Management of ecosystem resilience as optimal self-protection: A simple, but often non-convex problem

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Abstract: We interpret management of ecosystem resilience as a problem of finding the optimal level of self-protection. Economists commonly assume that the optimization problem within a simple self-protection framework is convex. We argue, however, that under reasonable assumptions on the ecosystem manager's risk-aversion and properties of the ecosystem this convexity assumption is not generally justified. By numerically scanning the parameter space for risk-aversion and properties of the ecosystem, we show that optimal investment in resilience often implies full self-protection or no self-protection.

JEL-Classification: Q57, D81, G11

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1 Introduction

Economists frequently employ a convexity assumption when considering optimization problems like "what is the optimal amount of effort spent on self-protection?". This ensures that the objective function of interest is "well-behaved" and interior solutions are obtained. However, as Dasgupta and Mäler (2003: 499) note, "the word "convexity" is ubiquitous in economics, but absent from ecology". The problem is that falsely assuming convexity in problems of environmental management results in distorted policy advice. By analyzing the case of ecosystem resilience, we investigate whether the assumption of a "well-behaved" objective function is justified in a simple self-protection framework, where "self-protection" (Ehrlich and Becker 1972) means that a person undertakes some real action that reduces the probability by which an unfavorable state of the world occurs.

The focus of the established literature on self-protection lies on comparative statics. That is, research centers on how the optimal level of self-protection varies with the subjective risk attitudes of the decision taker (risk-aversion, prudence) and objective parameters of the management problem (potential income loss, initial wealth). It is common practice to simply assume that the objective function is concave in self-protection effort – or to write the second-order condition of the problem without further investigating the requirements for this condition to be satisfied: Sweeney and Beard (1992: 302) analyze the impact of initial wealth and loss size on optimal self-protection and provide the "second-order necessary condition for an interior solution". Jullien et al. (1999: 23) focus on the effect of increasing risk-aversion on optimal self-protection. They write the second-order condition and add: "For the problem to be meaningful, we also assume that the optimal level of effort for U is interior ...". Eeckhoudt and Gollier (2005: 990) investigate the effect of increasing prudence on optimal self-protection and state: "We assume that V [expected utility] is concave in e [effort]".

These examples demonstrate how economic analysis of the self-protection problem focusses on comparative statics and regards the exclusive occurrence of interior solutions as a prerequisite to any further analysis. Beyond that, the statement of Jullien et al.

embodies the conviction that the existence of boundary solutions would render the whole problem "meaningless".

Yet, why should some management problem be devoid of meaning if it frequently delivers the result "full investment in x is optimal"? Determining the conditions for and the frequency of the (non-)occurrence of boundary solutions is an important requirement to formulate sound policy. Hence, our analysis extends the results of the established literature on self-protection in one important respect: We relax the assumption that the management's objective function is generally concave in self-protection effort and investigate under which conditions this assumption holds.

The self-protection problem we consider in this paper is the management of "ecosystem resilience". The concept of "ecosystem resilience" has been introduced to denote an ecosystem's ability to maintain its basic functions and controls under disturbances (Holling 1973, Carpenter et al. 2001). As a result of exogenous natural disturbances or human management, a system may flip from one stability domain into another one with different basic functions and controls (e.g. Levin et al. 1998). The economic relevance of ecosystem resilience derives from the fact that ecosystem flips may entail welfare losses. Indeed, such ecosystem flips may severely affect human well-being, which is why they have been termed "catastrophic shifts" (Scheffer et al. 2001). Since an increase in ecosystem resilience reduces the probability of a system flip to occur, this paper interprets investments in ecosystem resilience as efforts to self-protect.

This connection to the economic notion of self-protection fills a gap in the existing literature on management of ecosystem resilience. Until now, this research mainly focuses on (i) identifying the salient properties ecosystems must exhibit (e.g. modularity, diversity) to be resilient (Levin 1999, Folke et al. 2002), (ii) the management of particular ecosystems (e.g. Perrings and Walker 1997, 2004) and (iii) the social dimension of ecosystem management (Walker et al. 2002, Walker and Salt 2006).

We adhere to the original concept of ecosystem resilience (Holling 1973). That is, we reserve the notion of resilience for the level of ecosystems. We do not (re-)apply it to the social dimension and the broader institutional context of ecosystem management, a practice which has recently been criticized on epistemological grounds (Kirchhoff et al. 2010). Furthermore, we abstract from specific ecosystem processes and concentrate on the economic significance of ecosystem resilience as natural insurance against welfare losses from adverse ecosystem flips (Baumgärtner and Strunz 2009).

Specifically, our analysis builds on a model in which an ecosystem manager is endowed with some initial wealth and obtains income from ecosystem services. She faces a trade-off between decreasing the probability of an adverse ecosystem flip (i.e., investing in resilience) and preparing for the situation that such a fundamental environmental change occurs. The model explicitly includes the ecosystem manager's degree of risk aversion, the cost of investments in resilience, the characteristics of the ecosystem, and the size of the loss resulting from an adverse ecosystem flip.

In our analysis, we identify analytically which parameter combinations are likely to yield boundary solutions to the management problem. Subsequently, we numerically scan the parameter space and determine the share of parameter combinations that entail boundary solutions in all possible parameter combinations.

From this analysis we conclude that – in contrast to the conventional practice in economic analysis – assuming convexity of the optimization problem is not generally justified in the self-protection problem. We show that optimal management of ecosystem resilience as an income insurance often implies full self-protection (full investment in resilience) or no self-protection (full preparation for an ecosystem flip).

The remainder of this paper is organized as follows: in Sections 2 and 3 we introduce the stylized ecological-economic system and the corresponding management problem. In Section 4 we present our analytical and numerical analysis. Finally, we discuss our findings and draw conclusions in section 5.

2 Model: The Ecological-Economic System

We rely on the following stylized model of an ecological-economic system. Consider an ecosystem that potentially exhibits two different stability domains with respective levels of ecosystem services-production. One domain is characterized by a high level of ecosystem service provision and corresponding net income $y_H \in Y$; the other domain is characterized by a low level of ecosystem service provision and corresponding net income $y_L \in Y$; with $Y \subseteq \mathbb{R}_+$ and $y_L < y_H$, so that

$$\Delta y := y_H - y_L > 0 \ . \tag{1}$$

From the perspective of human ecosystem use, the former ecosystem state is therefore preferred over the latter.

Initially, the ecosystem is in the preferred high-production stability domain. In this domain, exogenous stochastic disturbances threaten to trigger a flip into the undesirable low-production stability domain. Such a flip may occur with probability p with $0 \leq p \leq 1$. Conversely, the ecosystem stays in the high-production domain with probability $1-p$.

In line with Holling's (1973) notion of resilience as the maximum amount of disturbance a system can absorb in a given stability domain while still remaining in that stability domain, we model resilience as a continuous state variable $R \in [0, \bar{R}]$ that determines the probability of the system flipping into another stability domain "given (a) its current state and (b) the disturbance regime" (Perrings and Walker 2004: 121):

$$p = p(R) \qquad \text{with} \qquad p'(R) \leq 0 \text{ for all } R \;\text{ and }\; p'(R) < 0 \text{ for all } R \in (0,1) \tag{2}$$

$$\text{and} \qquad p(0) = 1,\; p(1) = 0 \ . \tag{3}$$

In words, the higher the ecosystem's resilience in the high-production domain, the lower the probability that it flips into the low-production domain due to exogenous disturbance; with zero resilience, it flips for sure; and with a maximum resilience of one it will certainly not flip.

In order to give more ecological structure to our ecosystem model (2)–(3), we assume the following more specific model about the relationship between the level of resilience R and the flip probability p:

$$p(R) = 1 - R^{1-\sigma} \quad \text{with} \quad -\infty < \sigma < 1 \ . \tag{4}$$

This model has the fundamental resilience-defining properties (2) and (3). In addition, it has the analytically handsome property that $p'(\cdot)$ is a constant-elasticity function of R, where the parameter σ is the (constant) elasticity of $p'(\cdot)$,[1] i.e. σ specifies by how much

[1]Note that (4) implies $-p''(R)R/p'(R) = \sigma$.

(in percent) the flip probability's slope increases when the level of resilience increases by 1 %. For short, we will refer to σ as "the ecosystem's elasticity". As σ may be positive or negative, one has[2]

$$p''(R) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \text{ for all } R \in (0,1) \qquad \text{if and only if} \qquad \sigma \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \ .$$

Lacking strong ecological evidence on the value of σ, we will study the full range of theoretically possible values of σ. Notwithstanding this generality, the case of $\sigma = 0$ has an epistemically outstanding importance. For, one may argue that one can meaningfully define and measure the system's state variable "resilience" only through, and not independently of, the observable variable "flip probability". If the system's state space was one-dimensional, one could indeed meaningfully define and measure the system's resilience (sensu Holling 1973) independently of the system's flip probability, namely as the "distance" in state space – measured in units of the single state variable – between the current system state and the threshold between stability domains. However, if the system is characterized by more than one state variable, the "distance" in state space is not uniquely defined but requires some metric which is not naturally given. Then, the system's resilience in a given stability domain cannot be measured through some distance in state space.

One way to assess a system's resilience would be to directly measure its consequence in terms of flip probability. Such an epistemic equivalence between the state variable R and the observable p is exactly what is expressed by $\sigma = 0$. In this case, (4) reduces to a linear negative relationship, $p(R) = 1 - R$, so that resilience is measured directly in units of reduced flip-probability.[3]

[2]For $\sigma = 0$, $p''(R) = 0$ holds also for $R = 0$ and $R = 1$. Yet, for $\sigma < 0$, one has $p''(0) = 0$, and for $\sigma \to 1$, one has $p''(1) \to 0$.

[3]Research on alternative approaches to measure resilience is just developing. In general, in order to measure resilience it is necessary to specify resilience *of* what *to* what (Carpenter et al. 2001). If resilience is understood in a holistic, rather metaphorical way or even as a *perspective* (Walker and Salt 2006), the vagueness of the concept increases which makes it very difficult if not impossible to operationalize and to measure. Carpenter et al. (2005: 941) propose the use of surrogates to operationalize a holistic

We acknowledge that for multi-dimensional systems the question of how to measure resilience is not yet resolved and research on that issue is only developing. As our model abstracts from tangible ecosystem processes, we assume (1) that the system's flip probability is observable and (2) that resilience is measurable.

3 Model: The Management Problem

Given the above depicted ecological-economic system, there is an ecosystem manager who faces a binary income lottery $\{y_L, y_H; p(R), (1-p(R))\}$. We assume that the ecosystem manager only cares about (uncertain) income, and not directly about the underlying states of nature in terms of resilience. The ecosystem manager's preferences over income lotteries are represented by a von Neumann-Morgenstern expected utility function

$$U = \mathcal{E}_R[u(y)] \qquad \text{with} \qquad u'(y) > 0 \text{ and } u''(y) < 0 \text{ for all } y \ , \tag{5}$$

where $\mathcal{E}_R$ is the expectancy operator based on the probabilities of the binary income lottery, y is net income,[4] and $u(y)$ is a continuous and differentiable Bernoulli utility function which is assumed to be increasing and strictly concave, i.e. the ecosystem manager is non-satiated and risk averse. In order to study in the most simple way how optimal management of resilience depends on the ecosystem manager's degree of risk aversion, we assume that the ecosystem manager is characterized by constant absolute risk aversion in the sense of Arrow (1965) and Pratt (1964), i.e. $-u''(y)/u'(y) \equiv$ const., so that the Bernoulli utility $u(y)$ function is

$$u(y) = -e^{-\rho y} \qquad \text{with} \qquad \rho > 0 \ , \tag{6}$$

where the parameter ρ measures the ecosystem manager's risk aversion.

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concept of resilience: "By referring to surrogates, we acknowledge that important aspects of SES [social-ecological systems] may not be directly observable, but must be inferred indirectly." The work of Bennet et al. (2005) provides a guideline how to find such surrogates in practice.

[4]For notational simplicity, y denotes both the random variable income and income in a particular state of the world.

The ecosystem manager can influence the outcome of the income lottery by raising the ecosystem's level of resilience. In economic terminology, an increase in resilience by the ecosystem manager can be interpreted as a measure to "self-protect": "Self-protection" (Ehrlich and Becker 1972) means that a person undertakes some real action that reduces the probability by which an unfavorable – in terms of net income – state of the world occurs. An increase in the ecosystem's resilience by the manager is a real action and it provides self-protection by reducing the probability of an income loss due to a system flip. Investing in ecosystem resilience is thus equivalent to exerting effort in order to self-protect. In the following, we will use the terms "self-protection" and "investment in resilience" interchangeable.

Instead of investing in resilience, the ecosystem manager might also save income to be prepared for the case where the ecosystem does flip. Every unit of income not spent on self-protection raises the income available when the unfavorable state of the world occurs.[5] Hence, the ecosystem manager faces a trade-off between decreasing the probability of an adverse ecosystem flip (investment in resilience) and preparing for the situation that such a fundamental environmental change occurs (saving).

Formally, the ecosystem manager is endowed with some initial wealth w, which she may either invest in ecosystem resilience at a cost $c\ (R)$ or save. The cost of investments in resilience are given by

$$c\ (R) = \kappa R \quad \text{with} \quad c(0) = 0 \quad \text{and} \quad \kappa \geq 0 \tag{7}$$

where parameter κ regulates the slope of $c\ (R)$. The manager's problem is to find the level of investment in resilience that maximizes $\mathcal{E}_R[u]$, the expected utility of the ecosystem manager's income lottery R:

$$\mathcal{E}_R[u] = p\ (R)\ u(w\ - \Delta y\ - c\ (R)) + (1 - p\ (R))\ u(w\ - c\ (R)) \tag{8}$$

[5]This adaptation to the low productivity ecosystem domain could also be interpreted in non-monetary terms: the ecosystem manager might represent a community that chooses to spend effort rather for preparing its institutions for fundamental environmental changes than for developing management techniques that make a specific ecosystem more resilient to exogenous disturbances. Institutions that are able to cope with non-trivial environmental changes are said to exhibit "adaptive capacity" (e.g. Carpenter et al. 2001).

The optimal level of resilience that maximizes (8) is denoted by R^*.

4 Analysis and Results

4.1 Analytical results

In this section, we provide the analytical basis for our argument that boundary solutions to this maximization problem are not exceptional. First, we show that the condition for an interior solution given in the literature is not useful in the important case where the relationship between effort to self-protect and reduction in the probability of a loss is formalized by means of an iso-elastic function as in (4). Second, we show that interior solutions arise only for a restricted parameter set. We analyze which combinations of the parameters of the management problem are likely to induce boundary solutions.

1. A sufficient condition for the expected utility of a self-protection problem to be concave in effort is provided by Jullien, Salanié and Salanié (1999: 23). In our terminology of ecosystem resilience, the condition reads:

$$p''(R)\ p(R)\ \ - 2\ (p'(R))^2\ \ \geq 0 \tag{9}$$

This sharply restricts the type of ecosystems possibly to be considered since a necessary condition for the equation to be satisfied is $\sigma > 0$. All ecosystems with low elasticity, that require some buildup of resilience before the flip probability is significantly lowered, would be excluded from consideration. Yet, assuming $\sigma > 0$ is not sufficient for (9) to hold. Following our model of $p(R)$ as in (4), equation (9) can be translated and solved to:

$$R \leq (\frac{2-\sigma}{\sigma})^{\frac{1}{\sigma-1}} \tag{10}$$

Unless $\sigma = 1$, the right hand side of equation (10) is smaller than one. Thus, only for $\sigma \to 1$, equation (9) holds for all $R \in [0; 1]$. However, fixing the value of σ to 1 is not a reasonable option. This is for two reasons: First, an elasticity of 1 would imply an extreme level of ecosystem elasticity, where the first unit of resilience starting from $R = 0$ yields all the reduction in flip probability and all later units do not matter. For all other ecosystems, which are not characterized by an extreme elasticity of $\sigma = 1$, condition (9) could not be used to determine if the self-protection problem has a boundary or an interior solution. Second, the self-protection problem itself would reduce to a trivial exercise, in which it is always optimal to invest in the first unit of resilience unless the costs for this investment exceed the potential income loss.

We conclude that in the important case of iso-elastic functions representing the relationship between effort to self-protect and reduction in the probability of a loss, the sufficient condition of Jullien et al. (1999) is not a valuable instrument to determine wether the maximization program is globally concave.

2. As the literature does not provide a wholly satisfying answer to the question under which circumstances the self-protection problem exhibits an interior solution, we investigate the parameters of the decision problem: Which parameter combinations are likely to induce a boundary solution to the maximization of $\mathcal{E}_R[u]$ given by (8)?

Using (4), (6) and (7) to explicate (8) leads to the following expression:

$$\mathcal{E}_R[u] = -(1 - R^{1-\sigma})\ e^{-\rho\ (w\ -\ \Delta y\ -\ \kappa R)} -\ R^{1-\sigma}\ e^{-\rho\ (w\ -\ \kappa R)} \tag{11}$$

The location of the R^* that maximizes this equation depends in particular on the interplay of the parameters σ and ρ.

Proposition 1

(i) The probability of boundary solutions to the self-protection problem

$$\text{increases in risk aversion} \quad \rho, \tag{12}$$

$$\text{decreases in ecosystem elasticity} \quad \sigma. \tag{13}$$

$$\text{Boundary solutions inevitably arise when} \quad \rho \to \infty \quad \text{or} \quad \sigma \to -\infty. \tag{14}$$

Together, results (12) – (14) imply:

(ii) Interior solutions only occur for a restricted parameter set where $\rho < \hat{\rho}$ **and** $\sigma > \hat{\sigma}$.

The restrictions on ρ and σ vary with each other:

$$\hat{\sigma} = \sigma(\rho) \quad \text{and} \quad \hat{\rho} = \rho(\sigma) \tag{15}$$

with

$$\frac{d\,\hat{\sigma}}{d\rho} > 0 \qquad \text{and} \qquad \frac{d\,\hat{\rho}}{d\sigma} > 0 \tag{16}$$

Proof. See Appendix 5 □

Result (12) indicates that increasing aversion against risk raises the likelihood of extreme levels of self-protection, both of full investment in resilience and of no investment in resilience. This result follows intuitively from Jullien et al. (1999), although they do not consider boundary solutions. Their main result is that higher risk aversion entails higher (lower) levels of self-protection when the probability of a loss is next to 0 (1). In our terminology, it reads: For small flip probabilities the more risk-averse ecosystem manager chooses a higher level of resilience, whereas for high probabilities she prefers reducing the potential maximal loss and invests less in resilience. [6]

In other words, higher risk aversion has an ambiguous effect on the optimal amount of self-protection. The more risk-averse the decision maker, the less attractive are intermediate levels of self-protection compared to full (no) investment in resilience. It is straightforward to conclude that – unless you assume *a priori* that the solution will be an interior one, as Jullien et al. (1999) do – for a sufficiently high level of risk aversion, the optimal amount of investment in resilience lies at the boundary and either full investment is chosen or none at all. Wether such a risk-averse manager chooses $R = 1$ or $R = 0$ is determined by a comparison between $c(1)$ and Δy. Obviously, if the potential income loss is bigger than the costs of full self-protection, $R = 1$ is optimal, whereas for $c(1) > \Delta y$ no investment in resilience at all is preferred.

[6] Chiu (2000) provides a detailed examination of the switching level that determines wether the probability of a loss is high or low.

Result (13) states that increasing ecosystem elasticity diminishes the probability of boundary solutions. The intuition is as follows: for very low levels of σ only the last units of resilience next to $R = 1$ do significantly reduce the flip probability from 1 to 0, whereas all other units have a negligible effect. Hence, it seems reasonable either not to invest in resilience at all, or to opt for full investment in self-protection. Wether, for such inelastic ecosystems, $R = 1$ or $R = 0$ is optimal, follows from a comparison between $c(1)$ and Δy, as in the case of high risk-aversion: If the potential income loss is bigger than the costs of full self-protection, $R = 1$ is optimal, whereas for $c(1) > \Delta y$ no investment in resilience maximizes expected utility. With increasing ecosystem elasticity, this all-or-nothing intuition fades and eventually reverses. For $\sigma = 0$ all units of resilience contribute equally to a reduction in the flip probability and without knowledge of the management problem's other components no level of resilience is to be preferred. If the ecosystem elasticity is positive, the first units of resilience next to $R = 0$ do have a bigger impact on the probability reduction than the following ones. In the extreme, it's at a very low level of resilience that the bulk of the probability reduction occurs and all later units of resilience only have a negligible impact. Thus, the ecosystem manager will invest in these first units but abandon the option to further increase the level of resilience.

Result (14) indicates that boundary solutions inevitably occur for infinite risk-aversion or infinitely in-elastic ecosystems. Hence, there exist some threshold values for the ecosystem manager's risk-aversion ($\hat{\rho}$) and the ecosystem's elasticity ($\hat{\sigma}$) that must not be exceeded to ensure the existence of interior solutions. While both ρ and σ affect the optimization of expression (11), their impact on the probability of boundary solutions works in opposing directions. This means that the levels of risk-aversion and ecosystem elasticity that allow for an interior solution are not independent of each other, which is stated in result (15). The higher the ecosystem manager's risk-aversion, the more elastic the ecosystem must be to ensure an interior solution. The more inelastic an ecosystems is, the less risk-averse the decision taker must be to guarantee an interior solution. This interdependency of the restrictions on ρ and σ is formalized in result (16).

To sum up, the analytical analysis of expression (11) suggests that a combination of low risk-aversion and high ecosystem elasticity ensures interior solutions to the self-protection problem, whereas high risk-aversion or low elasticity lead to boundary solutions.

4.2 Numerical Simulation

In the following, we provide results from numerical analysis showing that boundary solutions of the above formulated management problem do frequently arise.

Proposition 2
For most possible combinations of parameter values, optimal investment in ecosystem resilience implies either full self-protection or no self-protection.

Our approach is as follows: a scan of the parameter space for risk-aversion ρ and ecosystem elasticity σ determines which combinations of both parameters lead to interior solutions and which combinations yield boundary solutions. It is important to acknowledge that there can be no absolute answer to the question which portion of the parameter space entails boundary solutions. The exact share of $\rho - \sigma$ combinations that yield boundary solutions in our model depends on the following factors:

i) the metrics used to perform the scan,

ii) the range of ρ and σ out of which possible $\rho - \sigma$ combinations are chosen and

iii) the size and relation of the other parameters Δy and κ.

ad i) Our aim is to determine a certain share of the parameter space, which means that if the parameters are scanned at the same resolution they contribute equally to the result. A priori it is not clear wether they should be analyzed at the same scale or wether they should be given distinct weight. Every metrics involves a value judgement about the importance of each parameter. For example, the parameter for ecosystem elasticity might be reduced to a 0/1–distinction "inelastic"–"elastic", whereas risk-aversion might be scrutinized at a very fine scale. Since we do not have a cogent reason why risk-aversion should have a stronger impact on the result then ecosystem elasticity (or the other way round), equal treatment implies that we scan the parameter space at the same resolution for both parameters.

ad ii) If ρ and σ were not bounded, a scan of the parameter space would yield the result that the share of ρ –σ combinations that entail interior solutions tends to zero. The reason is, of course, that for infinite risk-averse decision makers or infinite in-elastic ecosystems only boundary solutions materialize. In an unbounded parameter space, the cases of moderate risk-aversion and elastic ecosystems only occupy a negligible share of the whole parameter space. We therefore need to find restrictions on the ρ –σ parameter space, that may serve as a basis for our analysis.

To determine a "reasonable" range of values for the risk-aversion parameter ρ, we follow Babcock et al. (1993). In their paper, "a defensible range of risk-aversion coefficients is defined by the coefficients that correspond to risk premiums falling between 1% and 99% of the amount at risk [...] "(Babcock et al. 1993: 17). That is, for a given amount of income at stake, a "practical upper bound" for the risk-aversion coefficient is provided. Hence, to perform our analysis, we first choose the size of the potential income loss and the costs of investments in resilience and subsequently assign the corresponding "defensible range" of values for ρ.

With parameter σ for ecosystem elasticity we introduce a new concept to the self-protection literature. We are not aware of any research that indicates "reasonable" ranges of values for σ. Therefore, we proceed by excluding strongly negative values of σ that would trivially induce boundary solutions (cf. result 14) and consider only values of $\sigma > -1$.[7] Yet, we do allow for extreme elasticity ($\sigma \to 1$). The latter strongly induces interior solutions and is one subcase of the standard economic assumption that the probability of damage is a decreasing convex function of the self-protection effort (e.g. Jullien, Salanié and Salanié 1999: 23).

ad iii) Boundary solutions do trivially arise for extreme levels of potential income losses $\Delta y \to \infty$ (0) or extreme levels of the costs of investments in resilience $\kappa \to \infty$ (0):

[7] If an ecosystem is characterized by an elasticity of -1, a resilience level of 0.5 corresponds to a flip probability of 0.75. Thus, in the most inelastic ecosystem we consider, the first units of resilience reduce the system's flip probability less than the later units of resilience, yet their impact is not negligible.

If the potential income loss from an ecosystem flip is much bigger (smaller) than the costs of investments in resilience, optimal management implies full (no) self-protection. Only if both parameters are roughly of the same size, the management problem is non-trivial from an economic point of view.

Consequently, two relevant cases exist for the parameters Δy and κ. First, the case where $\Delta y > \kappa$. This excludes the possibility of a boundary solution at $R^* = 0$ since a rational individual will always exert *some* self-protection effort if the potential benefit of this action (prevention of an income loss) exceeds the costs at all levels of self-protection. Hence, only the condition for a boundary solution at $R^* = 1$ (cf. A.22) is to be considered in case the potential income loss is bigger than the costs of full investment in resilience.

Second, the case where $\kappa > \Delta Y$. This excludes the possibility of a boundary solution at $R^* = 1$ since optimal management cannot imply full self-protection if the costs for the latter exceed the potential income loss from an ecosystem flip. Hence, only the condition for $R^* = 0$ (cf. A.20) is of interest here.

Considering points i) to iii), we devise two relevant management scenarios in the following steps. First, we choose values for the potential income loss Δy and the cost parameter κ.[8] Second, we assign the corresponding appropriate range of risk-aversion values. Third, we define a relevant range of values for the ecosystem's elasticity. Finally, we scan the ρ–σ space for the relevant boundary condition. We summarize our approach in the following table.

Scenario	Δy	κ	ρ	σ	possible boundary solution
1	20.000	15.000	$\in (0, 0.0004)$	$\in (-1, 1)$	$R^* = 1$
2	12.000	15.000	$\in (0, 0.0004)$	$\in (-1, 1)$	$R^* = 0$

[8]The exact choice of Δy and κ is arbitrary. Yet, our aim is not to indicate an exact number concerning the share of the parameter space that yields boundary solutions: As outlined above, it is, due to the value judgements involved in every metrics, in principle not possible to produce an objectively exact number. Rather, we want to make an argument about the *order of magnitude* of the share of parameter combinations that yield boundary solutions and thus we concentrate on identifying two relevant scenarios.

Scenario 1

In this scenario, we investigate which ρ–σ combinations lead to a boundary solution with full investment in resilience. We assume that the potential income loss from an ecosystem flip ($\Delta y = 20.000$) exceeds the costs of self-protection ($\kappa = 15.000$) at all levels of resilience.

Following (Babcock et al. 1993), we assign the relevant range of values for the risk-aversion parameter $\rho \in (0, 0.0004)$ for income lotteries in this order of magnitude. The ecosystem's elasticity is considered in the range of $\sigma \in (-1, 1)$. Wether the ecosystem manager opts for both saving and investing in resilience or chooses full self-protection is determined by the following condition for a boundary solution at $R^* = 1$:

$$\mathcal{E}_1[u] > \mathcal{E}_R[u] \quad \text{for all} \quad R \tag{17}$$

If this condition is satisfied, full investment in resilience is optimal. Figure 1 displays the parameter scan we performed to determine which combinations of values for risk-aversion ρ and ecosystem elasticity σ satisfy this condition. Parameter combinations that entail boundary solutions are indicated in blue, combinations that entail interior solutions in red.

The clear result of our parameter scan is that only a fraction of parameter combinations yield an optimal management policy with both saving and investment in resilience. Only for risk-neutrality ($\rho = 0$, upper horizontal axis), extreme ecosystem elasticity ($\sigma = 1$, right vertical axis) or a combination of high elasticity and low risk-aversion does the optimization problem have an interior solution. In all other cases of moderate (high) risk-aversion and moderate (negative) elasticity the boundary solution at $R^* = 1$ arises.

Figure 2 illustrates this result with two examples. In the first case (figure 2, left picture), a combination of moderate ecosystem elasticity ($\sigma = 0.25$ indicates that a resilience level of $R = 0.4$ corresponds to flip probability of 50%) and moderate risk-aversion ($\rho = 0.000041$ means that an individual facing a lottery with equal chances of winning or loosing 10.000 \$ exhibits a risk premium of 0.2, Babcock et al. 1993: 21) yields full investment in ecosystem resilience as optimal management choice. In the second example (figure 2, right picture), lower risk-aversion ($\rho = 0.000024$ implies a risk

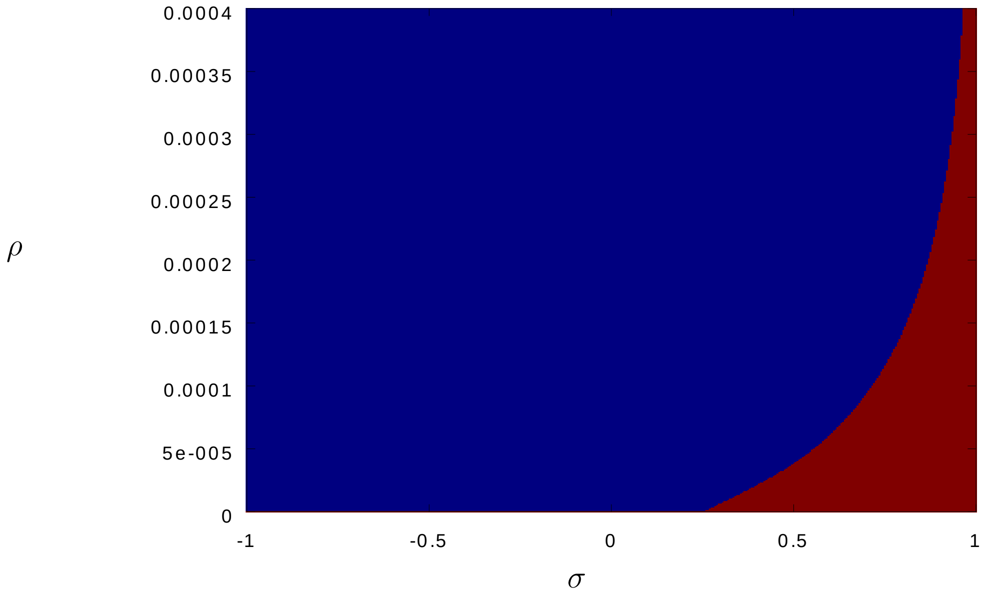

Figure 1: Parameter combinations that yield interior solutions (red), a boundary solution at $R^* = 1$ (blue)

premium of 0.12 to avoid a lottery with equal chances of winning or loosing 10.000$) and higher ecosystem elasticity ($\sigma = 0.5$ implies that a resilience level of 0.25 corresponds to a flip probability of 50%) lead to an interior solution. A shift in the parameter values that changes the optimal management strategy from full self-protection to a combination of self-protection and preparing for the ecosystem flip would in figure 1 be represented as a movement from a point in the blue segment to a point in the red segment of the parameter space.

Scenario 2

In this scenario, we analyze which ρ–σ combinations yield a boundary solution with full saving and no investment in resilience. We assume that the potential income loss from an ecosystem flip ($\Delta y = 12.000$) is not for all levels of resilience bigger than the costs of investing in resilience ($\kappa = 15.000$). The parameter for ecosystem's elasticity is considered in the range of $\sigma \in (-1, 1)$. In line with (Babcock et al. 1993), we investigate the risk-aversion parameter in the defensible range of $\rho \in (0, 0.0004)$. The relevant

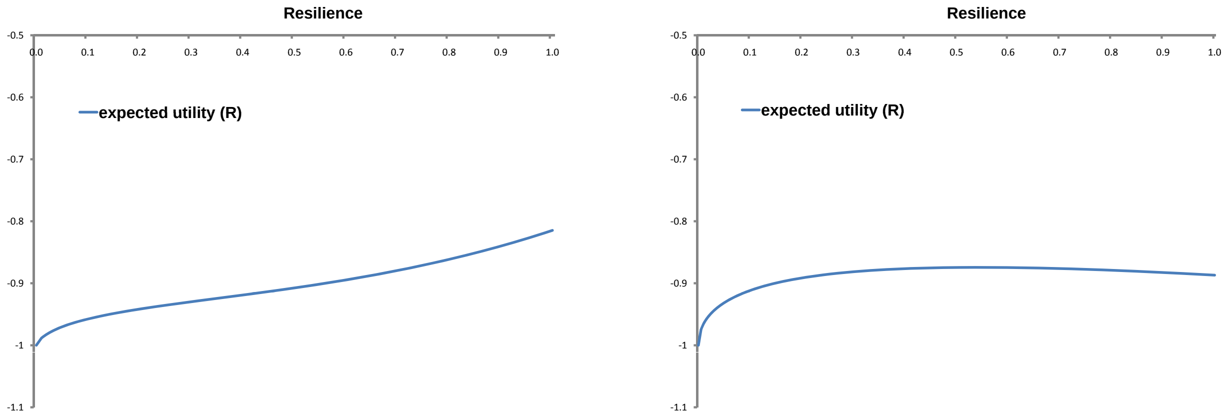

Figure 2: Expected utility in scenario 1 ($\Delta y > \kappa$) as a function of resilience for two different parameter combinations. Moderate risk-aversion and moderate ecosystem elasticity yield a boundary solution at $R^* = 1$ (left), low risk-aversion and high ecosystem elasticity an interior solution (right). (Parameter values, left: $\sigma = 0.25$, $\rho = 0.000041$; right: $\sigma = 0.5$, $\rho = 0.000024$; both: $\Delta y = 20000$, $\kappa = 15000$)

condition for a boundary solution at $R^* = 0$ is:

$$\mathcal{E}_0[u] > \mathcal{E}_R[u] \quad \text{for all} \quad R \tag{18}$$

Figure 3 displays the parameter scan we performed to determine which combinations of values for risk-aversion ρ and ecosystem elasticity σ satisfy this condition.

The numerical analysis of scenario 2 shows that the share of parameter combinations that yield a boundary solution at $R^* = 0$ (indicated in blue) in all combinations is less than in scenario 1. Comparing figures 1 and 3, the red segment representing interior solutions is considerably larger in scenario 2 than in scenario 1. However, interior solutions do only represent a minority of the whole parameter space. For low ecosystem elasticity interior solutions do not arise, unless risk-aversion is very small or tends towards risk-neutrality.

Figure 4 illustrates two different parameter combinations. The left picture displays a boundary solution with no self-protection, evoked by zero ecosystem elasticity (all units of resilience exert the same marginal impact on the ecosystem's flip probability) and moderate risk-aversion (a parameter value of $\rho = 0.000041$ indicates a risk premium of 0.2 for a lottery with equal chances of winning or loosing 10.000 \$, Babcock et al.

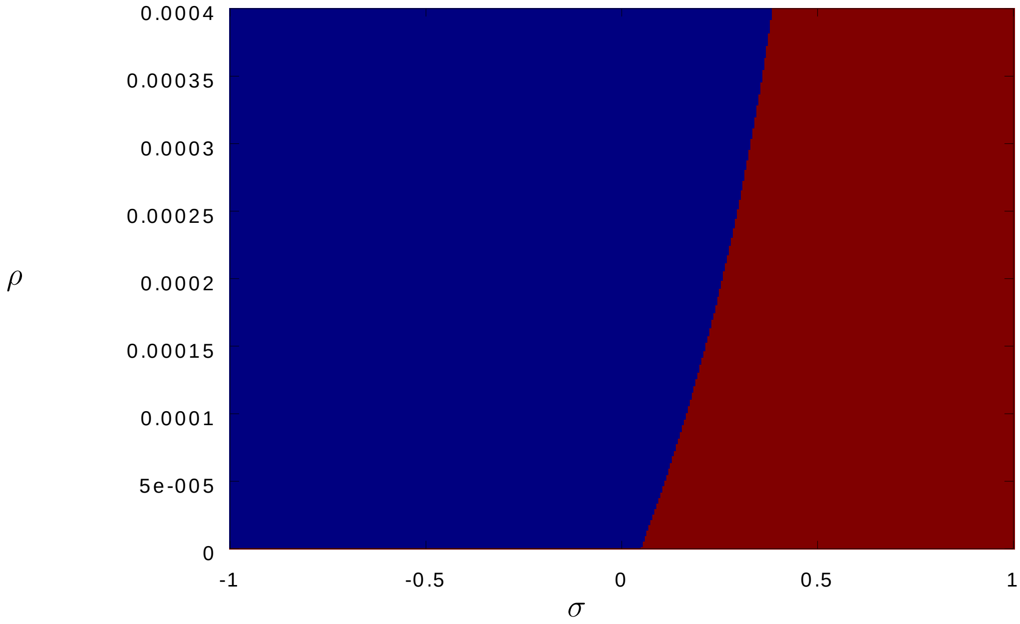

Figure 3: Parameter combinations that yield interior solutions (red), a boundary solution at $R^* = 0$ (blue)

1993: 21). Lower risk-aversion ($\rho = 0.000024$ implies a risk premium of 0.12 to avoid a lottery with equal chances of winning or loosing 10.000\$) and higher ecosystem elasticity ($\sigma = 0.5$ implies that a resilience level of 0.25 corresponds to a flip probability of 50%) entail an interior solution with both investment in resilience and saving, as illustrated in the right picture of figure 4.

5 Discussion and conclusion

Considering the case of ecosystem resilience, we have provided an example where the "convexity-assumption" frequently employed in economic analysis only covers a restrictive subset of possible cases. We have interpreted investments in ecosystem resilience as efforts to self-protect following the established economic literature on self-protection (e.g. Sweeney and Beard 1992, Eeckhoudt and Gollier 2005). Our analysis built on a stylized model of an ecological-economic system with parameters for risk-aversion, ecosystem elasticity, potential income loss due to a flip of the ecosystem and the costs of investments in resilience. We proposed two salient results:

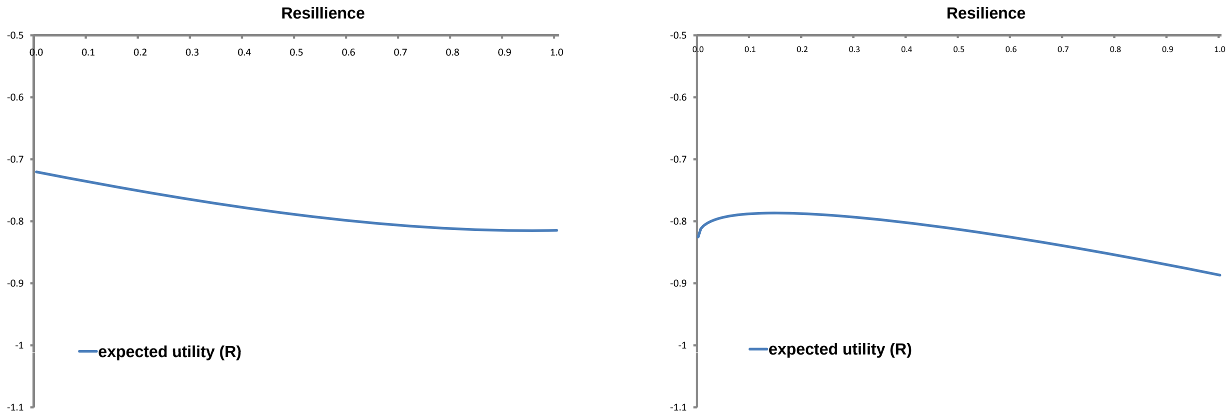

Figure 4: Expected utility in scenario 2 ($\kappa > \Delta y$) as a function of resilience for two different parameter combinations. Moderate risk-aversion and zero ecosystem elasticity yield a boundary solution at $R^* = 1$ (left), low risk-aversion and high ecosystem elasticity an interior solution (right). (Parameter values, left: $\sigma = 0$, $\rho = 0.000041$; right: $\sigma = 0.5$, $\rho = 0.000024$; both: $\Delta y = 12000$, $\kappa = 15000$)

First, choosing the level of ecosystem resilience in order to maximize the expected utility from the income lottery leads to interior solutions only when the set of parameter values of the management problem is restricted. We showed that if the ecosystem manager's risk aversion is sufficiently high or the ecosystem is sufficiently inelastic, optimal resilience management implies full or no self-protection.

Second, boundary solutions to the self-protection problem do frequently occur. Numerical analysis showed that reasonable assumptions about the ecosystem manager's risk aversion and properties of the ecosystem do not guarantee convexity of the decision problem. In particular, we investigated two scenarios. In the first scenario the potential income loss from an ecosystem flip exceeds the costs of full investment in resilience. In this case, full self-protection is optimal for most combinations of the parameters for the ecosystem manager's risk-aversion and ecosystem elasticity. Interior solutions to the management problem occupy only a small fraction of the whole parameter space. In the second scenario, the costs of full self-protection exceed the potential damage from an ecosystem flip. Our parameter scan showed that in a majority of all possible parameter combinations, optimal self-protection means full saving and no investment in resilience.

However, the finding in scenario 2 is less clear than in scenario 1, with interior solutions accounting for a considerable fraction of the whole parameter space.

These results are particularly important in two respects. First, there is an implication for the correct policy formulation for ecosystem management: Ecosystem flips need not to be "catastrophic shifts" (Scheffer et al. 2001) to warrant policies aiming at the highest possible level of self-protection. For a rather risk-averse ecosystem manager and low ecosystem elasticity, full self-protection is optimal as long as the potential income loss from a system flip exceeds the costs of full investment in resilience. Thus, we refute the common wisdom that optimally trading off two possible strategies in a maximization problem results in a mix of those policies.

Second, our results contest the economic practice of assuming "well-behaved" objective functions in seemingly simple cases as the self-protection problem. Since our stylized model so far abstracts from explicit ecosystem processes, our main finding is not conditioned on a specific ecological background. While it is acknowledged that a "convexity assumption" is overly simplistic (Dasgupta and Mäler 2003) for management problems involving non-linear ecosystem behavior, we show that intricate ecologic processes are not necessary to invalidate this convexity assumption. The self-protection problem is an example where standard economic assumptions on risk preferences and objective characteristics of the decision problem are not sufficient to guarantee the desired properties of the objective function.

For future research on ecosystem resilience as self-protection we reckon two leads to be especially promising: First, the analysis of more complex ecosystem structures with explicit, non-linear system attributes would expand the simple self-protection framework. For the price of a less general framework, more specific resilience management policies could be analyzed. Second, modeling Knightian uncertainty (Knight 1921) about ecosystem flips instead of the clear observability of the flip probability p assumed here, might constitute a more accurate representation of ecosystem management under environmental uncertainty.

Acknowledgments

Appendix

Proof. The condition for an optimal solution at $R^* = 0$, implying no investment in resilience, is:

$$\mathcal{E}_0[u] > \mathcal{E}_R[u] \quad \text{for all} \quad R \tag{A.19}$$

Explicating this condition by using (4), (6), (7) and (8) leads, after rearranging, to:

$$1 < e^{\rho\kappa R}(1 - R^{1-\sigma} + R^{1-\sigma}e^{-\rho\Delta y}) \quad \text{for all} \quad R \tag{A.20}$$

The condition for an optimal solution at $R^* = 1$, implying full investment in resilience, is:

$$\mathcal{E}_1[u] > \mathcal{E}_R[u] \quad \text{for all} \quad R \tag{A.21}$$

Explicating this condition by using (4), (6), (7) and (8) leads, after rearranging, to:

$$1 < [(1 - R^{1-\sigma})e^{\rho\Delta y} + R^{1-\sigma}]e^{-\rho\kappa(1-R)} \quad \text{for all} \quad R \tag{A.22}$$

Differentiating the right hand side of equations (A.20) and (A.22) with respect to ρ and σ yields the tendencies stated in results (12) and (13). We need to show however, that conditions (A.20) and (A.22) are not vacuous and there are parameter values for which they hold, respectively do not hold. Accordingly, we investigate (A.20) and (A.22) separately for ρ and σ in their limits.

- ρ

  For $\rho \to 0$, equations (A.20) and (A.22) both collapse to $1 < 1$ and do not hold.

  For $\rho \to \infty$, condition (A.20) behaves as follows: If $R \in (0, 1)$ the term in brackets reduces to $(1-R^{1-\sigma})$, the term $e^{\rho\kappa R}$ tends to infinity, and hence the right hand side is bigger than 1. If $R = 1$ however, the whole right hand side can be reduced to $e^{\rho(\kappa-\Delta y)}$. Thus, for condition (A.20) to be satisfied for all R, $\kappa > \Delta y$ is necessary.

  Now consider condition (A.22) for $\rho \to \infty$. A comparison between the two exponential terms shows that condition (A.22) holds if $\Delta y > \kappa(1 - R)$: Then, the first term grows faster than the second shrinks and there will be some ρ for which condition (A.22) holds.

To sum up, an increase in ρ

(i) raises the probability of a boundary solution at $R^* = 0$ if $\kappa > \Delta y$

(ii) raises the probability of a boundary solution at $R^* = 1$ if $\Delta y > \kappa$.

This proofs result (12).

- σ

  For $\sigma \to -\infty$, condition (A.20) reduces to $1 < e^{\rho\kappa R}$ if $R \in (0,1)$. For $R = 1$, however, it reduces to $1 < e^{\rho(\kappa-\Delta y)}$. Thus, for condition (A.20) to hold, $\kappa > \Delta y$ is necessary.

  Now consider condition (A.22). For $\sigma \to -\infty$, its right hand side becomes $e^{\rho(\Delta y-\kappa(1-R))}$. Thus, condition (A.22) is violated if there is a R such that $\kappa(1-R) > \Delta y$ but it holds if $\forall R \quad \Delta y > \kappa(1-R)$.

  For $\sigma \to 1$, condition (A.20) reduces to $1 < e^{\rho(\kappa R-\Delta y)}$, which is violated because $\kappa R > \Delta y$ in general does not hold for all R (the exception is of course $\Delta y = 0$, when the ecosystem manager naturally chooses $R^* = 0$).

  Now consider equation (A.22). For $\sigma \to 1$, it is violated since the term in brackets on the right hand side reduces to 1 but the term $e^{-\rho\kappa(1-R)}$ is smaller than 1.

  To sum up, a decrease in σ

  (i) raises the probability of a boundary solution at $R^* = 0$ if $\kappa > \Delta y$

  (ii) raises the probability of a boundary solution at $R^* = 1$ if $\Delta y > \kappa$.

  Hence, result (13) is true.

Comparing the above investigations of conditions (A.20) and (A.22) shows that for $\sigma \to -\infty$ or $\rho \to \infty$ a boundary solution inevitably arises. This proofs result (14). Which boundary solution occurs, however, is determined by the relation between Δy and κ: If $\kappa > \Delta y$, $R^* = 0$, whereas $\Delta y > \kappa$ yields $R^* = 1$.

□

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