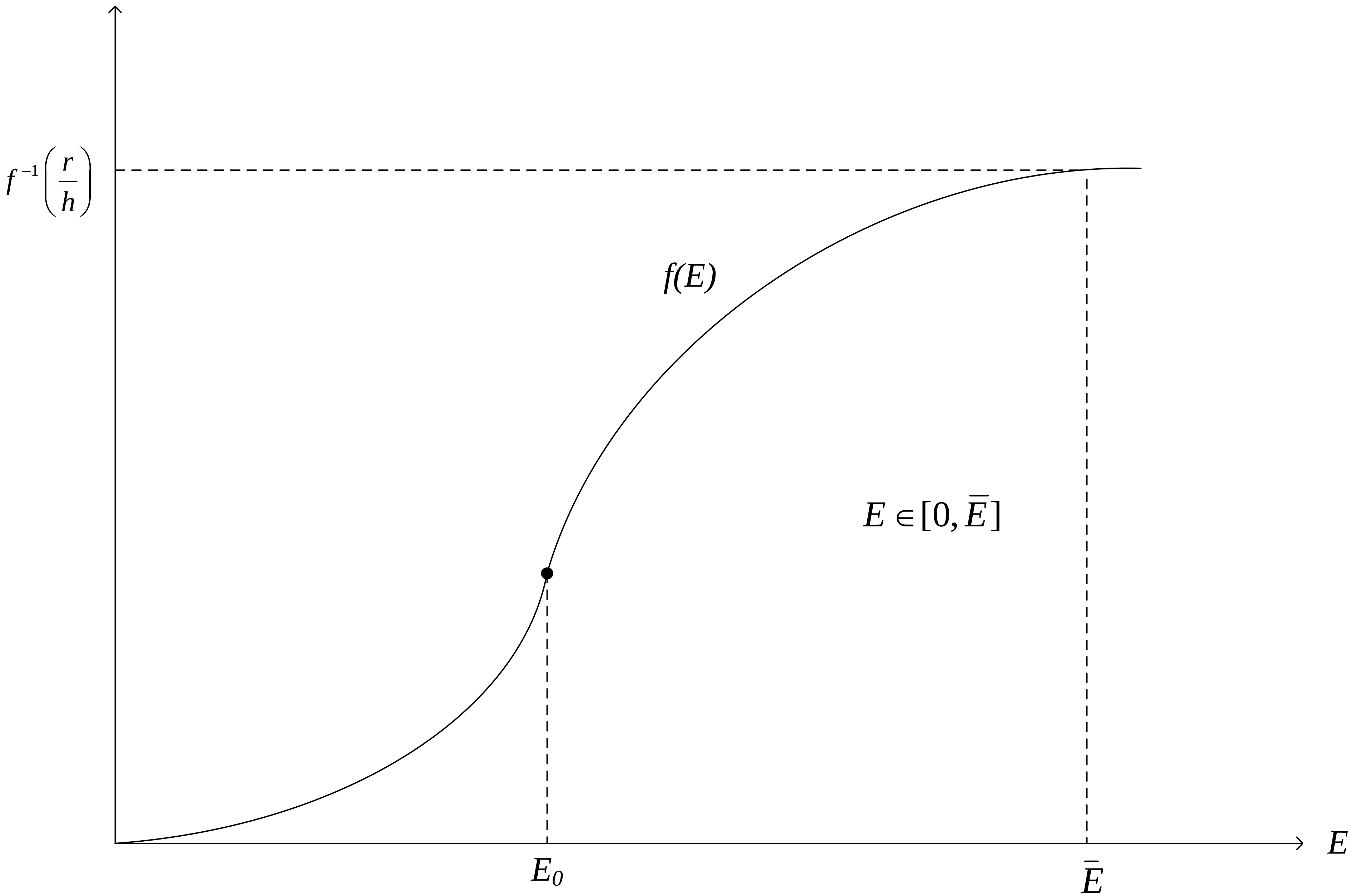

Fig. 1 The shape of *f(E)*

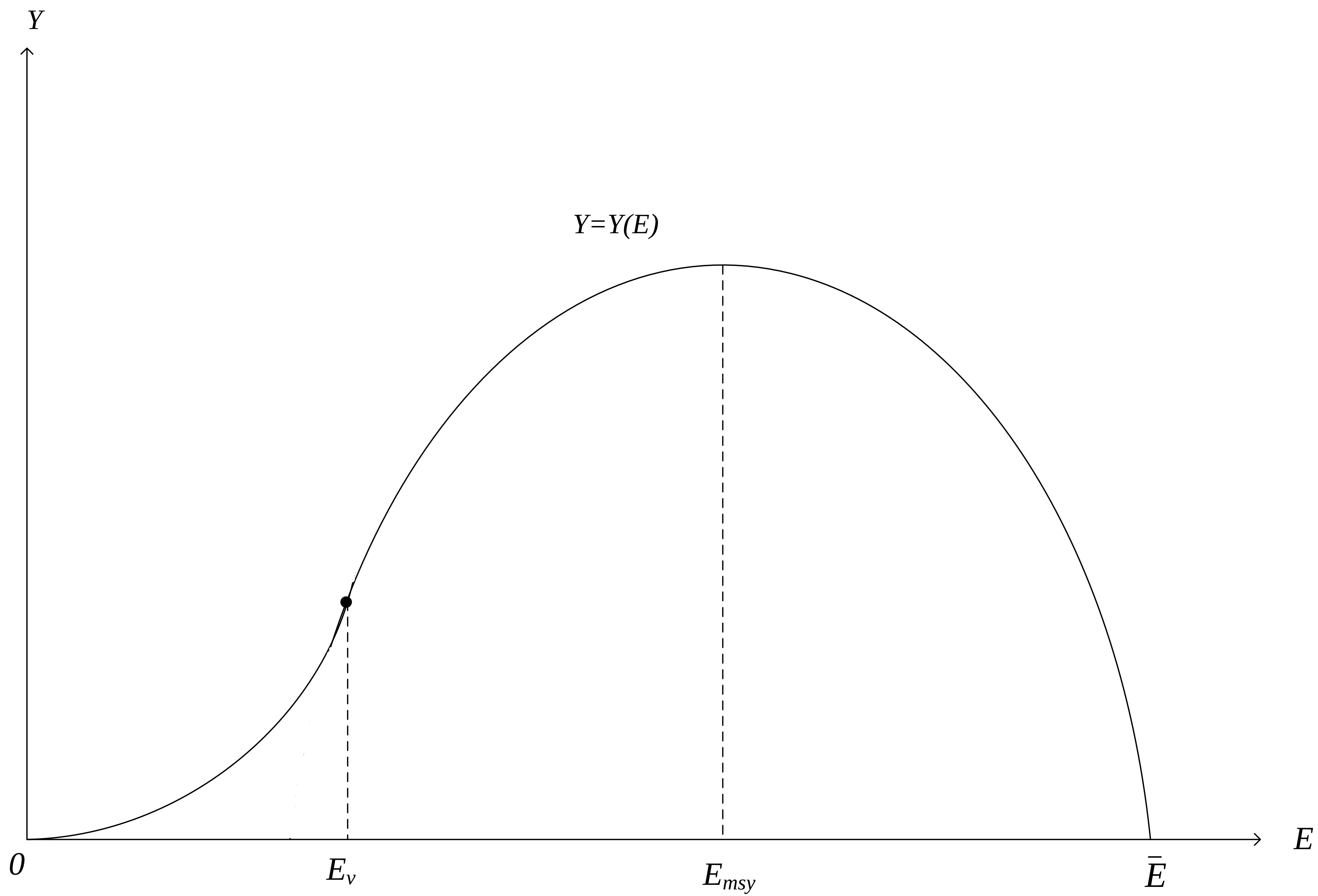

Fig. 2 *Y(E)* under assumptions 1 and 3.

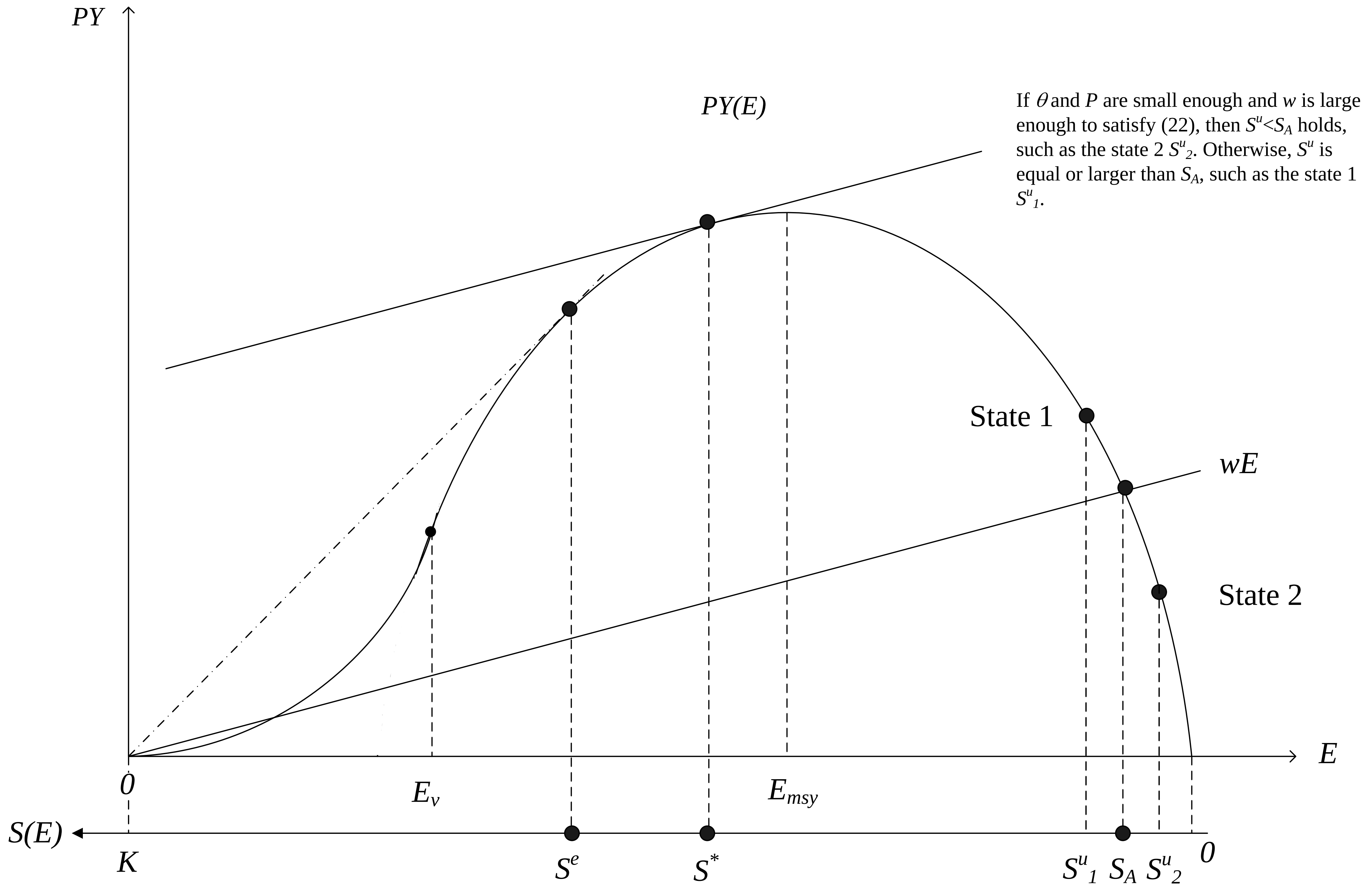

Fig. 3 Graphical expression of the determination of 4 equilibria.