

The maximum sustainable yield management of an age-structured salmon population: Fishing vs. conservation

Anders Skonhoft

Department of Economics

Norwegian University of Science and Technology

(Anders.skonhoft@svt.ntnu.no)

Abstract

This paper develops a sustainable yield harvesting model for the wild Atlantic salmon (*Salmo salar*), where the population comprises different age classes. It is shown that the weight–fecundity relationship of the spawning population, comprising young and old fish, is crucial for the maximum sustainable yield fishing composition. First, the optimal harvest is found when fecundity is approximated by weight. Second, we analyze what happens when weight is an inaccurate approximation for fertility. Finally, the conservation perspective is taken into account when the maximum sustainable yield is maximized subject to a minimum viable size of the harvestable, or spawning, population. It is shown that the conservation perspective does not affect the qualitative structure of the optimal harvest policy in which the old mature fish are more aggressively targeted than the young mature population.

Key words: Salmon fishery, age classes, maximum sustainable yield

JEL: Q22; Q57; C61

.....

Thanks to Yajie Liu for assistance with preparation of this paper.

1. Introduction

For many years, the North Atlantic salmon (*Salmo salar*) has been one of the most important fish species in Norway because of its social, cultural, and economic importance. It was traditionally harvested for food, but today is most important to recreational anglers.

Norwegian rivers are the most important spawning rivers for the East Atlantic stock, and about 30% of the remaining stock spawns there. The wild salmon are harvested by commercial and recreational fisheries. The marine harvest is commercial and semi-commercial, whereas the harvest in the spawning rivers is recreational (NOU 1999). The amount harvested in marine and river fisheries has been more or less similar over the last few years. However, the value of river fishery is much higher, because of the higher willingness to pay for sport fishing (NOU 1999, Olaussen and Skonhøft 2008).

However, the abundance of wild salmon stocks has been declining during the last few decades. Stock development has been especially disappointing since the 1990s because of a combination of various factors, such as sea temperature, diseases, and human activity, both in the spawning streams and through the strong growth of salmon sea farming (NASCO 2004). As the wild stock began to decrease during the 1980s, the Norwegian government imposed gear restrictions to limit the marine harvest. Drift net fishing was banned in 1989, and the fishing season of bend net fishing, taking place in the fjords and close to the spawning rivers, has been restricted several times. At the same time, the sport fishing season in the spawning rivers has been subject to various restrictions (NOU 1999). However, despite all these measures taken to secure and rebuild the stock, the abundance of wild salmon seems to be at only half the level experienced in the 1960s and 1970s. It is believed that the rapid expansion of the farmed salmon industry has played the most important role in this decline and today, farmed salmon is regarded as the main threat to the viability of the wild salmon population because of the spread of diseases, escapees, and environmental pollution (Hindar et al. 2006).

Wild salmon fishing has been studied in many papers from an economic perspective. Routledge (2001) studied a mixed stock versus single stock fishery related to Pacific salmon while Laukkanen (2001) analyzed the northern Baltic salmon fishery in a sequential fishing biomass model. Olaussen and Skonhøft (2008) also analyzed a sequential harvesting biomass model, but with recreational fishery in the rivers as its focus. The economics of the Baltic salmon fishery is studied in an age-structured model in Kulmala et al. (2008). This is a dynamic model, comprising migration and seasonal harvest and competing harvesting by

commercial and recreational fishermen. Uncertainty is also included, and the model is parameterized and solved numerically for a Finnish river stock. In the following few pages, an age-structured wild salmon bioeconomic salmon model is analyzed as well, but within a much simpler framework than that of Kulmala et al. The goal is to study, from a theoretical point of view, how the harvesting of different age classes influences recruitment and stock abundance, and the main focus is to find the harvest composition that maximizes the yield in biological equilibrium.. Neither possible interactions from the farmed salmon sector nor the problem of nonselectivity in the harvest is considered. However, the conservation perspective is taken into account as the maximum sustainable yield policy given a minimum viable spawning population is studied.

Age-structured models are far more complex than biomass models. On the one hand, it is relatively straightforward to formulate a reasonably good age-structured model and numerically simulate the effects of variations in fishing mortality between age classes and over time (e.g., Kulmala et al. 2008). On the other hand, it is notoriously difficult to understand the various biological and economic forces at work in such models. Tahvonen (2008, 2009) has recently published papers dealing with some of these issues, in which he finds some results in a dynamic setting, but under quite restrictive assumptions. Early contributions analyzing age-structured models include Reed (1980), who studied the maximum sustainable yield problem. He found that optimal harvesting comprises, at most, two age classes. Further, if two age classes are harvested, the elder is harvested completely. Getz and Haight (1988) reviewed various age-structured models, and formulated the solution for the maximum sustainable yield problem as well as the maximum yield problem over a finite planning horizon. The following analysis has similarities with Reed (1980) and Getz and Haight (1988), but we study a different biological system in which all the spawning fish, i.e., salmon, die after spawning. This contrasts with Reed's model, where the spawning fish (e.g., cod) survive and enter an older year class after spawning. As will be seen, this difference has important implications for the optimal harvesting policy. While our analysis is directly related to Atlantic salmon, we will find that it fits various Pacific salmon species, such as pink and chum salmon, which also die after spawning (see, e.g., Groot and Margolis 1991).

The paper is organized as follows. In the next section, the population model is formulated. The model is somewhat stylized ('generic') as we consider only two harvestable, and hence

two spawning, age classes. In section three, we find the maximum sustainable yield fishing policy under different assumptions, whereas in section four, we take the fishing value into account. The question of conservation is analyzed in section five and the theoretical reasoning is numerically illustrated in section six. Section seven summarizes and concludes the paper.

2. Population model

Atlantic salmon is an anadromous species that has a complex life cycle with several distinct phases. Freshwater habitat is essential in the early development stages, as this is where it spends the first one to four years from spawning to juvenile rearing before undergoing smoltification and seaward migration. Then, it stays for one to three years in the ocean for feeding and growing and, when mature, returns to the natal or ‘parent’ rivers to spawn. After spawning, most salmon die, as less than 10% of the female salmon spawn twice (Mills 1989). The Atlantic salmon is subject to fishing when it migrates back to its parent river. In Norway, most sea fishing takes place in fjords and inlets with wedge-shaped seine and bend nets. This fishing is commercial or semi commercial. In the rivers, salmon are caught by recreational anglers with rods and hand lines. As indicated, the recreational fishery is by far the most important from an economic point of view.

In what follows, a specific salmon population (with its native river) is considered in terms of a number of individuals at time t structured into recruits $N_{0,t}$ ($yr < 1$), three young age classes, $N_{1,t}$ ($1 \leq yr < 2$), $N_{2,t}$ ($2 \leq yr < 3$) and $N_{3,t}$ ($3 \leq yr < 4$), and two adult, spawning classes, $N_{4,t}$ ($4 \leq yr < 5$) and $N_{5,t}$ ($5 \leq yr \leq 6$). Recruitment is endogenous and density dependent, and the old spawning salmon has higher fertility than does the young spawning salmon (more details are provided below). Natural mortality is fixed and density independent and, as an approximation, it is assumed that the whole spawning population dies after spawning. It is further assumed that the proportion between the two mature age classes is fixed. This proportion may be influenced by a number of factors, such as the type of river (‘small’ salmon river vs. ‘large’ salmon river) and environmental factors (NOU 1999). As fishing takes place when the fish returns back to its native river (see also above), only the mature salmon $N_{4,t}$ and $N_{5,t}$ are subject to fishing. Figure 1 presents the model life cycle of a single cohort of the salmon population considered. A far more detailed description of the life cycle of the Atlantic salmon is found in, e.g., Verspoor et al. (2003).

Figure 1 about here

With B_t as the size of the spawning population, adjusted for different fertility among the two spawning classes (see below), the stock recruitment relationship is defined by:

$$(1) \quad N_{0,t} = R(B_t).$$

$R(B_t)$ may be a one-peaked value function (i.e., of the Ricker type) or it may be increasing and concave (i.e., of the Beverton–Holt type). In both cases, zero stock means zero recruitment, $R(0) = 0$. Next, the number of young, depending on natural mortality, is as follows:

$$(2) \quad N_{a+1,t+1} = s_a N_{a,t}; \quad a = 0, 1, 2,$$

where s_a is the age-specific natural survival rate, assumed to be density independent and fixed over time. Finally, we have the mature age classes that are subject to fishing mortality (marine as well as river fishing), in addition to natural mortality. With $0 < \sigma < 1$ as the proportion of the mature stock that returns to spawn in the first year, the number of spawning fish of this part of the adult population (young) is:

$$(3) \quad N_{4,t+1} = s_3 N_{3,t} \sigma (1 - f_{4,t}),$$

where $f_{4,t}$ yields the fishing mortality. Accordingly, $H_{4,t} = s_3 N_{3,t} \sigma f_{4,t}$ is the number of harvested young mature fish in year t . As indicated, the parameter σ may depend on various factors, but is considered as fixed and exogenous.

The rest of this cohort $s_3 N_{3,t} (1 - \sigma)$ stays one year more in the ocean. When subject to natural mortality, as well as subsequent fishing mortality, on migration back to the home river, the size of the next year's (old) spawning population becomes:

$$(4) \quad N_{5,t+2} = s_3 N_{3,t} (1 - \sigma) s_4 (1 - f_{5,t+1}).$$

Hence, $H_{5,t-1} = s_3 N_{3,t} (1 - \sigma) f_{5,t+1}$ is the number of harvested old mature salmon in the cohort year $t+1$. With γ_4 and γ_5 as the fecundity parameters of the young and old mature population, respectively, and where the old mature class is more productive than the young mature class, $\gamma_5 > \gamma_4$, the spawning population in year t may be written as

$$B_t = \gamma_4 N_{4,t} + \gamma_5 N_{5,t}, \text{ or}$$

$$(5) \quad B_t = \gamma_4 s_3 N_{3,t-1} \sigma (1 - f_{4,t-1}) + \gamma_5 s_3 N_{3,t-2} (1 - \sigma) s_4 (1 - f_{5,t-1}).$$

The fecundity parameters will be considered as dimensionless parameters; that is, when scaling the fertility of the young mature population to one, $\gamma_4 = 1$, the fertility of the old mature population, $\gamma_5 > 1$, simply indicates that B_t is measured as the fertility-weighted number of spawning salmon. Equation (2) implies that $N_{3,t+3} = s_0 s_1 s_2 N_{0,t}$, or

$$(6) \quad N_{3,t+3} = sR(B_t),$$

when also using equation (2) and where $s = s_0 s_1 s_2$ comprises the previous years' survival rates. Subsequently, $N_{3,t}$ will be referred to as the potentially harvestable population, or simply the harvestable population. For given fishing mortalities, equations (6) and (5) yield a system of two difference equations of degree five for the two variables $N_{3,t}$ and B_t .

As already indicated, we are concerned only with equilibrium fishing, or sustainable harvesting, in this paper. The population equilibrium for fixed fishing mortalities is defined for $N_{3,t} = N_3$ and $B_t = B$ for all t such that:

$$(5') \quad B = [\gamma_4 s_3 \sigma (1 - f_4) + \gamma_5 s_3 (1 - \sigma) s_4 (1 - f_5)] N_3,$$

and

$$(6') \quad N_3 = sR(B).$$

In what follows, (5') is referred to as the *spawning constraint*, whereas (6') represents the *recruitment constraint* (see also Skonhøft et al. 2010). An internal equilibrium ($N_3 > 0$ and $B > 0$) holds only if either f_4 or f_5 , or both, are below one; that is, to exclude depletion, both mature classes cannot be totally fished down. Notice that this is a necessary but not sufficient condition. Figure 2 illustrates the internal, unique equilibrium when the recruitment function is of the Beverton–Holt type, i.e., $R(0) = 0$, $\partial R / \partial B_t = R' > 0$ and $R'' < 0$ (see also section six). As the spawning constraint (5') is a linear function, whereas the recruitment constraint (6') is a strictly concave function with Beverton–Holt recruitment, an internal solution requires that the slope of the spawning constraint is less steep than that of the recruitment constraint for a zero stock level.

Figure 2 about here

In line with intuition, we find that higher fishing mortalities shift the spawning constraint up (5') and yield smaller equilibrium stocks. On the other hand, higher survival rates yield more fish as the spawning constraint (5') shifts down (both through s_3 and s_4), and the recruitment constraint (6') shifts up (through s). For a larger fraction of the young mature stock through a higher value of σ , we find more spawning fish as well as more harvestable fish if the mortality-corrected fertility parameter is higher for the young mature stock than for the old stock, i.e., $\gamma_4(1-f_4) > \gamma_5 s_4(1-f_5)$. For equal targeted stocks, $f_4 = f_5$, this simplifies to $\gamma_4 / s_4 > \gamma_5$, indicating that the 'biologically discounted' fertility of the young mature stock dominates the old mature stock fertility.

3. The maximum sustainable yield harvesting program

Initially, we analyze the optimal sustainable harvesting program without any conservation concerns. With w_5 and w_4 as the fixed weights (kg per fish) of the young and old mature population, respectively, where $w_5 > w_4$, the equilibrium biomass harvested (in kg) is defined by $Y = w_4 H_4 + w_5 H_5 = [w_4 s_3 \sigma f_4 + w_5 s_3 (1-\sigma) s_4 f_5] N_3$. Then, the maximum sustainable yield problem is described by finding the fishing mortalities that maximize Y subject to the spawning constraint (5') and the recruitment constraint (6'). The Lagrangian of this problem may be written as

$$L = [w_4 s_3 \sigma f_4 + w_5 s_3 (1-\sigma) s_4 f_5] N_3 - \lambda [N_3 - sR(B)] - \mu \{B - [\gamma_4 s_3 \sigma (1-f_4) + \gamma_5 s_3 (1-\sigma) s_4 (1-f_5)] N_3\},$$

where $\lambda > 0$ and $\mu > 0$ (both in kg per fish) are the shadow prices of the recruitment and spawning constraints, respectively. Following the Kuhn–Tucker theorem, the first-order necessary conditions (assuming that $N_3 > 0$ and $B > 0$) are:

$$(7) \quad \partial L / \partial f_4 = N_3 (w_4 - \mu \gamma_4) \begin{matrix} \geq \\ < \end{matrix} 0; \quad 0 \leq f_4 \leq 1,$$

$$(8) \quad \partial L / \partial f_5 = N_3 (w_5 - \mu \gamma_5) \begin{matrix} \geq \\ < \end{matrix} 0; \quad 0 \leq f_5 \leq 1,$$

$$(9) \quad \partial L / \partial N_3 =$$

$$w_4 s_3 \sigma f_4 + w_5 s_3 (1-\sigma) s_4 f_5 - \lambda + \mu [\gamma_4 s_3 \sigma (1-f_4) + \gamma_5 s_3 (1-\sigma) s_4 (1-f_5)] = 0,$$

and

$$(10) \quad \partial L / \partial B = \lambda s R'(B) - \mu = 0.$$

Control condition (7) indicates that the fishing mortality of the young mature population should take place at the point where the marginal biomass gain is equal, below or above its marginal biomass harvest loss, determined by the fecundity parameter and evaluated by the spawning constraint shadow price. Condition (8) is analogous for the old mature population. The stock condition (9) says that the harvestable population should be managed so that the recruitment constraint shadow price λ is equal to the total marginal harvest gain plus the total marginal spawning biomass gain, evaluated at its shadow price. Finally, stock condition (10) indicates that the recruitment growth, evaluated at its shadow price, should be equal to the spawning constraint shadow price μ .

From the control conditions (7) and (8), it is observed that only the weight–fecundity ratio w_i / γ_i ($i = 4, 5$) determines the fishing mortality and the fishing composition and, hence, no other factors play a *direct* role. This outcome differs from the seminal paper by Reed (1980), who found that weight together with natural mortality directly determined the fishing composition. As already indicated, the reason for this discrepancy is the different biological characteristics of the fish stocks, and the fact that the mature fish die after spawning in our salmon model, whereas the spawning fish survive and enter older age classes in the Reed model. The above solution is analyzed in two steps. First, we look at the situation where weight is considered as an approximation for fertility. Second, we analyze the optimal harvest option when the weight–fecundity ratio is highest for the old mature stock.

3.1 Proportional weight–fecundity relationship

Weight and fertility are related, and larger, heavier, and older fish, in most instances, if not always, indicate higher fertility (e.g., Getz and Haight 1989). When first assuming that weight may serve as an approximation for fertility, indicating a proportional relationship between weight and fertility, $w_5 / w_4 = \gamma_5 / \gamma_4$ or $w_4 / \gamma_4 = w_5 / \gamma_5$, both control conditions (7) and (8) must hold as equations. That is, $w_i - \mu \gamma_i = 0$ with $0 < f_i < 1$ ($i = 4, 5$). This is the only possibility, because $w_4 / \gamma_4 = w_5 / \gamma_5 > \mu$ and, therefore, $f_i = 1$ indicates extinction, whereas $w_4 / \gamma_4 = w_5 / \gamma_5 < \mu$ and $f_i = 0$ ($i = 4, 5$) imply a zero biomass yield. Given this weight–fecundity assumption, the spawning constraint shadow price μ^* is determined by condition (7) (or 8) as $\mu^* = w_4 / \gamma_4 = w_5 / \gamma_5$ (the superscript ‘*’ indicates optimal values). Next, from

equation (9), after some small rearrangements, we find the recruitment constraint shadow price from $\lambda^* = s_3[w_4\sigma + w_5(1-\sigma)s_4]$. Therefore, when inserting μ^* and λ^* into condition (10), the size of the optimal spawning biomass is described by

$$R'(B^*) = \mu^* / s\lambda^* = \frac{w_5 / \gamma_5}{s s_3 [w_4\sigma + w_5(1-\sigma)s_4]}, \text{ or } R'(B^*) = \frac{1}{s s_3 [\gamma_4\sigma + \gamma_5(1-\sigma)s_4]}.$$

When further plugging in the recruitment constraint (6'), the sustainable yield maximizing harvestable population is determined by $N_3^* = sR(B^*)$. Therefore, somewhat surprisingly, we find that the optimal spawning population as well as the size of the harvestable population are determined independently of the fishing mortalities. Further, it is seen that higher values of both the fecundity parameters reduce $R'(B^*)$ and, hence, yield a higher spawning as well as a harvestable population. In line with this intuition, higher survival rates yield similar effects, whereas generally the effects of a higher σ are ambiguous.

The task of finding the optimal fishing mortalities remains. Because, under the proportional weight–fecundity assumption, conditions (7) and (8) give the same information, there is one degree of freedom in the system of equations defining the maximum sustainable yield policy. Therefore, the remaining equation describing the optimal policy, the spawning constraint (5'), must be satisfied for all combinations of fishing mortalities that keep this equation in balance. When rewriting (5'), we then find that all $0 < f_i^* \leq 1$ ($i = 4, 5$) (but not $f_4^* = f_5^* = 1$) satisfying

$$(11) \quad f_5^* = 1 - \frac{(B^* / N_3^*)}{\gamma_5 s_3 (1-\sigma) s_4} + \frac{\gamma_4 \sigma}{\gamma_5 (1-\sigma) s_4} (1 - f_4^*)$$

are in accordance with the maximum sustainable yield fishing policy. This is stated as the following proposition.

Proposition 1. When fecundity is approximated by weight, fishing both adult subpopulations will represent the maximum sustainable yield harvesting policy. This optimal policy can be reached by an infinite number of combinations of fishing mortalities.

The above optimal fishing mortality frontier (11) describes f_5^* as a decreasing function of f_4^* .

Both $0 < f_4^* \leq f_5^* < 1$ and the opposite case can be in accordance with this maximum

sustainable harvesting policy. Differentiation of (11) yields $df_5^* = -\frac{\gamma_4 \sigma}{\gamma_5 (1-\sigma) s_4} df_4^*$,

indicating that a one percentage point increase of the young adult fishing mortality must be accompanied by a reduction of $\frac{\gamma_4 \sigma}{\gamma_5 (1 - \sigma) s_4}$, or $\frac{w_4 \sigma}{w_5 (1 - \sigma) s_4}$, of the old adult stock to sustain the optimal harvesting policy. This reduction may be above or below the one percentage point increase, but it will certainly be below it if the proportion σ of the stock that return to spawn in the first year is ‘small’. With similar fishing mortalities, $f_4^* = f_5^* = f^*$, we find that

$$f^* = 1 - \frac{B^* / N_3^*}{s_3 [\gamma_5 (1 - \sigma) s_4 + \gamma_4 \sigma]}.$$

Therefore, not surprisingly, a higher optimal

spawning–harvestable population ratio must be accompanied by a less aggressive similar fishing mortality among the two stocks.

3.2 Weight as an inaccurate approximation for fecundity

According to McGinnity et al. (2003), weight is a questionable fertility approximation for the wild Atlantic salmon. Instead, they assume fertility to be described by a strictly concave function of weight (and age), indicating that the weight–fertility ratio *increases* with weight. With $w_5 / \gamma_5 > w_4 / \gamma_4$ and, hence, a higher marginal gain–loss ratio for fishing the old spawning fish, the maximum sustainable yield harvesting policy, given by conditions (7) and (8), indicates a higher fishing mortality for the old than the young mature subpopulation. This is stated as the following proposition.

Proposition 2. With a higher weight–fecundity ratio for the old adult subpopulation, the maximum yield harvesting policy is governed by a higher fishing mortality of the old subpopulation.

There are three possible cases, all corner solutions, that may represent this optimal policy: i) $f_5^* = 1$ and $0 < f_4^* < 1$, ii) $f_5^* = 1$ and $f_4^* = 0$, and iii) $0 < f_5^* < 1$ and $f_4^* = 0$. The spawning constraint (5') will be steeper in case i) than in case ii), which again will be steeper than that in case iii). Therefore, when taking the recruitment constraint (6') into account (again, see Figure 2), we find that the size of the spawning population as well as the harvestable stock will be highest with harvest option iii) and lowest if case i) represents the optimal policy.

In case i), the spawning constraint shadow price is determined through condition (7) as $\mu^* = w_4 / \gamma_4$. In combination with equation (9), this yields $\lambda^* = s_3[w_4\sigma + w_5(1-\sigma)s_4]$. Therefore, as both shadow price values are similar to the above proportional weight–fecundity relationship in section 3.1, equation (10) $\lambda^* sR'(B) = \mu^*$ together with the recruitment constraint (6') indicates similar sizes of the optimal spawning biomass B^* , and the harvestable population N_3^* as well. If case ii) where $f_5^* = 1$ and $f_4^* = 0$ represents the maximum sustainable yield policy, the spawning constraint (5') simply reads as $B = \gamma_4 s_3 \sigma N_3$. In this case, we find that this equation together with the recruitment constraint (6') $N_3 = sR(B)$ alone determines N_3^* and B^* . Hence, the optimal stock sizes are not directly influenced now either by the fertility parameter of the old mature group or the fish weights. In case iii), where $0 < f_5^* < 1$ and $f_4^* = 0$, the spawning constraint shadow price is fixed as $\mu^* = w_5 / \gamma_5$, whereas the recruitment constraint shadow price becomes $\lambda^* = s_3 w_5 [\gamma_4 \sigma / \gamma_5 + (1-\sigma)s_4]$. Therefore, just as in case i), the size of the harvestable population and the spawning biomass are determined through condition (10) and the recruitment constraint (6'). The yield maximizing fishing mortality is again determined through the spawning constraint (5'), in this case as $B^* = s_3 [\gamma_4 \sigma + \gamma_5 (1-\sigma)s_4 (1-f_5)] N_3^*$.

If w_5 / γ_5 is substantially higher than w_4 / γ_4 , intuitively, we may suspect that it is beneficial for the manager to invest in the salmon population by leaving the young mature population unexploited and harvesting the whole old adult population. Hence, case ii), where $f_5^* = 1$ and $f_4^* = 0$, should maximize the sustainable yield. On the other hand, with a 'small' weight–fertility difference, either case i), with harvesting of both mature populations, or case iii), with harvesting of the old mature population only, may represent the optimal solution. Therefore, values of γ_4 and γ_5 that make the gap $w_5 / \gamma_5 > w_4 / \gamma_4$ larger may indicate that case ii) is more likely to take place. Moreover, for given fecundity parameters, we can find the same outcome when the gap widens through a higher weight discrepancy. However, as will be demonstrated in the numerical section, other parameter values, such as natural survival rates, will play a role as well.

4. The maximum economic yield policy

As indicated, the Atlantic salmon is targeted both by commercial fishermen and sport fishers, with the sport fishing taking place in the rivers with rods, whereas the commercial or semicommercial fishery takes place in the fjords and inlets, with wedge-shaped seine and bend nets. When p_i ($i = 4, 5$) are the fishing values (NOK per kg) and when the value is higher in the sport fishery than in the commercial fishery, but $p_5 \geq p_4$ in both fisheries (Olaussen and Liu 2010), then $\pi = [p_4 w_4 s_3 \sigma f_4 + p_5 w_5 s_3 (1 - \sigma) s_4 f_5] N_3$ describes the yearly gross money value in our salmon fishery. Because, typically, stock dependent variable costs in both fisheries are small and negligible (for a description of these fisheries, see, e.g., NOU 1999), the gross money value may approximate gross profit when considering the fish prices as ‘net’ prices. See Olaussen and Skonhøft (2008) for a further discussion of the cost and benefit structure of recreational salmon fishery.

The control conditions for maximizing the sustainable economic yield value π subject to the spawning and recruitment constraints are as follows:

$$(12) \quad \partial L / \partial f_4 = N_3 (p_4 w_4 - \mu \gamma_4) \begin{matrix} \geq \\ < \end{matrix} 0; \quad 0 \leq f_4 \leq 1,$$

and

$$(13) \quad \partial L / \partial f_5 = N_3 (p_5 w_5 - \mu \gamma_5) \begin{matrix} \geq \\ < \end{matrix} 0; \quad 0 \leq f_5 \leq 1,$$

where the spawning constraint shadow price μ is now measured in NOK per fish. The stock conditions of this new problem are basically the same as in the above maximum sustainable yield problem. Equation (10) is similar, whereas equation (9) changes slightly as the marginal benefit is now measured in NOK per fish. With $w_5 / \gamma_5 > w_4 / \gamma_4$ and $p_5 \geq p_4$, we have

$p_5 w_5 / \gamma_5 > p_4 w_4 / \gamma_4$ and corner solutions with a higher fishing mortality of the old than of the young adult population. Therefore, the structure of the solution of this problem is the same as the previous solution for the situation where weight is considered to be an inaccurate approximation for fecundity. The only thing to note is that case ii) is more likely to represent the optimal policy, as the marginal gain–loss ratio of the old mature population may become relatively higher.

5. The conservation perspective

So far, the maximum sustainable fishing policy has been analyzed when only harvest values have been taken into account. However, wild Atlantic salmon typically also carry

nonconsumptive benefits such as existence and biodiversity values (see Verspoor et al. 2003, and also the classical Krutilla 1967). These stock values may have increased during the last few decades not only because of lower fish abundance and the extinction of some stocks (each river has its own stock), but also because of interbreeding and genetic pollution from escaped farmed salmon (cf. introductory section; see also Verspoor et al. 2003).

There are at least two different ways to take the stock value and conservation perspective into account within our social planner framework. One possibility involves attaching a nonconsumptive value to the population size and finding fishing mortalities that maximize the composite utility, comprising the harvesting (flow) value and population (stock) value. Another possibility is to impose a minimum population size constraint, and find fishing mortalities that maximize the sustainable economic yield subject to this constraint, in addition to the biological constraints. Here, we considering the second option, in line with the upcoming management practice in Norway whereby the Environmental Department (through the Directorate for Natural Resource Management) aims to regulate salmon fishery in rivers with weak and threatened populations by imposing a minimum viable size of the spawning population (Hindar et al. 2007). In our analysis, this type of policy is formulated by demanding that the following holds:

$$(14) \quad N_3 \geq \bar{N}_3.$$

Notice that this restriction is similar to imposing a minimum size of the spawning population B (see also below).

Now, the planning problem is to maximize $\pi = [p_4 w_4 s_3 \sigma f_4 + p_5 w_5 s_3 (1 - \sigma) s_4 f_5] N_3$ subject to the spawning and recruitment constraints (5') and (6'), in addition to the minimum harvestable population size constraint (14). The Lagrangian of this problem is

$$L = [p_4 w_4 s_3 \sigma f_4 + p_5 w_5 s_3 (1 - \sigma) s_4 f_5] N_3 - \lambda [N_3 - sR(B)] - \mu \{ B - [\gamma_4 s_3 \sigma (1 - f_4) + \gamma_5 s_3 (1 - \sigma) s_4 (1 - f_5)] N_3 \} - \theta (\bar{N}_3 - N_3) \text{ with } \theta \geq 0 \text{ as the stock}$$

(conservation) shadow price (NOK per fish). Now, we find that (12) and (13) also describe the first-order control conditions, whereas the stock conditions (with $N_3 > 0$ and $B > 0$) are given by (10) together with:

$$(15) \quad \partial L / \partial N_3 = p_4 w_4 s_3 \sigma f_4 + p_5 w_5 s_3 (1 - \sigma) s_4 f_5 - \lambda + \mu [\gamma_4 s_3 \sigma (1 - f_4) + \gamma_5 s_3 (1 - \sigma) s_4 (1 - f_5)] + \theta = 0.$$

Therefore, still under the assumption that $p_3 w_5 / \gamma_5 > p_4 w_4 / \gamma_4$, the optimal harvesting policy should again be governed by a higher fishing mortality of the old than of the young adult stock either through case i) $f_5^* = 1$ and $0 < f_4^* < 1$, case ii) $f_5^* = 1$ and $f_4^* = 0$, or case iii) $0 < f_5^* < 1$ and $f_4^* = 0$ (but see below). This is stated as the following proposition.

Proposition 3. Imposing a minimum viable population size constraint reflecting the conservation perspective leaves the structure of the optimal harvest composition unaffected.

The interesting situation, of course, is when this new constraint binds. When it binds, we have $N_3^* = \bar{N}_3$ together with $\theta^* > 0$. Then, the spawning population is determined directly through the recruitment constraint (6') as $\bar{N}_3 = sR(B^*)$. Furthermore, the fishing mortalities must satisfy the spawning constraint (5') written as

$B^* = [\gamma_4 s_3 \sigma (1 - f_4) + \gamma_5 s_3 (1 - \sigma) s_4 (1 - f_5)] \bar{N}_3$ (see also Figure 2). Therefore, case ii) with $f_5^* = 1$ and $f_4^* = 0$ cannot represent the optimal solution (except by accident). Moreover, because case i) with $f_5^* = 1$ and $0 < f_4^* < 1$ indicates a more aggressive fishing policy than case iii) with $0 < f_5^* < 1$ and $f_4^* = 0$ (see also section three above), we find that this last case yields the only possible optimal solution when the stock regulation policy is tight and, hence, the stock constraint \bar{N}_3 is 'high'. With this case as the optimal solution, the old adult fishing mortality is determined through $B^* = [\gamma_4 s_3 \sigma + \gamma_5 s_3 (1 - \sigma) s_4 (1 - f_5^*)] \bar{N}_3$. On the other hand, with case i) as the optimal option, the young adult fishing mortality is determined through $B^* = \gamma_4 s_3 \sigma (1 - f_4^*) \bar{N}_3$. In the various cases, we have $\partial \pi^* / \partial \bar{N}_3 = -\theta^*$ (envelope theorem).

6. Numerical illustration

6.1 Data

The above theoretical reasoning will now be illustrated numerically. Hansen et al. (1996) estimated a salmon recruitment function for a small river in Norway (the Imsa River) based on the Shepherd recruitment function, which includes three parameters. In our generic model, we choose a simpler approach and use the Beverton–Holt function (cf. Figure 2). This

function may be specified as $R(B) = r \frac{B}{1 + B/K}$, with $r > 0$ as the intrinsic growth rate, or

maximum number of recruits per (fertility adjusted) spawning salmon, and $K > 0$ as the stock

level for which density-dependent mortality equals density-independent mortality. The size of rK yields the maximum number of recruits and scales the system ('size of the river'), which is assumed to be 40,000 (number of recruits). The value of r indicates the 'quality' of the river, and we choose $r = 400$ (number of recruits per spawning salmon). Then, we find $K = 100$.

Table 1 shows these values, as well as the other baseline parameter values used in the numerical analysis. When normalizing the fertility parameter for the young to one, $\gamma_4 = 1$, and using the fertility–weight function in McGinnity et al. (2003), we find that $\gamma_5 = 2.4$ under the assumption of (average) fishing weights of $w_4 = 2.0$ and $w_5 = 5.5$ (kg/salmon). These weights fit a 'typical' medium-sized Norwegian salmon river (NOU 1999). Therefore, for the given weight and fecundity values, the weight–fertility ratio is higher for the adult stage than for the young stage, $w_5 / \gamma_5 > w_4 / \gamma_4$. The survival parameters are based on NOU (1999), whereas the fishing prices are related to recreational fishery, which, as indicated, is far more important economically than the marine fishery. The assumption here is that the fishing permit price in a reasonably good river is about 200 NOK per day (see also Olaussen and Liu 2010). Based on average catch success, this permit price may translate into fishing prices in the range of 100–400 (NOK/kg), or even higher. We assume the same price for old and young and use $p_4 = p_5 = 150$ (NOK/kg).

Table 1 about here

6.2 Results

Because the weight–fertility ratio is highest for the old adult population and the fish price is similar in the baseline scenario, and hence $p_5 w_5 / \gamma_5 = 343.8 > p_4 w_4 / \gamma_4 = 300$ (NOK/fish), the economic yield maximizing fishing mortality will be highest for the old adult population (Proposition 2). Table 2 (first row) demonstrates where case i) with $f_5^* = 1$ and $f_4^* = 0.31$ yields the optimal fishing mortality. Increasing γ_5 , while keeping all the other parameters fixed, eventually leads to similar marginal gain–loss ratio with $\gamma_5 = 2.75$,

$p_5 w_5 / \gamma_5 = p_4 w_4 / \gamma_4 = 300$ (row two). Then, the maximum sustainable economic yield policy can be reached by an infinite number of combinations of fishing mortalities obeying equation (11) (Proposition 1), ranging from 0.31 to 1.00 for the young and from 1.00 to 0.50 for the old

mature population. As analyzed above (section three), the harvestable population size N_3^* and the size of the spawning population B^* are similar to those of case i) (first row). This will be the case for the yield as well as for the profit. On the other hand, reducing the gain–loss ratio of the young mature subpopulation by lowering p_4 while keeping all other parameters at their baseline values, so that the discrepancy between $p_5 w_5 / \gamma_5$ and $p_4 w_4 / \gamma_4$ increases, leads to the optimal fishing policy described by case ii), with no harvesting of young fish (row three).

Table 2 about here

The last row in Table 2 indicates what happens when the natural survival rate of the young s is reduced while all other parameters are kept at their baseline values. Such a reduction may be the result of infection through transmission of lice from farmed salmon. Indeed, as already indicated, this is considered to be one of the most important threats to the wild Atlantic salmon (see, e.g., Verspoor et al. 2003). A 40% reduction yields quite dramatic effects. The spawning biomass declines significantly and the profit is reduced by more than 50%. Again, case i) with harvesting of the entire old adult population represents the optimal fishing policy.

Table 3 demonstrates what happens when conservation through the harvestable population constraint (14) (section five) is taken into account. The qualitative structure of the harvesting composition with more aggressive fishing of the old mature stock is unchanged (Proposition 3). The optimal harvesting policy is determined by case i) with $f_5^* = 1$ and $0 < f_4^* < 1$ when the conservation constraint starts to bind. Increasing \bar{N}_3 further eventually leads to case iii) with $0 < f_5^* < 1$ and $f_4^* = 0$ because case ii), as analyzed above (section five), cannot represent an optimal fishing option when conservation is taken into account. (However, this case becomes the optimal harvest option by accident when $\bar{N}_3 = 1,600$, although this is not shown in the table). The cost of conservation eventually becomes quite significant (column seven), as also indicated by the shadow price value θ^* (last column). Therefore, increasing the minimum viable size of the harvestable population by one single fish from, say, $\bar{N}_3 = 1,700$, reduces the profit by NOK 452.

Table 3 about here

7. Concluding remarks

In this paper, we have studied, from a theoretical point of view, the maximum sustainable yield management of an age-structured wild Atlantic salmon (*Salmo salar*) population with two spawning and harvestable classes. The basic finding is that the weight–fecundity ratio discrepancy between the harvestable classes determines the optimal fishing mortality and the fishing composition, and no other factors play a *direct* role. This outcome differs from the seminal paper by Reed (1980), who found that weight together with natural mortality directly determined the maximum sustainable yield fishing composition. The reason for this discrepancy is the different biological characteristics of the fish stocks, as the mature salmon die after spawning in our model, whereas in the Reed model, a fixed fraction of the spawning fish survive and enter older age classes. Our analysis and findings are based on the Atlantic salmon, but the results will also apply to, e.g., the various Pacific salmon stocks, which also die after spawning.

Our model is analyzed in three steps. First, the situation with a proportional weight–fecundity relationship is considered. Second, the optimal harvest options are analyzed when the weight–fecundity ratio is highest for the old mature stock. Finally, conservation through a minimum viable size of the harvestable population is also taken into account. The analysis provides three propositions about the maximum sustainable fishing yield for the Atlantic salmon. Our model may also be extended to the possibly more realistic situation with three harvestable age classes. Given that the weight–fecundity ratio increases with weight (and age) (McGinnity et al. 2003), we find that the maximum sustainable yield policy includes fishing of the two oldest year classes at most. This will be the straightforward outcome when analyzing the Kuhn–Tucker control conditions of this new, extended problem.

One important management implication of our analysis is that a ‘balanced’ fishing policy that implies equal fishing mortalities among the fishable stocks will generally lead to economic losses, and that such losses will typically increase with the weight–fecundity discrepancy among the spawning fish stocks. Hence, any quota policy should be based on specified fishing mortalities for the harvestable classes, which will generally be different, and highest for the old mature salmon. This will also be the case if the management policy includes imposing a minimum viable size of the spawning population. Selecting between harvesting old and young mature fish is difficult, but can be influenced by factors such as fishing gear and mesh size, as well as seasonal regulation. Seasonal regulation will affect fishing selectivity, because young

mature and old mature salmon to some extent migrate back to the parent river sequentially, with the broad pattern being that the young mature return before the old (see, e.g., NOU 1999). Hence, seasonal regulation combined with mesh-size regulation in the marine fishery are possible ways to more efficiently target the Atlantic salmon fish stock.

References

Getz, W. and R. Haight 1988. Population Harvesting. Princeton University Press, Princeton

Groot, G. and L. Margolis (eds.) 1991. Pacific salmon life histories. UBC press, Vancouver

Hansen, L. P. , B. Jonsson and N. Jonsson 1996. Overvåking av laks fra Imsa og Drammenselva (in Norwegian). NINA Oppdragsmelding 401, Trondheim

Hindar, K., I. Flemming, P. McGinnity and O. Diserud 2006. Genetic and ecological effects of farmed salmon on native salmon: modeling from experimental results. ICES Journal of Marine Science 63: 1234-1247

Hindar, K. et al. 2007. Gytebestandsmål for laksebestander i Norge (in Norwegian). NINA Report 226, Trondheim

Krutilla, J. 1967. Conservation reconsidered. American Economic Review 57: 605-619

Kulmala, S., M. Laukkanen and C. Michielsens 2008. Reconciling economic and biological modelling of a migratory fish stock: Optimal management of the Atlantic salmon fishery in the Baltic Sea. Ecological Economics 64: 716-728

Laukkanen, M. 2001. A bioeconomic analysis of the Northern Baltic salmon fishery. Environmental and Resource Economics 188: 293-315

McGinnity, P. et al. 2003. Fitness reduction and potential extinction of wild populations of Atlantic salmon, *Salmo salar*, as a result of interactions with escaped farm salmon. Proceedings of the Royal Society B 270: 2443-2450

Mills, D. 1989. Ecology and management of Atlantic salmon. Chapman and Hall, New York

NASCO 2004. Report on the activities of the North Atlantic salmon conservation organization. 2002-2003. (www.nasco.int)

NOU 1999. Til laks åt alle kan ingen gjera? (in Norwegian) Norges Offentlige utredninger 1999:9, Oslo.

Olaussen, J. O. and Y. Liu 2010. Does the recreational angler care? Escaped farmed versus wild Atlantic salmon. Working paper Department of Economics NTNU, Trondheim

Olaussen, J.O. and A. Skonhoft 2008. A bioeconomic analysis of a wild Atlantic salmon recreational fishery. Marine Resource Economics 23: 119- 139

- Reed, W. J. 1980. Optimum age-specific harvesting in a nonlinear population model. *Biometrics* 36: 579–593.
- Routledge, R. 2001. Mixed-stock vs. terminal fisheries: A bioeconomic model. *Natural Resource Modeling* 14: 523-539
- Skonhøft, A., N. Vestergaard and M. Quaas 2010. Optimal harvest in an age structured model with different fishing selectivity. Submitted
- Tahvonen, O., 2008. Harvesting an age-structured population as biomass: does it work? *Natural Resource Modeling* 21: 525-550
- Tahvonen, O. 2009. Economics of harvesting age structured fish populations. *Journal of Environmental Economics and Management* 58: 281-269
- Verspoor, E., L. Stradmeyer and J. Nielsen (eds.) 2003. *The Atlantic Salmon. Genetics, Conservation and Management*. Blackwell Publishing, New York

Table 1. Biological and economic baseline parameter values

Parameter	Description	Value
s	Natural survival rate young	0.05
s_3	Natural survival rate young adult	0.5
s_4	Natural survival rate old adult	0.5
r	Intrinsic growth rate recruitment function	400 (# of recruits/ fertility adjusted spawner)
K	Scaling parameter recruitment function	100 (# of spawners)
σ	Migration parameter	0.5
w_4	Weight young adult	2.0 (kg/fish)
w_5	Weight old adult	5.5 (kg/fish)
γ_4	Fecundity parameter young adult	1.0
γ_5	Fecundity parameter old adult	2.4
p_4	Fish price young adult	150 (NOK/ kg)
p_5	Fish price old adult	150 (NOK/kg)

Table 2: The maximum sustainable economic yield problem

	f_4^*	f_5^*	N_3 (#)	B (#)	H_4^* (#)	H_5^* (#)	π^* (1000 NOK)
Baseline values	0.31	1.00	1,420	245	110	178	179
15% increase fertility coefficient old ($\gamma_5 = 2.75$)	[0.31 - 1.00]	[1.00 - 0.50]	1,420	245	[355 - 114]	[182 - 89]	179
200% reduction price young adult ($p_4 = 50$)	0.00	1.00	1,600	400	0	200	165
40% reduction natural survival rate young ($s = 0.03$)	0.11	1.00	751	167	21	94	84

Table 3: The maximum sustainable economic yield and conservation

\bar{N}_3	f_4^*	f_5	B (#)	H_4^* (#)	H_5^* (#)	π^* (1000 NOK)	θ^* (NOK/fish)
1,400	0.31	1.00	245	110	178	179	0
1,500	0.20	1.00	300	75	188	177	108
1,610	0.00	0.98	412	0	197	162	332
1,700	0.00	0.72	567	0	153	126	452
1,800	0.00	0.17	900	0	38	32	1,564

Figure 1. Schematic representation of the life cycle of a wild Atlantic salmon for a single cohort (the time index is omitted). See main text for definition of symbols.

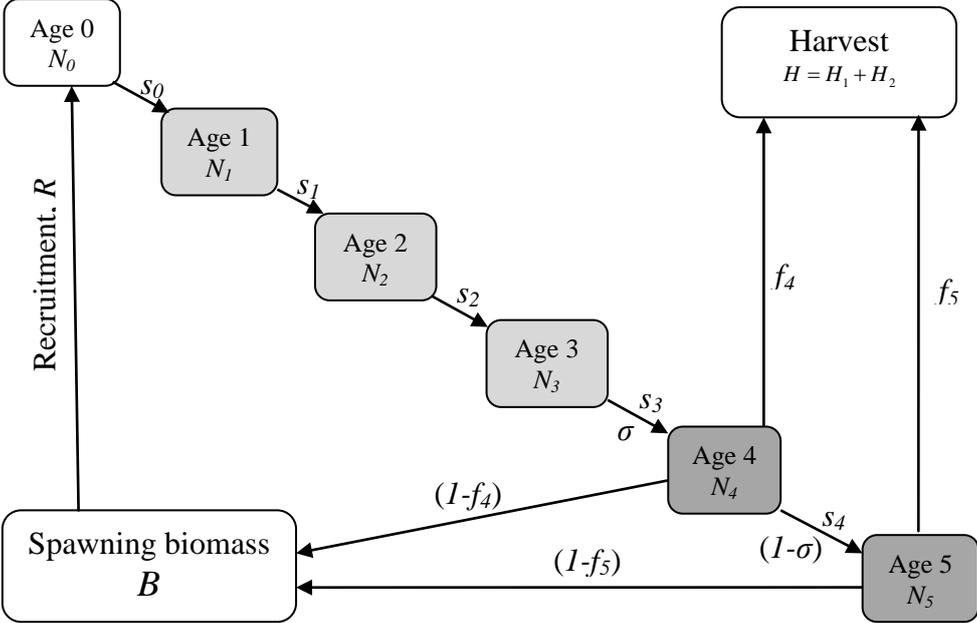


Figure 2. Internal equilibrium for fixed fishing mortalities $0 \leq f_4 \leq 1$, $0 \leq f_5 \leq 1$ (but not $f_4 = f_5 = 1$) . Beverton-Holt type recruitment function.

