An Equilibrium Model of Habitat Conservation under Uncertainty and Irreversibility

Luca Di Corato* Michele Moretto† Sergio Vergalli‡

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Abstract

In this paper stochastic dynamic programming is used to investigate habitat conservation by a multitude of landholders under uncertainty about the value of environmental services and irreversible development. We study land conversion under competition on the market for agricultural products when voluntary and mandatory measures are combined by the Government to induce adequate participation to a conservation plan. We determine analytically the impact of uncertainty and policy optimal conversion dynamic and discuss different policy scenarios on the basis of the relative long-run expected rate of deforestation. Finally, some numerical simulations are provided to illustrate our findings.

KEYWORDS: OPTIMAL STOPPING, DEFORESTATION, PAYMENTS FOR ENVIRONMENTAL SERVICES, NATURAL RESOURCES MANAGEMENT.

JEL CLASSIFICATION: C61, D81, Q24, Q58.

1 Introduction

As human population grows the human-Nature conflict has become more severe and natural habitats are more exposed to conversion. On the one hand, clearing land to develop it may lead to the irreversible reduction or loss of valuable environmental services (hereafter, ES) such as biodiversity conservation, carbon sequestration, watershed control and provision of scenic beauty for recreational activities and ecotourism. On the other hand, conserving land in its pristine state has a cost opportunity in terms of foregone profits from economic activities (e.g. agriculture, commercial forestry) which can be undertaken once land has been cleared.

By balancing marginal social benefit and cost of conservation, the social planner is required to destine the available land to conservation or development which are usually two competing and mutually exclusive uses. Despite its theoretical appeal, the idea of a social planner that, once defined a socially optimal habitat conversion rule, can implement it by simply commanding the constitution of protected areas is far from reality. In fact, since the most part of remaining ecosystems are on land privately owned then the economic and political cost of such intervention would make unlikely the adoption of command mechanisms by Governments (Langpap and Wu, 2004).

At least initially, Governments have privileged an indirect approach in conservation policies. The main idea behind it was to divert, through programs such as integrated conservation and development projects,

*Corresponding address: Department of Economics, Swedish University of Agricultural Sciences, Box 7013, Johan Brauners väg 3, Uppsala, 75007, Sweden. Email: luca.di.corato@ekon.slu.se. Telephone: +46(0)18671758. Fax: +46(0)18673502

†Corresponding address: Dipartimento di Scienze Economiche "Marco Fanno", University of Padua, Via del Santo 33, 35123, Padova, Italy. Email: michele.moretto@unipd.it. Telephone: +390498274265.

‡Corresponding address: Department of Economics, University of Brescia, via S. Faustino 74/b, 25122, Brescia, Italy. E-mail: vergalli@eco.unibs.it.

community-based natural resource management or other environmental friendly commercial ventures, the allocation of labour and capital from ecosystem damaging activities toward conservative ones (Wells et al., 1992, Ferraro and Simpson, 2002). However, despite the initial enthusiasm, effectiveness and cost-efficiency concerns have lead to abandon this approach in favour of compensations to be paid directly to the landholders providing conservative services (see e.g. Ferraro, 2001; Ferraro and Kiss; 2002, Ferraro and Simpson, 2005). A direct approach, mainly represented by Payments for environmental services – like schemes (hereafter, PES) has become increasingly common in both developed and developing countries. Under a PES program, a provider delivers to a buyer a well-defined ES (or corresponding land use) in exchange for an agreed payment.[1] Unfortunately, also the efficacy of PES programs has been questioned since their performance has not always met the targeted conservation goals.[2] In particular, lack of additionality in the conservation efforts induced by the programs has often been suspected.[3] In other words, it seems that landholders have been practically paid for conserving the same extent of land they would have conserved without the program. Considering the limited amount of money for conservation initiatives and the perverse effect that wasting it may have on future funding further research is needed to increase our understanding of the actual impact of PES programs on economic agents' conversion decision.

The literature investigating land allocation under irreversibility and uncertainty over the net benefits attached to conservation represents a significant branch of environmental and resource economics. A unifying aspect in this literature is the stress on the effect that irreversibility and uncertainty have on decision making. In fact, since irreversible development under uncertainty over future prospects may be later regretted, conversion may be postponed to benefit from option value attached to the maintained flexibility (Dixit and Pindyck, 1994). Pioneer papers such as Arrow and Fisher (1974) and Henry (1974) have been followed by several other contributions which have improved the modelling effort and solved the technical problems posed by increasingly complex model set-up.[4] However, these analyses, privileging a central planner perspective on the allocative problem, miss the complexity of challenges characterizing conservation efforts undertaken by Governments. Therefore, we aim to contribute developing a dynamic model investigating the impact of introducing a payment scheme for conservation in a decentralized economy populated by a multitude of homogenous landholders. Each landholder manages a portion of total available land and may conserve or develop it by affording some conversion cost. If land is conserved, society benefits from conservation for a value proportional to the area conserved which follows a geometric Brownian motion.[5] If the parcel is developed, land enters as an input into the production of good or services (agricultural products, oil palm products, commercial timber, etc.) and the farmer must compete with other farmers on the market. In this context, the Government introduces a land use policy which aims to balance conservation and development. The policy is based on two main pillars: first, the landholder conserving the entire parcel is entitled to compensation, second, the landholder is allowed to develop his/her parcel but only partially since a compensated restriction is imposed.[6]

Our framework is general enough to include different conservation targets such as old-growth forests or habitat surrounding wetlands, marshes, lagoons or by the marine coastline and meet several spatial

[1] In this respect we follow Wunder (2005, p. 3) where a PES is defined as "(i) a voluntary transaction where (ii) a well-defined ES (or a land-use likely to secure that service) (iii) is being "bought" by a (minimum one) ES buyer (iv) from a (minimum one) ES provider (v) if and only if the ES provider secures ES provision (conditionality)".

[2] As reported by Ferraro (2001), this may be due to several reasons such as lack of funding, failures in the institutional design, poor definition and weak enforcement of property rights and strategic behaviour by potential ES providers. See Ferraro (2008) on information failures and Smith and Shogren (2002) on specific contract design issues.

[3] We refer in particular to government-financed programs. On the performance of user vs. government-financed interventions see Pagiola (2008) on PSA program in Costa Rica and Wunder et al. (2008) for a comparative analysis of PES programs in developed and developing countries. See Ferraro and Pattanayak (2006) for a call on empirical monitoring of conservation programs.

[4] Among them it is worth to cite see for instance Conrad (1980), Conrad (1997), Conrad (2000), Clarke and Reed (1989), Reed (1993), Bulte et al. (2002), Leroux et al. (2009).

[5] We share this assumption with most part of the existing literature. See Leroux et al. (2009) for an exception.

[6] As in the reality where conservation is needed also beyond the boundaries of protected areas (Sierra and Russman, 2006).

requirements. For instance, the conservation target may be represented by an area divided into homogenous parcels running along a river or around a lake or a lagoon where to maintain a significant provision of ecosystem services a portion of each parcel must be conserved. In this case the conservation program may be induced by implementing a payment contract schedule differentiating for the state of land i.e. totally conserved vs developed within the restriction enforced through environmental law. However, we are also able to consider the opposite case where the landholder may totally develop his/her plot but an upper limit is fixed on the total extent of land which can be cleared in the region.[7]

We solve for the conversion path taking a real option approach but we depart from previous literature internalizing the role of market entry dynamics. Under competition on the market for agricultural goods, profits from agriculture decrease as land is converted. This dynamic is anticipated by the landholders who consequently have a lower incentive to develop. Hence, we determine the conversion path on the basis of a long-run zero profit condition. In this respect, we study the impact of different conservation policies on the market entry process. Two clear effects emerge. First, by compensating conservation on the undeveloped plot, keeping open the option to convert land has a higher value and conversion is delayed since its cost opportunity is higher. We can show that, as suggested by Ferraro (2001), a landholder may conserve the entire plot even if partially compensated for the provided ES. Second, we find that the existence of a limit on the total developable surface may induce an endogenous conversion run able to exhaust immediately such land stock. In fact, depending on the compensation policy chosen by the Government, landholders may perversely react to the policy by generating a conversion runs as the surface cleared approaches the upper limit.

We show how conversion may be compatible with the conservation target fixed by the Government and that such target can be met by compensating only for a part of foregone development proceeds. We also determine the long-run average expected rate of conversion and we use it to illustrate our findings and the implications that different conservation policies may have in terms of expected conversion duration.[8] We find that when the policy menu offers a higher land unit payment to landholders conserving the whole plot then increasing uncertainty over payments increases the long-run average rate of conversion. Whilst, the opposite effect occurs when the policy, reversing the payment rates, rewards more generously farmers conserving a portion of their plot to fulfil the Government restrictions.

Finally, we complete our analysis providing numerical illustrations based on the conversion dynamic of forested areas in Costa Rica.

The remainder of the paper is organized as follows. In Section 2 the basic set-up for the model is presented. In Section 3 we study the equilibrium in the conversion strategies under two policy scenarios. In Section 4, we discuss issues related to the PES voluntary participation and contract enforceability. Section 5 is devoted to the derivation of the long-run average rate of conversion. In Section 6 we illustrate through numerical exercises our main findings. Section 7 concludes.

2 A Dynamic Model of Land Conversion

Consider a country where at time period $t = 0$ the total land available, L, is allocated as follows:

$$L = A_0 + F \tag{1}$$

where A_0 is the surface cultivated and F is the portion still in its pristine natural state covered by primary forest.[9] Assume that F is divided into small and homogenous parcels of equal extent held by a multitude of

[7] This could be the case for an area covered by a tropical forest (Bulte et al., 2002; Leroux et al., 2009), or a protected area where farmers located next to the site may sustainably extract natural resources (Tisdell, 1995; Wells et al., 1992).

[8] Differently from Leroux et al. (2009) who exogenously assumes a maximum annual conversion rate (2.5% for the case of Costa Rica forests), we calculate it optimally on the basis of land currently converted and information on current and future payments.

[9] As in Bulte et al. (2002) A_0 may represent the best land which has yet been converted to agriculture.

identical risk-neutral agents.[10] By normalizing such extent to 1 hectare, F denotes also the number of agents in the economy.[11] Natural habitats provide valuable environmental goods and services at each time period t.[12] Let $B(t)$ represent the per-parcel value of such goods and services and assume it randomly fluctuates according to the following geometric Brownian motion:

$$dB(t) = \alpha B(t)dt + \sigma B(t)dz(t) \tag{2}$$

where α and σ are respectively the drift and the volatility parameters, and $dz(t)$ is the increment of a Wiener process.[13]

At each t, two competitive and mutually exclusive destinations may be given to forested land: conservation or irreversible development. Once the plot is cleared, the landholder becomes a farmer using land as an input for agricultural production (or commercial forestry).[14]

2.1 The Government

To induce conservation the Government offers to each agent a contract to be accepted on a voluntary basis. A compensation equal to $\eta_1 B(t)$ with $\eta_1 \in [0,1]$ is paid if the entire plot is conserved.[15] On the contrary, if the landholder aims to develop his/her parcel,[16] a restriction is imposed in that a portion of the total surface, $0 \leq \lambda \leq 1$, must be conserved.[17] In this case, a payment equal to $\lambda\eta_2 B(t)$ with $\eta_2 \in [0,1]$ may be offered to compensate the landholder.[18] Since ES have usually public-good nature, payment rates, η_1 and η_2, may be interpreted as the levels of appropriability that the society is willing to guarantee on the value generated by conserving, i.e. $B(t)$ and $\lambda B(t)$ respectively.[19] In addition, besides λ the Government fixes an upper level $\bar{A}$ on total land conversion. This limit may preclude land development for some landholders. The number of landholders for who conserving the entire plot may become compulsory depends on the magnitude of λ. In fact, note that λ may be low enough to allow every landholder to clear land. However, the definition of

[10] At the moment for the sake of generality we refer to landholders. Later we will discuss the implications of our model with respect to property rights issues.

[11] None of our results relies on this assumption. In fact, provided that no single agent has significant market power, we can obtain identical results allowing each agent to own more than one unit of land. See e.g. Baldursson (1998) and Grenadier (2002).

[12] They may include biodiversity conservation, carbon sequestration, watershed control, provision of scenic beauty for recreational activities and ecotourism, timber and non-timber forest products. See e.g. Conrad (1997), Conrad (2000), Clarke and Reed (1989), Reed (1993), Bulte et al. (2002).

[13] Conrad (1997, p. 98) considers a geometric Brownian motion for the amenity value as a plausible assumption to capture uncertainty over individual preferences for amenity. Bulte et al. (2002, p.152) points out that "parameter α can be positive (e.g., reflecting an increasingly important carbon sink function as atmospheric CO2 concentration rises), but it may also be negative (say, due to improvements in combinatorial chemistry that lead to a reduced need for primary genetic material)".

[14] In the following, by "landholder" we will refer to an agent conserving land while by farmer to an agent cultivating it.

[15] As pointed out by Engel et al. (2008), by internalizing external non-market values from conservation PES schemes have attracted increasing interest as mechanism to induce the provision of ES.

[16] Although most of the PES programs in developing countries were introduced as *quid pro quo* for legal restrictions on land clearing, there are no specific contract conditions preventing the landholder from clearing the area enrolled under the program (Pagiola, 2008, p. 717).

[17] Note that our analysis is general enough to include also the case where λ is not imposed but it is endogenously set by each landholder. In fact, due for instance to financial constraints limiting the extent of the development project, the landholders may find optimal not to convert the entire plot.

[18] Note that our contract scheme is in line with Ferraro (2001, p. 997) where the author states that conservation practitioners "may also find that they do not need to make payments for an entire targeted ecosystem to achieve their objectives. They need to include only "just enough" of the ecosystem to make it unlikely, given current economic conditions, infrastructure, and enforcement levels, that anyone would convert the remaining area to other uses". In addition, taking a different perspective, our frame seems supported also by wildlife protection programs which rarely pay farmers more than a fraction of the losses due to wildlife (See Rondeau and Bulte, 2007).

[19] As η_1 and η_2 are constant, also payments follow a geometric Brownian motion (trivially derivable from (2)). However, this is different from the way payments are modelled in Isik and Yang (2004) where they depend also on the fluctuations in the conservation cost opportunity (profit from agriculture, changes in environmental policy, etc.).

λ does not need to meet such requirement since other issues may be prioritized, i.e. habitat fragmentation, ecological critical thresholds, enforcement and transaction costs for the program implementation, etc. Thus, denoting by $\bar{N} = \frac{\bar{A}}{1-\lambda}$ the number of potential farmers involved in the conversion process, we assume $\bar{N} \leq F$.

2.2 The Landholders

Developing the parcel is an irreversible action which has a sunk cost, $(1-\lambda)c$, including cost for clearing and settling land for agriculture.[20] Denoting by $A(t)$ the total land developed at time t, the number of farmers must be equal to $N(t) = \frac{A(t)}{1-\lambda}$ and since $1-\lambda$ is fixed, the conversion dynamic must mirror the variation in the number of farmers, i.e. $dN(t) = \frac{dA(t)}{1-\lambda}$. Therefore, assuming that the extent of each plot is small enough to exclude any potential price-making consideration, we may use either $N(t)$ or $A(t)$ when evaluating the individual decision process.[21] Competition on the market for agricultural products implies that at each time period t the optimal number of farmers (or the optimal total land developed) is determined by the entry zero profit condition. In addition, since the per-parcel value of services, $B(t)$, makes symmetric all agents, some random mechanism must be used to select which landholder develops first.

We assume a constant elasticity demand function for agricultural products. Since supply depends on the surface cultivated then let demand be specified as $P_A(t) = \delta A(t)^{-\gamma}$ with $A(0) = A_0 (> 1)$, and where δ is a parameter illustrating different positions of the demand and $-\gamma$ is the inverse of the demand elasticity.

Now, let solve for the conversion process taking η_1, η_2 and λ as exogenously given parameters. Denoting by $P_A(t)$ the marginal return as land is cleared over time, the farmer instantaneous profit function is then given by:

$$\pi(A(t), B(t); \bar{A}) = (1-\lambda)P_A(t) + \lambda\eta_2 B(t) \tag{3}$$

The discounted present value of the net benefits over an infinite horizon is:[22]

$$\begin{aligned} E_0 \left[\int_0^T \eta_1 B(t)e^{-rt}dt \; + \int_T^\infty \pi(A(t), B(t); \bar{A})e^{-rt}dt \;\mid B(0) = B \right] &= \\ = \frac{\eta_1 B}{r-\alpha} + E_0 \left[\int_T^\infty \Delta\pi(A(t), B(t); \bar{A})e^{-r(t-T)}dt \right] \end{aligned} \tag{4}$$

where r is the constant risk free interest rate,[23] $\Delta\pi(A(t),B(t); \bar{A})=(1-\lambda)P_A(t) + (\lambda\eta_2 - \eta_1)B(t)$ and T is the stochastic conversion time.[24]

In (4) the first term represents the perpetuity paid by the Government if the parcel is conserved forever, while the second term represents the extra profit that each landholder may expect if s/he clears the land and becomes a farmer. The extra profit is given by the crop yield sold on the market plus the difference in the payments received by the Government. As soon as the excess profit from land development equals the deforestation cost the landholder may clear the parcel. This implies that the optimal conversion timing depends only on the second term in (4).

[20] Bulte et al., (2002, p. 152) defines c as "the marginal land conversion cost". It "may be negative if there is a positive one-time net benefit from logging the site that exceeds the costs of preparing the harvested site for crop production". We also assume, without loss of generality, that the conversion cost is proportional to the surface cleared.

[21] To consider infinitesimally small agents is a standard assumption in infinite horizon models investigating dynamic industry equilibrium under competition. See for instance Jovanovic (1982), Dixit (1989), Hopenhayn (1992), Lambson (1992), Dixit and Pindyck (1994, chp. 8), Bartolini (1993), Caballero and Pindyck (1996), Dosi and Moretto (1992) and Moretto (2008).

[22] See Harrison (1985, p. 44).

[23] The introduction of risk aversion does not change the results since the analysis can be developed under a risk-neutral probability measure for $B(t)$. See Cox and Ross (1976) for further details.

[24] Note that the expected value is taken accounting for $A(t)$ increasing over time as land is cleared.

3 The Equilibrium

Denote by $V(A(t),B(t);\bar{A}\ )$ the value function of a farmer.[25] By (4), the optimal conversion time, τ, solves the following maximization problem:

$$V(A(t),B(t);\bar{A}) = \max_{\tau} E_0 \left[\int_0^{\infty} \Delta\pi(A(t),B(t);\bar{A})e^{-rt}dt - I_{[t=\tau]}(1-\lambda)c \right] \tag{5}$$

where $I_{[t=\tau]}$ is an indicator function and the expectation is taken considering that the total land developed $A(t)$ (i.e. the number of farmers $N(t)$), may vary as time rolls over. The indicator function states that, due to competition among farmers on the market, at the time of conversion the value from converting land must equal the cost of land clearing. In the real option literature the problem we must solve is referred to as "optimal stopping" (Dixit and Pindyck, 1994). The idea is that at any point in time the value of immediate investment (stopping) is compared with the expected value of waiting dt (continuation), given the information available at that point in time (the value of the stochastic variable $B(t)$ and the stock of land developed, $A(t)$) and the knowledge of the two processes. If the initial size of the active farmers is $A \geq A_0$, we expect that the converting process works as follows: for a fixed number of farmers, profits in (3) move stochastically driven by $B(t)$. As soon as the per-parcel value of ES reaches a critical level, says B^C, development (i.e. entry in the agricultural market) becomes feasible. This implies an increase, $dA(t)$, in cultivated land and a drop in revenues from agriculture along the function $P_A(t)$. The value of services will then continue to move stochastically until the next entry occurs.

In this setting the (competitive) equilibrium bounding the profit process for each farmer can be constructed as a symmetric Nash equilibrium in entry strategies and we can determine it by simply analysing the single farmer clearing policy which is defined regardless of future entry decisions (see e.g. Leahy 1993; Bartolini 1993 and Dixit and Pindyck, 1994). In fact, consider a short interval dt where any conversion takes place. Over this interval $A(t)$ is constant and each farmer holds an asset paying $\Delta\pi(A,B(t);\bar{A})dt$ as cash flow and $E[dV(A,B(t);\bar{A}\ )]$ as capital gain. If the farmer is active then the cash flow and the expected capital gain must equal the normal return, that is $rV(A,B(t);\bar{A})]dt = \Delta\pi(A,B(t);\bar{A})dt + E[dV(A,B(t);\bar{A})]$.

Let $V(A,B(t);\bar{A}\ )$ be twice-differentiable in $B(t)$,[26] and expand $dV(A,B(t);\bar{A}\ )$ using Ito's Lemma. Then, in the region where non new conversion takes place (i.e. for $B(t) \neq B^C$), the solution to (5) must solve the following differential equation:

$$\begin{aligned} &\frac{1}{2}\sigma^2 B^2 V_{BB}(A,B;\bar{A}) + \alpha B V_B(A,B;\bar{A}) - rV(A,B;\bar{A}) + \\ &\quad + \left[(1-\lambda)\delta A^{-\gamma} + (\lambda\eta_2 - \eta_1)B\right] = 0 \end{aligned} \tag{6}$$

This is an ordinary differential equation since the number of farmers is constant. Using standard arguments the general solution is (see Dixit and Pindyck, 1994):

$$V(A,B;\bar{A}) = Z_1(A)B^{\beta_1} + Z_2(A)B^{\beta_2} + (1-\lambda)\frac{\delta A^{-\gamma}}{r} + (\lambda\eta_2 - \eta_1)\frac{B}{r-\alpha} \tag{7}$$

where $1 < \beta_1 < r/\alpha$, $\beta_2 < 0$ are the roots of the characteristic equation $\Psi(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r = 0$ and Z_1, Z_2 are two constants to be determined.

[25] As we show in the appendix the problem can be equivalently solved considering a landholder evaluating the option to develop.

[26] In the following we will drop the time subscript for notational convenience.

3.1 Case with $\eta_1 > \lambda\eta_2$

Suppose that a lower payment is offered for conservation once land is converted, i.e. $\eta_1 > \lambda\eta_2$.[27] To determine the optimal conversion threshold, $B^C(A) = B^*(A)$, the landholder must consider benefits and costs attached to conversion. According to (7), the profit accruing from the crop yield, $(1-\lambda)\frac{\delta A^{-\gamma}}{r}$, is counterbalanced by the difference in the payments, $(\lambda\eta_2 - \eta_1)\frac{B}{r-\alpha}$, received for conservation. In addition, note that as landholders convert land and become farmers profit from agriculture decreases. This negative effect on the value of converted land is accounted in (7) by the second term ($Z_2(A) \leq 0$ for $A \leq \bar{A}$). In fact, since $\eta_1 > \lambda\eta_2$ then only an expected reduction in B can induce conversion. This implies that to keep $V(A,B;\bar{A}\ )$ finite we must drop the first term by setting $Z_1 = 0$, i.e. $\lim_{B\to\infty} V(A,B;\bar{A}\ ) = 0$. Hence, (7) reduces to:

$$V(A,B;\bar{A}) = Z_2(A)B^{\beta_2} + (1-\lambda)\frac{\delta A^{-\gamma}}{r} + (\lambda\eta_2 - \eta_1)\frac{B}{r-\alpha} \tag{8}$$

To determine $Z_2(A)$ and $B^*(A)$ some suitable boundary conditions on (8) are required. First, development by increasing the number of competing farmers in the market keeps the value of being an active farmer below $(1-\lambda)c$. Second, since the agent's size is infinitesimal, then the trigger $B^*(A)$ must be a decreasing function of A. These considerations can be formalized by the following proposition.

Proposition 1 *Provided that each agent rationally forecasts the future dynamics of the market for agricultural goods for land to be converted the following condition must hold*

$$V(A, B^*(A); \bar{A}) = (1-\lambda)c \tag{9}$$

where

(i) if $\hat{A} \leq \bar{A}$ then

$$B^*(A) = \frac{\beta_2}{\beta_2 - 1}(r-\alpha)\frac{1-\lambda}{\eta_1 - \lambda\eta_2}\left[(\frac{\hat{A}}{A})^\gamma - 1\right]c \quad \text{for } A_0 < A \leq \hat{A} \tag{10}$$

(ii) if $\hat{A} > \bar{A}$ then

$$B^*(A) = \begin{cases} \frac{\beta_2}{\beta_2-1}(r-\alpha)\frac{1-\lambda}{\eta_1-\lambda\eta_2}\left[(\frac{\hat{A}}{A})^\gamma - 1\right]c, & \text{for } A_0 < A \leq A^* \quad \text{(a)} \\ (r-\alpha)\frac{1-\lambda}{\eta_1-\lambda\eta_2}\left[(\frac{\hat{A}}{A})^\gamma - 1\right]c, & \text{for } A^* < A \leq \bar{A} \quad \text{(b)} \end{cases} \tag{10 bis}$$

with $\hat{A} = (\frac{\delta}{rc})^{1/\gamma}$ and $A^* = [\frac{(\beta_2-1)\bar{A}^{-\gamma} + \hat{A}^{-\gamma}}{\beta_2}]^{-\frac{1}{\gamma}}$.

Proof. See appendix A.1. ■

For conversion to be optimal, the dynamic zero profit condition in (9) must hold at the threshold, $B^*(A)$. Let analyse such condition by rearranging (9) as follows:

$$Z_2(A)B^*(A)^{\beta_2} + (1-\lambda)\frac{\delta A^{-\gamma}}{r} + \lambda\eta_2\frac{B^*(A)}{r-\alpha} = (1-\lambda)c + \eta_1\frac{B^*(A)}{r-\alpha}$$

This means that benefits from clearing land and become a farmer must match the cost opportunity of conversion, i.e. the cost for clearing and settling land plus the payment perpetuity which is implicitly given up converting.

[27] Note that this may occur even if $\eta_1 < \eta_2$, i.e. the payment rate for unit of land conserved is more generous when a portion of the plot has been developed. We will discuss more deeply this case in the next section.

By Proposition 1 the whole conversion dynamics is characterized in terms of B. In both figure 1 and 2 in the region below the curve, $B^*(A)$, conversion is optimal. As B crosses $B^*(A)$ from above, a discrete mass of landholders will enter in the agricultural market developing (part of) their land. Since this reduces the profits accruing from agriculture, entries take place until the threshold curve $B^*(A)$ is reached. In the region above the curve conservation is optimal since B is not low enough to trigger conversion. Each landholder conserves up to the time where B driven by (2) fall down to $B^*(A)$. Here, again a mass of landholders enters the market. Individual farmer profit lowers and implicitly prevents B from crossing $B^*(A)$.

Depending on $\bar{A}$, we obtain two different scenarios (see figure 1 and 2):

(1) if $\hat{A} \leq \bar{A}$, the conversion process stops at $\hat{A}$ since this is the last parcel for which conversion makes economic sense ($\frac{\delta}{r}\hat{A}^{-\gamma} - c = 0$). This in turn implies that the surface, $\bar{A} - \hat{A} \geq 0$, is conserved forever at a total cost equal to $\eta_1 \frac{B}{r-\alpha}(\bar{A} - \hat{A})$.

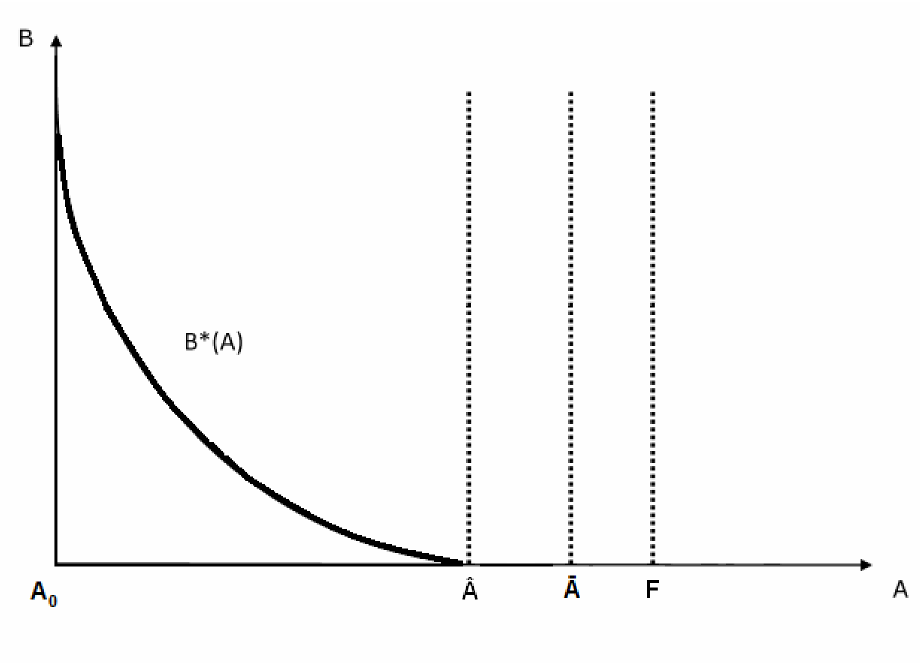

Figure 1: *Optimal conversion threshold with* $\eta_1 > \lambda\eta_2$ *and* $\hat{A} \leq \bar{A}$

(2) if $\hat{A} > \bar{A}$, land is converted smoothly up to A^* following the curve (10 bis (a)). If the surface of cultivated land falls in the interval $A^* \leq A \leq \bar{A}$, when B hits the threshold $B^*(A)$, the landholders start a run for conversion up to $\bar{A}$. Differently from the previous case, here the limit imposed by the Government binds and restricts conversion on a surface, $\bar{A} - \hat{A} > 0$, where development would be profitable from the landholder's viewpoint. The intuition behind this result is immediate if we take a backward perspective. When the limit imposed by the Government $\bar{A}$ is reached then it must be $Z_2(\bar{A}) = 0$ since no market entry may occur. Hence, condition (9) reduces to $V(\bar{A}, B^*(\bar{A}); \bar{A}) = (1-\lambda)\frac{\delta \bar{A}^{-\gamma}}{r} + (\lambda\eta_2 - \eta_1)\frac{B^*(\bar{A})}{r-\alpha} = (1-\lambda)c$ from which we obtain (10bis (b)) as optimal trigger. This implies that at $\bar{A}$ marginal rents induced by future reduction in B are not null, i.e. $V_B(\bar{A}, B; \bar{A}) < 0$, and they would be entirely captured by market incumbents. Since each single landholder realizes the benefit from marginally anticipating its entry decision then an entry run occurs to avoid the restriction imposed by the Government. However, by rushing, the rent attached to information on the market profitability, collectable by waiting, vanishes.

Therefore there will be a land extent (i.e. a number of farmers), $A^* < \bar{A}$, such that for $A < A^*$ no landholder finds convenient rushing since the marginal advantage from a future reduction in B are lower than the option value lost.[28] Note also that, as A^* is given by $B^*(A^*) = B^*(\bar{A})$, the threshold that triggers the run is equal to the traditional NPV break-even rule (see Appendix A.1).[29]

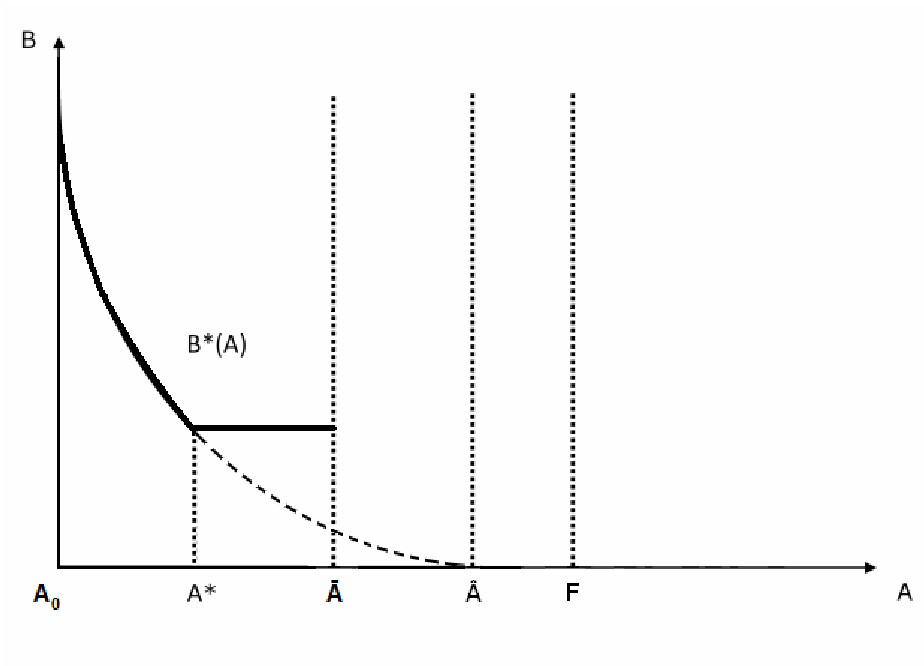

Figure 2: *Optimal conversion threshold with* $\eta_1 > \lambda\eta_2$ *and* $\hat{A} > \bar{A}$

As shown in table 1 the definition of the last plot, $\hat{A}$, which is worth to convert depends on parameters regulating the demand for agricultural goods, the interest rate and the land unit conversion cost. A higher δ illustrating a higher demand for agricultural products and/or a more rigid demand moves $\hat{A}$ forward since higher profits support the conversion for a larger land surface. Similarly, as $c \to 0$, all the available land will be cultivated ($\hat{A} \to \bar{A}$).[30] With a higher r future returns from agriculture becomes relatively lower with respect to the cost of clearing land and land conversion is less attracting.

δ	c	r	γ
>0	<0	<0	<0

Table 1: *Comparative statics on* $\hat{A}$

Considering the impact that changes in the exogenous parameters have on the definition of $B^*(A)$ we note that a higher δ induces the barrier, $B^*(A)$, to shift upward due to higher profits from agriculture. This in

[28] This means the A^*th is the last landholder for who $V_B(A^*, B^*(A^*); \bar{A}) = 0$.

[29] In Bartolini (1993) a similar result is obtained. The author studies decentralized investment decision in a market where a limit on aggregate investment is present. Under linear adjustment costs and stochastic returns, investment cost is constant up to the investment limit where it becomes infinite. As a reaction to this external effect recurrent runs may occur under competition as aggregate investment approaches the ceiling. See also Moretto (2008).

[30] Note that by setting $\eta_1 = 1$, $\eta_2 = 0$ and $\lambda = 0$ then (10) collapses into the conversion strategy of the social planner (Bulte et al., 2008). That is, a competitive equilibrium evolves as maximizing solution for the expected present value of social welfare in the form of consumer surplus (Lucas and Prescott, 1971; Dixit and Pindyck, 1994, ch.9).

turn implies that the region where conversion is optimal enlarges. The same effect may also be produced by a relatively more inelastic demand. On the contrary, we get the opposite effect as c increase since a higher conversion cost discourages conversion.

δ	c	r	γ	
≥ 0	≤ 0	≤ 0	≤ 0	

α	σ^2	η_1	η_2	λ
≤ 0	≤ 0	≤ 0	≥ 0	$< 0_{(\frac{\eta_1}{\eta_2}>1)}$ $\geq 0_{(\frac{\eta_1}{\eta_2}>1)}$

Table 2: *Comparative statics on* $B^*(A)$

With an increase in the interest rate the exercise of the option to convert should be anticipated but this effect is too weak to prevail over the effect that a higher r has on the cost opportunity of conversion. Studying the effect of volatility, σ, and of growth parameter, α, the sign of the derivatives is in line with the standard insight in the real options literature. An increase in the growth rate and volatility of B determines a postponed exercise of the option to convert. This can be explained by the need to reduce the regret from taking an irreversible decision under uncertainty. Since the cost of this decision is growing at a faster rate and there is uncertainty about its magnitude waiting to collect information about future prospects is a sensible strategy.

As expected an increase in η_1 pushes the barrier downward since it makes more profitable to conserve the plot and keep open the option to convert. In line with this result, the barrier responds in the opposite way to an increase in η_2 which implicitly provides an incentive to conversion. Changes in λ have a non-monotonic effect on the barrier which depends on the ratio between the two payment rates. A higher λ defines a stricter requirement on development that may push the barrier downward for two reasons. First, a lower return from agriculture since less land is cultivated which is however balanced by a lower cost for clearing land, and second, as $\frac{\eta_1}{\eta_2} > 1$ a higher payment on the marginal unit which the farmer is required to set aside is guaranteed if the plot is totally conserved. The case where $\frac{\eta_1}{\eta_2} \leq 1$ and the barrier shifts upward can be easily explained inverting the second argument.

These considerations mostly hold for both (10) and (10 bis). Clearly, over the interval $A^* < A \leq \bar{A}$ since the option multiple, $\frac{\beta_2}{\beta_2-1}$, drops out, the barrier $B^*(A)$ is not affected by σ. The derivative with respect to the benefit drift α maintains the sign in table 2 while the comparative statics on r reveals:

$$\frac{\partial B^*(A)}{\partial r} = \begin{cases} > 0 \text{ for } r < \alpha(\frac{\hat{A}}{\bar{A}})^\gamma \\ \leq 0 \text{ for } r \geq \alpha(\frac{\hat{A}}{\bar{A}})^\gamma \end{cases} \quad \text{for } A^* < A \leq \bar{A}$$

Finally, since by (10bis) the same level of B triggers the entry of a positive mass of landholders, i.e. $B^*(A^*) = B^*(\bar{A})$, it is worth to highlight that the surface triggering a conversion rush its independent on the definition of η_1, η_2 and λ. The Government policy may either speed up or slown down the conversion dynamic but it cannot alter A^* which depends only on the choice of $\bar{A}$ with respect to $\hat{A}$. Note that $\frac{\partial A^*}{\partial \bar{A}} > 0$ which reasonably means that as $\bar{A} \to \hat{A}$ the run would be triggered only by relatively a lower level for B. In other words, since in expected terms a higher $\bar{A}$ implies a less strict threat of being regulated then landholders are less willing to give up information rents collectable by waiting. Not surprisingly, $\frac{\partial A^*}{\partial \hat{A}} < 0$. A lower $\hat{A}$ implies a faster fall in the profit from agriculture as A increases and then a lower incentive for the conversion run.

3.2 Case with $\eta_1 < \lambda\eta_2$

Now, assume that $\eta_1 < \lambda\eta_2$. In this case it must necessarily be $\eta_2 > \eta_1$, that is, the payment rate for unit of land conserved is more generous when a portion of the plot has been developed. This could be the case for a Government that, having run out of funding for the conservation program, may be willing to sacrifice some pristine habitat to finance more generously conservation on a smaller extent.[31] Differently, this choice may also be reasonably explained thinking of a Government wishing to indirectly induce, offering a more favourable rate, a switch toward a certain agricultural or forestal practises. For instance, the Government may choose to favour timber harvest and successive reforestation as land-use to cash funding on carbon markets and finance conservation on the remaining habitat.[32] In this case, our model permits to frame competition between these two "green" but however mutually exclusive land destinations, i.e. secondary forests vs. primary forests. Finally, the Government could simply consider fair and/or politically convenient to reward better conservation as soon as the restriction on development is binding and the real conservation cost opportunity is implicitly revealed.

As in the previous section, the optimal conversion threshold, $B^C(A) = B^{**}(A)$, must be determined by matching benefits and costs from conversion. Differently from the previous case, when developing land in addition to the profit accruing from agriculture, $(1-\lambda)\frac{\delta A^{-\gamma}}{r}$, the landholder can earn a higher payment for ES provision since $(\lambda\eta_2 - \eta_1)\frac{B}{r-\alpha} > 0$. Hence, it makes sense to clear land as B increases. However, as above market competition has a negative effect on the value from farming which being entry free lies below $(1-\lambda)c$. This effect is accounted by the first term ($Z_1(A) \leq 0$ for $A \leq \bar{A}$) in (7) since as $\lim_{B\to 0} V(A, B; \bar{A}) = 0$ then to keep $V(A, B; \bar{A})$ finite we must set $Z_2 = 0$. It follows that (7) reduces to:

$$V(A, B; \bar{A}) = Z_1(A)B^{\beta_1} + (1-\lambda)\frac{\delta}{r}A^{-\gamma} + (\lambda\eta_2 - \eta_1)\frac{B}{r-\alpha} \tag{11}$$

As in the previous case, we determine $Z_1(A)$ and $B^{**}(A)$ by imposing the free entry condition. That is,

Proposition 2 *Provided that each agent rationally forecasts the future dynamics of the market for agricultural goods for land to be converted the following condition must hold*

$$V(A, B^{**}(A); \bar{A}) = (1-\lambda)c \tag{12}$$

where

(i) if $\hat{A} \leq \bar{A}$ then

$$B^{**}(A) = \begin{cases} 0, \quad \text{for } A_0 < A \leq \hat{A} \quad \text{(a)} \\ \frac{\beta_1}{\beta_1 - 1}(r-\alpha)\frac{1-\lambda}{\lambda\eta_2 - \eta_1}\left[1 - (\frac{\hat{A}}{A})^{\gamma}\right]c, \quad \text{for } \hat{A} < A \leq A^{**} \quad \text{(b)} \\ (r-\alpha)\frac{1-\lambda}{\lambda\eta_2 - \eta_1}\left[1 - (\frac{\hat{A}}{\bar{A}})^{\gamma}\right]c, \quad \text{for } A^{**} < A \leq \bar{A}, \quad \text{(c)} \end{cases} \tag{13}$$

(ii) if $\hat{A} > \bar{A}$ then

$$B^{**}(A) = 0, \quad \text{for } A_0 < A \leq \bar{A} \tag{13 bis}$$

[31] Note that this scenario does not exclude to push toward a form of development which is perceived as environmental friendly.

[32] Under the CDM (Clean Development Mechanism) of the Kyoto Protocol forest conservation/avoided deforestation efforts were not considered in the first commitment period (2008-2012) (IPCC, 2007). On the contrary, through the CDM investment in tree planting projects has been undertaken (Santilli et al., 2005; van Vliet, 2003). In our model, this would imply a η_1 lower than η_2 in relative terms. Only recently, at the December 2009 United Nations Framework Convention on Climate Change (UNFCCC) meeting in Copenhagen this controversial issue has been discussed and finally forest conservation should now be allowed to qualify (Phelps et al., 2010). See also Fargione et al. (2008) on land clearing and biofuel carbon debt. On palm oil trees vs. primary forests see Butler et al., (2009), Fitzherber et al., (2008) and Koh and Ghazoul (2008).

with $\hat{A} = (\frac{\delta}{rc})^{1/\gamma}$ and $A^{**} = [\frac{(\beta_1 - 1)\bar{A}^{-\gamma} + \hat{A}^{-\gamma}}{\beta_1}]^{-\frac{1}{\gamma}}$.

Proof. See appendix A.3. ■

Equation (12) defines the dynamic zero profit condition which must hold at $B^{**}(A)$. Proposition 2 illustrates the conversion dynamic as B fluctuates according to (2). Here, differently from the previous case the threshold $B^{**}(A)$ is an increasing continuous function of A and the conversion region is above the barrier. Development is worth only if B crosses $B^{**}(A)$ from below. In the conservation region the landholder conserves as B is not high enough to trigger conversion and s/he waits until the stochastic process B moves up to $B^{**}(A)$. At that point, a mass of landholders enters the market keeping profits low enough to push upward the barrier.

Also in this case, depending on the value of $\bar{A}$, two different scenarios emerge (see figure 3 and 4). That is,

(1) if $\hat{A} \leq \bar{A}$ then

- a surface equal to $\hat{A}$ is converted independently from the value taken by B. In the interval $A_0 < A \leq \hat{A}$ landholders rush as agricultural profits are so high that there is no reason for waiting. Moreover, they know that no matter which value B takes they are paid more for conserving less since $\lambda\eta_2 > \eta_1$;
- once the surface $\hat{A}$ has been converted, landholders convert smoothly up to A^{**} according to (13b). Note that over the interval $\hat{A} < A \leq A^{**}$ as land is converted an increasing B is required to trigger conversion. This is due to the fact that profit from agriculture does not cover the cost of clearing and settling land for cultivation. Hence, landholders convert only if the payment for conservation is high enough to cover the gap.
- if B is high enough to support conversion up to A^{**} then a run activate and the remaining land is cleared up to the upper limit. This dynamic is due to having fixed an upper limit, $\bar{A}$, for development. In fact, since for $A > \bar{A}$ the value attached to conversion vanishes then to cash it each landholder must run to anticipate the others. However, as in the previous case, the run dissipates the rent that the landholders earn by postponing conversion and collecting information on the market profitability, i.e. at $B^{**}(A^{**}) = B^{**}(\bar{A})$, $(1-\lambda)\frac{\delta}{r}\bar{A}^{-\gamma} + (\lambda\eta_2 - \eta_1)\frac{B^{**}(\bar{A})}{r-\alpha} = (1-\lambda)c$. Note that also in this case the occurence of a run is due to the external effect ($V_B(\bar{A}, B; \bar{A}) > 0$) induced by the presence of a ceiling

on land development.

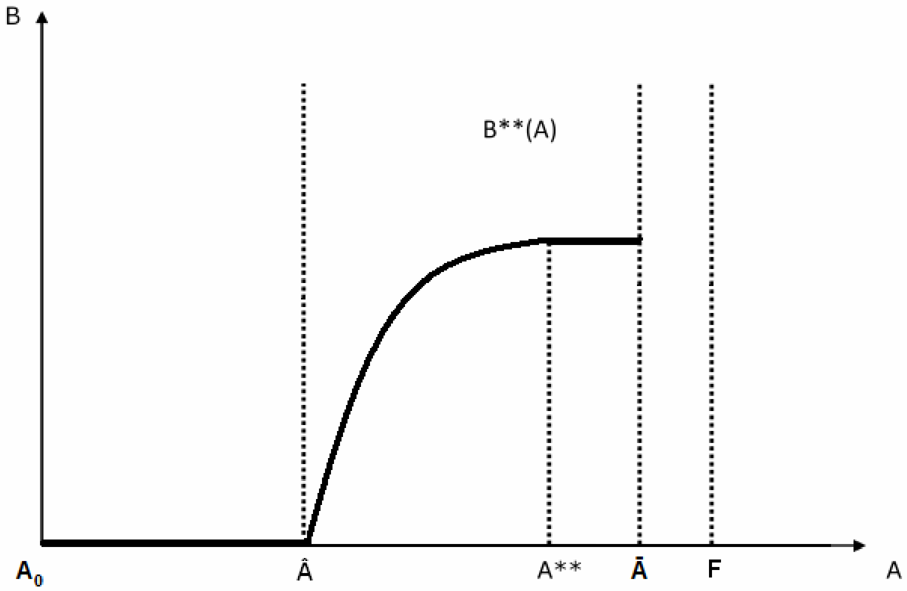

Figure 3: *Optimal conversion threshold with* $\eta_1 < \lambda\eta_2$ *and* $\hat{A} \leq \bar{A}$

(2) If $\hat{A} > \bar{A}$ a surface equal to $\bar{A}$ is converted for any B. As above, in the interval $A_0 < A \leq \bar{A}$, landholders rush for two reasons, namely high agricultural profits and a more generous transfer to compensate conservation. Also in this case the limit, $\bar{A}$, restricts profitable land conversion over the surface $\hat{A} - \bar{A} > 0$ where land would be converted for any B even without a conservation payment.

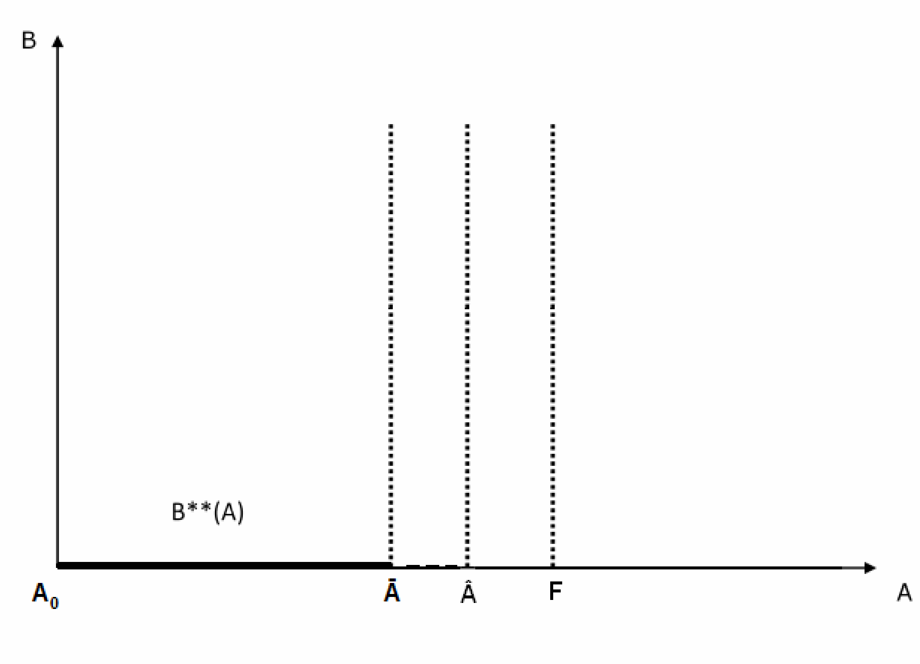

Figure 4: *Optimal conversion threshold with $\eta_1 < \lambda\eta_2$ and $\hat{A} > \bar{A}$*

The analysis of the case where $\hat{A} > \bar{A}$ is trivial since $B^{**}(A) = 0$. Let then focus on the case $\hat{A} \leq \bar{A}$. In (13) over the interval $A_0 < A \leq \hat{A}$, $B^{**}(A) = 0$ and then the only interesting comparative statics are the ones regarding $\hat{A}$ provided in table 1 and previously discussed. For $\hat{A} < A \leq \bar{A}$ the discussion provided in section 3.1 applies as the two barriers $B^{*}(A)$ and $B^{**}(A)$ are symmetric and react to changes in the parameters in the opposite way. However, this implies that the considerations above on the impact of a change in the parameters are still valid.

δ	c	r	γ	
≤ 0	≥ 0	≥ 0	≥ 0	

α	σ^2	η_1	η_2	λ
≥ 0	≥ 0	≥ 0	≤ 0	< 0

Table 3: *Comparative statics on $B^{**}(A)$*

An increase in λ has a monotonic downward shifting effect on the barrier.[33] This makes sense since for any level of B the payment rate on the additional marginal land unit to be set aside is higher if land is developed ($\eta_2 > \eta_1$). We complete the analysis of (13) studying the barrier $B^{**}(A)$ for $A^{**} < A \leq \bar{A}$. Since the option value multiple, $\frac{\beta_1}{\beta_1 - 1}$, drops out the barrier is not affected by σ. We note also that differently from the results in table 3, $\frac{\partial B^{**}(A)}{\partial \alpha} \leq 0$ and:

$$\frac{\partial B^{**}(A)}{\partial r} = \begin{cases} \geq 0 \text{ for } r \geq \alpha(\frac{\hat{A}}{A})^{\gamma} \\ < 0 \text{ for } r < \alpha(\frac{\hat{A}}{A})^{\gamma} \end{cases} \quad \text{for } \; A^{**} < A \leq \bar{A}$$

[33] Note that $\frac{\partial B^{**}(A)}{\partial \lambda} = \frac{\eta_1 - \eta_2}{\eta_2 \lambda - \eta_1} B^{**}(A) < 0$ since for $\eta_2 \lambda > \eta_1$ it must be $\eta_2 > \eta_1$.

Finally, also in this case, the policy parameters η_1, η_2 and λ are neutral in the definition of A^{**} in (13 bis). This level depends only on $\hat{A}$ and the ceiling $\bar{A}$. We find that $\frac{\partial A^{**}}{\partial \bar{A}} > 0$ and $\frac{\partial A^{**}}{\partial \hat{A}} > 0$. If the limit $\bar{A}$ is less strict the landholders are less willing to dissipate information rents and participate to the run only for high level of B. A lower $\hat{A}$ implies a faster fall in the profit from agriculture as A increases and then a higher incentive for developing land as soon as B is high enough. Since this consideration is anticipated by all landholder the run start at a lower A^{**}.

4 Voluntary participation or contract enforceability?

A peculiar feature of the PES programs is represented by the acceptance of the conservation contract on a voluntary basis (Wunder, 2005). In this section we present under which conditions such property holds in our frame. In this respect, two elements must be considered. First, the dynamic of the whole conversion process involving all the landholders who enrolled under conservation program. Second, the restrictions on land development that the Government may wish to impose.

Focusing on the second element, it is conceivable that the Government may find desirable that the landholder develops only partially his/her plot i.e. $0 < \lambda \leq 1$. On the contrary, from (9) and (12) it results that the landholder may consider profitable to develop the entire plot, i.e. $\lambda = 0$. Therefore, the conservation contract may be accepted on a voluntary basis only if each landholder is better of signing it than not. As one can easily see the acceptance will depend on the expectation about the ability of the Government in imposing a $\lambda > 0$. Let formalize this consideration. Since by propositions 1 and 2 the conversion is optimal at $B^C(A)$ with $C =^*,^{**}$, then to sign the contract it must be:

$$\frac{\eta_1}{r-\alpha}B^C(A) + V(A, B^C(A); \bar{A}) \geq E_t\left[\int_t^\infty e^{-r(s-t)}(1-\theta\lambda)\delta A(s)^{-\gamma}ds\right] \quad \text{for } C =^*,^{**} \tag{14}$$

where $\theta \in [0,1]$ is the probability of regulation, i.e. the restriction λ holds also for landholders not signing the contract. In (14) the LHS describes the position of a landholder within the program while on the RHS we have the expected present value for a landholder not accepting the contract and developing land at time t. Note that in the last case the conversion option is exercised as soon as the expected cost of conversion, $(1-\theta\lambda)c$, equals the expected benefit from conversion. Rearranging (14) yields:

$$\frac{\eta_1}{r-\alpha}B^C(A) + (1-\lambda)c \geq (1-\theta\lambda)c \tag{15}$$

which holds if

$$\eta_1 B^C(A) - \lambda(1-\theta)(r-\alpha)c \geq 0 \quad \text{for } C =^*,^{**}$$

where $(r-\alpha)c$ is the annualized conversion cost. Depending on the parameters this condition may not hold for some A. Note in fact that since $B^*(A)$ is a decreasing function of A, while $B^{**}(A)$ is increasing, (15) implies that:

Proposition 3 *If $\theta \in [0,1)$ then contract acceptance can be voluntary for some but not all the landholders in the conservation program.*

Proof. Straightforward from propositions 1 and 2. ■

Segerson and Miceli (1998) shows that an agreement can always signed on a voluntary basis if the probability of future regulation is positive. By Proposition 3 we show that this result does not hold in our frame. In fact, uncertainty about future regulation does not allow capturing all the agents who can be potentially regulated. A similar result is obtained by Langpap and Wu (2004) in a regulator-landowner two-period model for conservation decisions under uncertainty and irreversibility. In their paper, since contract pay-offs are uncertain and signing is an irreversible decision then under certain conditions a landholder may not accept it to stay flexible. Differently from them, we find that the under the same threat of regulation a contract can be voluntary signed by some landholders and not by others. This outcome is due to the competition on the market for agriculture products which raises the cost opportunity of conversion as land is progressively cleared. Note that in this respect the scenario $\eta_1 < \lambda\eta_2$ is the most problematic. In fact, under uncertain regulation in this case switching to competitive farming is so convenient for the first landholders entering the market that they never accept the contract. A mass of landholders will run to convert their entire plots. This implies that (1) should be restated as $L' = A_0' + F$ with $A_0' > A_0$ since by (15) only a lower number of landholder may enter the program on a voluntary basis.

5 The long-run average rate of forest conversion

In line with Ferraro (2001, p. 997) we have shown that even not entirely compensating for the ES provided by a targeted ecosystem ($\eta_1 \leq 1$) we may be able to induce landholders to conserve their plot. However, we believe that this result address only "statically" the conservation/development dilemma. Hence, in this section we aim to study the temporal implications of the optimal conversion policy, i.e. how long it takes to clear the target surface $\bar{A}$, and the impact of increasing uncertainty about future environmental benefits, B, and conversion cost, c, on conversion speed under the two policy scenarios characterized above. To do it we derive through a robust linear approximation the long-run average growth rate of forest conversion (see Appendix A.4 and A.5).

5.1 Case with $\eta_1 > \lambda\eta_2$

Let consider the case where $\widehat{A} \leq \bar{A}$. This represents the case of more interest since the analysis below is still valid also for the opposite case over the range $A < A^*$. Note in fact that for $A \geq A^*$ the long-run rate of reforestation must trivially tend to infinity due to the conversion run.

Let now focus our attention on the long-run average growth rate of forest conversion. Rearranging (10) yields:

$$\xi = \frac{\beta_2}{\beta_2 - 1}(1-\lambda)\frac{P_A(A)}{r} - \frac{\eta_1 - \lambda\eta_2}{r - \alpha}B \quad \text{for } \xi < \hat{\xi} \tag{16}$$

where $\hat{\xi} = \frac{\beta_2}{\beta_2 - 1}(1-\lambda)c$.

The first term on the RHS of (16) represents the expected discounted profit from the cultivation of land conditional on the number of farmers remaining constant. The multiple $\frac{\beta_2}{\beta_2-1} < 1$ accounts for the presence of uncertainty and irreversibility. The second term is the expected discounted flow of payments implicitly given up developing land net of the payments for conservation paid for setting aside λ as required by the Government. Note that ξ can be defined as a regulated process in the sense of Harrison (1985, chp. 2) with $\hat{\xi}$ as upper reflecting barrier. This implies that when a reduction of B drives ξ upward toward $\hat{\xi}$ some landholders find profitable to convert land. New entries in the market, however, determines a drop along $P_A(A)$ which balancing for the effect of B prevents ξ from rising above $\hat{\xi}$. On the contrary, if $\xi < \hat{\xi}$ no entries occur and consequently the deforestation rate is null. In this case in fact, the level of B is still high enough to support conservation. Our aim is then, given a generic level of B, to define a steady-state (long-run) distribution for A and determine the long-run average growth rate of forest conversion. Since A and B enter additively on (16) some manipulation is required to apply the well-known properties of log-normal distribution to show that ξ is log-normally distributed. Denoting by $\frac{1}{dt}E(d\ln A)$ the measure of the average growth rate of forest conversion, in the appendix we prove that:

Proposition 4 *When $\eta_1 > \lambda\eta_2$ the average or expected long-run growth rate of deforestation for a generic initial point $(\tilde{B}, \tilde{A})$ can be approximated by:*

$$\frac{1}{dt}E\left[d\ln A\right] \simeq -\frac{\alpha - \frac{1}{2}\sigma^2}{\gamma}(1 - \frac{c}{\frac{\delta}{r}\tilde{A}^{-\gamma}}) \quad \text{for } \alpha < \frac{1}{2}\sigma^2 \tag{17}$$

Proof. See Appendix A.5. ■

By (17) it is immediate to verify that the rate is increasing in the volatility of future payments. The intuition behind this result should be illustrated using (16). Since ξ is a log-normal process [34] with an upper reflecting barrier at $\hat{\xi}$, a higher volatility has two distinct effects. First, it pushes downward the barrier $\hat{\xi}$, second, by increasing the positive skewness of the distribution of ξ, it raises the probability that the barrier is reached. Clearly, both effects induce a higher rate of deforestation in the short-run as well as in the long-run. Furthermore, we also note that a higher conversion cost c induces a lower long-run average rate of deforestation. Two effects must be recognized. The first is immediate and driven by the higher c. The second is more subtle. A higher c prevents from converting now for a certain B. Since conversion in the future will be triggered by a decreasing B then the landholder can benefit from an implicit advantage paying a lower $(\eta_1 - \lambda\eta_2)B$ which is a conversion cost opportunity.

5.2 Case with $\eta_1 < \lambda\eta_2$

Consider the interval $\widehat{A} \leq \bar{A}$.[35] Rearranging (13) yields:

$$\varsigma = \frac{\beta_1}{\beta_1 - 1}\left(1 - \lambda\right)\frac{P_A(A)}{r} + \frac{\lambda\eta_2 - \eta_1}{r - \alpha}B \qquad \text{for } \varsigma < \hat{\varsigma} \tag{18}$$

where $\hat{\varsigma} = \frac{\beta_1}{\beta_1 - 1}\left(1 - \lambda\right)c$.

The first term on the RHS of (18) is the expected discounted profit from the cultivation of land if any further conversion occurs. The multiple $\frac{\beta_1}{\beta_1 - 1} > 1$ accounts for uncertainty and irreversibility. Differently from (16), since $\eta_1 < \lambda\eta_2$ the second term stands for the expected discounted flow of payments received when developing and taking apart λ net of the payments implicitly given up. Again, ς can be characterized as a regulated process having $\hat{\varsigma}$ as upper reflecting barrier. Whenever an increase of B leads ς upward toward $\hat{\varsigma}$ new plots are cleared. This will produce an increase in the supply of agricultural goods and consequently a drop along $P_A\left(A\right)$ preventing ς from passing $\hat{\varsigma}$. It follows that for keeping unchanged the surface conserved it must be $\varsigma > \hat{\varsigma}$.

As shown in the appendix:

Proposition 5 *When $\eta_1 < \lambda\eta_2$ the average or expected long-run growth rate of deforestation for a generic intial point $(\tilde{B}, \tilde{A})$ is given by:*

$$\frac{1}{dt}E\left[d\ln A\right] \simeq \frac{\alpha - \frac{1}{2}\sigma^2}{\gamma}(\frac{c}{\frac{\delta}{r}\tilde{A}^{-\gamma}} - 1) \quad \text{for } \alpha > \frac{1}{2}\sigma^2 \tag{19}$$

Proof. See Appendix A.5. ■

The long-run average rate of deforestation is decreasing in the volatility of future payments. Again, we remind that ς is a log-normal process with an upper reflecting barrier at $\hat{\varsigma}$. As volatility soars up the barrier $\hat{\varsigma}$ moves upward and positive skewness of the distribution of ς increases. Whilst the first has a reducing

[34] Technically the log-normality is a property of the process for ξ linearized around an initial point $(\tilde{B}, \tilde{A})$. See Appendix A.5 for further details.

[35] The same discussion provided in the previous section applies for $A \geq A^{**}$.

effect on the rate, the second raises the probability of hitting $\hat{\varsigma}$ and consequently the rate of deforestation. In addition, since the expected discounted profit from competitive farming decreases as land is converted, then the former effect prevails in the long-run. On the contrary, we find that the rate is increasing in c. This may seem surprising at a first sight. As a higher c prevents conversion we would expect landholders holding on the decision to develop. But postponing conversion is costly since one should give up the per-period increase in payments $(\lambda\eta_2 - \eta_1) B > 0$. Since the weight of expected discounted higher payments accruing if conversion is anticipated prevails over the expected pay-off from delay then the average rate of deforestation is increasing in c.[36] More formally, a higher c, by inducing a shift upward for the barrier, should definitely decrease the probability of hitting it. However, as c increases then $\hat{A}$ decreases and so does A^{**}. This means that a run will start at a lower surface A^{**} as soon as $B^{**}(\bar{A})$ has been reached. Since the run will exhaust the stock $\bar{A}$ and lower drastically the profit from agriculture then $A^{**} - \hat{A}$ landholders will prefer to anticipate the conversion. Note that since $\frac{\partial(A^{**}-\hat{A})}{\partial\hat{A}} > 0$ then, even for a $B < B^{**}(\bar{A})$, they would prefer to convert to trade-off the dramatic effect on the profit due to the run with a higher profit from farming. This latter effect justifies a higher deforestation rate in the long-run.

6 The Costa Rica case study

In this section we provide a numerical example by calibrating the model to fit the characteristics of the humid Atlantic zone of Costa Rica for which detailed data are provided by Bulte et al. (2002) and Conrad (1997). We use the following parameters taken by Bulte et al. (2002, page 155): $\delta = 6990062$; $r = 0.07$; $\alpha = 0.05$; $\gamma = 0.887$; $\tilde{B} = \$75/ha$, $\tilde{A} = 25000$, $\bar{A} = 320000$[37].

We study the case with $\eta_1 > \lambda\eta_2$ by distinguishing among five different policy scenarios. In detail:

Scenario 1:	$\eta_1 = 1$	$\eta_2 = 0$	$\lambda = 0$
Scenario 2:	$\eta_1 = 0.7$	$\eta_2 = 0$	$\lambda = 0$
Scenario 3:	$\eta_1 = 1$	$\eta_2 = 0$	$\lambda = 0.3$
Scenario 4:	$\eta_1 = 0.7$	$\eta_2 = 0.5$	$\lambda = 0.3$
Scenario 5:	$\eta_1 = 0.7$	$\eta_2 = 1$	$\lambda = 0.3$

In particular, in the following tables, we show three results for different values of α, σ and c:

a) **The starting agricultural land** for a given current forest value. By assuming in equation (10) that the annual forest value $\tilde{B} = \$75/ha$ is the current forest value, we obtain which is the optimal converted (agricultural) land $A\left(\tilde{B}\right)$. By subtracting $A\left(\tilde{B}\right)$ from $\bar{A}$, we obtain the forest stock $(\bar{A} - A\left(\tilde{B}\right))$ that might be cut in future;

b) **Deforestation rate** with respect the current forest value. After having obtained the current (starting) agricultural land for a given current forest value, we subtitute this value in equation (17) and we get the optimal long-run deforestation rate. An useful *caveat* about this result consists in stressing that the long-run deforestation rate changes with uncertainty and the policy scenarios only because changes the starting point $A\left(\tilde{B}\right)$;

c) **Deforestation rate** with respect the same starting point $\tilde{A} = 25000$. This value is obtained by substituting $\tilde{A}$ in equation (17). By using the same starting land, we are able to stress how α, σ and c

[36] See Bentolilla and Bertola (1990) for a similar effect on firing decision when firing costs are higher.

[37] Specifically, as in Bulte et al. (pages 154-155), forests are valued ($\tilde{B}$) annually by taking into account only the production function and not the regulatory function and existence values. Therefore, $\tilde{B}$ corresponds to the current value of the forest.

$\tilde{A}$ is assumed by Bulte et al. (footnote 4, page 153) as the starting plot to ensure convergence.

$\bar{A}$ is the area that Bulte et al. (page 153) use for their linear programming model. We assume that it corresponds to the upper level $\bar{A}$ on total land conversion fixed by Government.

can affect the deforestation rate, *ceteris paribus*. Moreover we calculate the optimal timing for the complete deforestation of the area $\bar{A}$.

A further *caveat* useful to undestrand our numerical results, is that the expected growth rate of deforestation defined in equations (17) and (19) is an average level in the long-run. This means that the different government policies adopted (i.e, the five scenarios) do not affect the deforestation rate, but only the initial point $\tilde{A}$.

By using $\tilde{B} = \$75/ha$, let us show our results with $c = 0$ in the following tables (4,5,6,7,8) corresponding to the scenarios (1,2,3,4,5), respectively.

	α=0.00			α=0.025			α=0.05		
σ	Ã	Ā-Ã	Def rate	Ã	Ā-Ã	Def rate	Ã	Ā-Ã	Def rate
0	320000	0	0,0000	243316	76684	0,0000	97526	222474	0,0000
0,025	320000	0	0,0000	240004	79996	0,0000	96845	223155	0,0000
0,05	320000	0	0,0000	231210	88790	0,0000	94873	225127	0,0000
0,075	319539	461	0,0032	219191	100809	0,0000	91804	228196	0,0000
0,1	296505	23495	0,0056	205789	114211	0,0000	87897	232103	0,0000
0,125	275222	44778	0,0088	192114	127886	0,0000	83426	236574	0,0000
0,15	255569	64431	0,0127	178760	141240	0,0000	78639	241361	0,0000
0,175	237433	82567	0,0173	166023	153977	0,0000	73735	246265	0,0000
0,2	220703	99297	0,0225	154045	165955	0,0000	68865	251135	0,0000
0,225	205277	114723	0,0285	142875	177125	0,0004	64134	255866	0,0000
0,25	191055	128945	0,0352	132516	187484	0,0070	59611	260389	0,0000
0,275	177947	142053	0,0426	122944	197056	0,0144	55338	264662	0,0000
0,3	165864	154136	0,0507	114121	205879	0,0225	51336	268664	0,0000
0,325	154727	165273	0,0595	106002	213998	0,0314	47612	272388	0,0032

Table 4: current used land, optimal stock and long-run average rate of deforestation in scenario 1.

	α=0.00			α=0.025			α=0.05		
σ	Ã	Ā-Ã	Def rate	Ã	Ā-Ã	Def rate	Ã	Ā-Ã	Def rate
0	320000	0	0,0000	320000	0	0,0000	145800	174200	0,0000
0,025	320000	0	0,0000	320000	0	0,0000	144782	175218	0,0000
0,05	320000	0	0,0000	320000	0	0,0000	141835	178165	0,0000
0,075	320000	0	0,0000	320000	0	0,0000	137245	182755	0,0000
0,1	320000	0	0,0000	307651	12349	0,0000	131404	188596	0,0000
0,125	320000	0	0,0000	287207	32793	0,0000	124721	195279	0,0000
0,15	320000	0	0,0000	267242	52758	0,0000	117564	202436	0,0000
0,175	320000	0	0,0000	248202	71798	0,0000	110233	209767	0,0000
0,2	320000	0	0,0000	230294	89706	0,0000	102952	217048	0,0000
0,225	306885	13115	0,0285	213595	106405	0,0004	95879	224121	0,0000
0,25	285624	34376	0,0352	198108	121892	0,0070	89117	230883	0,0000
0,275	266027	53973	0,0426	183799	136201	0,0144	82730	237270	0,0000
0,3	247964	72036	0,0507	170609	149391	0,0225	76747	243253	0,0000
0,325	231313	88687	0,0595	158471	161529	0,0314	71179	248821	0,0032

Table 5: current used land, optimal stock and long-run average rate of deforestation in scenario 2.

	α=0.00			α=0.025			α=0.05		
σ	Ã	Ā-Ã	Def rate	Ã	Ā-Ã	Def rate	Ã	Ā-Ã	Def rate
0	267754	52246	0,0000	162756	157244	0,0000	65235	254765	0,0000
0,025	248404	71596	0,0004	160540	159460	0,0000	64780	255220	0,0000
0,05	230402	89598	0,0014	154657	165343	0,0000	63461	256539	0,0000
0,075	213741	106259	0,0032	146618	173382	0,0000	61408	258592	0,0000
0,1	198334	121666	0,0056	137653	182347	0,0000	58794	261206	0,0000
0,125	184097	135903	0,0088	128506	191494	0,0000	55804	264196	0,0000
0,15	170951	149049	0,0127	119573	200427	0,0000	52602	267398	0,0000
0,175	158820	161180	0,0173	111054	208946	0,0000	49321	270679	0,0000
0,2	147629	172371	0,0225	103041	216959	0,0000	46064	273936	0,0000
0,225	137310	182690	0,0285	95569	224431	0,0004	42899	277101	0,0000
0,25	127798	192202	0,0352	88640	231360	0,0070	39874	280126	0,0000
0,275	119029	200971	0,0426	82237	237763	0,0144	37016	282984	0,0000
0,3	110947	209053	0,0507	76336	243664	0,0225	34339	285661	0,0000
0,325	103497	216503	0,0595	70905	249095	0,0314	31848	288152	0,0032

Table 6: current used land, optimal stock and long-run average rate of deforestation in scenario 3.

	α=0.00			α=0.025			α=0.05		
σ	Ã	Ā-Ã	Def rate	Ã	Ā-Ã	Def rate	Ã	Ā-Ã	Def rate
0	320000	0	0,0000	319337	663	0,0000	127997	192003	0,0000
0,025	320000	0	0,0000	314990	5010	0,0000	127104	192896	0,0000
0,05	320000	0	0,0000	303448	16552	0,0000	124516	195484	0,0000
0,075	320000	0	0,0000	287674	32326	0,0000	120487	199513	0,0000
0,1	320000	0	0,0000	270085	49915	0,0000	115359	204641	0,0000
0,125	320000	0	0,0000	252138	67862	0,0000	109492	210508	0,0000
0,15	320000	0	0,0000	234610	85390	0,0000	103209	216791	0,0000
0,175	311615	8385	0,0173	217895	102105	0,0000	96773	223227	0,0000
0,2	289659	30341	0,0225	202174	117826	0,0000	90381	229619	0,0000
0,225	269412	50588	0,0285	187514	132486	0,0004	84171	235829	0,0000
0,25	250748	69252	0,0352	173918	146082	0,0070	78236	241764	0,0000
0,275	233543	86457	0,0426	161356	158644	0,0144	72628	247372	0,0000
0,3	217686	102314	0,0507	149777	170223	0,0225	67375	252625	0,0000
0,325	203069	116931	0,0595	139121	180879	0,0314	62488	257512	0,0032

Table 7: current used land, optimal stock and long-run average rate of deforestation in scenario 4.

	α=0.00			α=0.025			α=0.05		
σ	Ã	Ā-Ã	Def rate	Ã	Ā-Ã	Def rate	Ã	Ā-Ã	Def rate
0	320000	0	0,0000	320000	0	0,0000	183283	111717	0,0000
0,025	320000	0	0,0000	320000	0	0,0000	182003	112997	0,0000
0,05	320000	0	0,0000	320000	0	0,0000	178298	116702	0,0000
0,075	320000	0	0,0000	320000	0	0,0000	172529	122471	0,0000
0,1	320000	0	0,0000	320000	0	0,0000	165186	129814	0,0000
0,125	320000	0	0,0000	320000	0	0,0000	156785	138215	0,0000
0,15	320000	0	0,0000	320000	0	0,0000	147788	147212	0,0000
0,175	320000	0	0,0000	312011	0	0,0000	138572	156428	0,0000
0,2	320000	0	0,0000	289499	5501	0,0000	129419	165581	0,0000
0,225	320000	0	0,0000	268507	26493	0,0004	120528	174472	0,0000
0,25	320000	0	0,0000	249039	45961	0,0070	112029	182971	0,0000
0,275	320000	0	0,0000	231051	63949	0,0144	103999	191001	0,0000
0,3	311711	0	0,0507	214470	80530	0,0225	96477	198523	0,0000
0,325	290780	4220	0,0595	199211	95789	0,0314	89478	205522	0,0032

Table 8: current used land, optimal stock and long-run average rate of deforestation in scenario 5.

Our first comments are that: i) the rate is increasing in the volatility of future payments as we expect from equation (17); ii) the initial plot $(A(\tilde{B}))$ is increasing with σ. In fact, when σ increases, the threshold level (10) increases. Therefore, fixing a given current forest value, the trigger corresponds to a greater converted land; iii) the long-run average rate and the the initial plot $(A(\tilde{B}))$ decrease with respect the drift (α) value. This is not a surprising result. Indeed, on the one hand, an increase of α leads ς downward far from $\hat{\varsigma}$, so it reduces the average rate. On the other hand, α increases the optimal trigger (10), so the current forest value $\tilde{B}$ converts a lower area.

As far as the Government's policies is concerned, the long-run deforestation rate is not affected by the different scenarios when c is nil, as we expect, while the starting converted areas change with the policy parameters $\eta_{1,}$ $\eta_{2,}\lambda$.

In order to focus on the parameter λ we compare scenario 1 and 3 (table 4 and 6, respectively). We remember that in table 6 λ is equal to 0.3, while in table 4 is equal to 0. In both the cases, η_2 is equal to zero. This means that in scenario 3 there is a restriction λ over the convertible portion of the total surface but, in this case, the Government does not offer a compensation payment to the landholder. This implies that the agricultural profit of scenario 3 $((1-\lambda)P_A(t))$ is lower than scenario 1 $(P_A(t))$. Therefore, it is better to convert a greater parcel in scenario 1, ceteris paribus.

Focussing on the role of the parameter η_1 , we compare scenario 1,3 with scenario 2 (table 5). The main result is that the initial cleared plot $A(\tilde{B})$ is greater when the parameter η_1 reduces. That is, the lower the compensation paid if the entire plot is conserved, the greater the converted land.

The effect of the compensation payment in case of conversion (η_2) comes from the comparison between table 7 and 8 (scenario 4 and 5 respectively) and the table 5 (scenario 2), characterized by the parameter η_1 equal to 0.7. In some detail, in scenario 4 η_2 is equal to 0.5, while in scenario 5 is equal to 1. In both the scenarios the parameter λ is equal to 0.3, while in scenario 2, λ is equal to 0. We observe two results: in scenario 4 the converted plot (i.e. $A(\tilde{B})$) for $\tilde{B}$ is smaller than in scenario 2, while the opposite is observable comparing scenarios 5 and 2. The main insight is that there are two counterbalancing effects: on the one hand the reduction of agricultural profit due to a lower available parcel in the case of conversion $(1-\lambda)$, on the other hand the compensation payment that raises profits. The first effect dominates when we compare scenario 2 with scenario 4, while the second dominates with respect scenario 2 and 5.

Other important results arise in the following tables, where again $\tilde{B} = \$75/ha$, and $c = 200$.

	α=0.00			α=0.025			α=0.05		
σ	A0	Abar-A0	Def Rate	A0	Abar-A0	Def Rate	A0	Abar-A0	Def Rate
0	320000	0	0,0000	214133	105867	0,0000	91977	228023	0,0000
0,025	309741	10259	0,0003	211526	108474	0,0000	91367	228633	0,0000
0,05	290434	29566	0,0012	204572	115428	0,0000	89599	230401	0,0000
0,075	272206	47794	0,0028	194983	125017	0,0000	86839	233161	0,0000
0,1	255030	64970	0,0049	184175	135825	0,0000	83314	236686	0,0000
0,125	238876	81124	0,0078	173018	146982	0,0000	79264	240736	0,0000
0,15	223712	96288	0,0113	161993	158007	0,0000	74907	245093	0,0000
0,175	209498	110502	0,0133	151357	168643	0,0000	70422	249578	0,0000
0,2	196195	123805	0,0203	141242	178758	0,0000	65945	254055	0,0000
0,225	183760	136240	0,0259	131711	188289	0,0003	61575	258425	0,0000
0,25	172149	147851	0,0321	122784	197216	0,0066	57377	262623	0,0000
0,275	161318	158682	0,0391	114458	205542	0,0136	53392	266608	0,0000
0,3	151223	168777	0,0467	106717	213283	0,0212	49643	270357	0,0000
0,325	141821	178179	0,0551	99534	220466	0,0297	46141	273859	0,0031

Table 9: current used land, optimal stock and long-run average rate of deforestation in scenario 1.

		α=0.00			α=0.025			α=0.05	
σ	A0	Abar-A0	Def rate	A0	Abar-A0	Def rate	A0	Abar-A0	Def rate
0	320000	0	0,0000	304325	15675	0,0000	134217	185783	0,0000
0,025	320000	0	0,0000	300781	19219	0,0000	133346	186654	0,0000
0,05	320000	0	0,0000	291309	28691	0,0000	130819	189181	0,0000
0,075	320000	0	0,0000	278207	41793	0,0000	126870	193130	0,0000
0,1	320000	0	0,0000	263380	56620	0,0000	121821	198179	0,0000
0,125	320000	0	0,0000	248007	71993	0,0000	116009	203991	0,0000
0,15	317317	2683	0,0108	232748	87252	0,0000	109745	210255	0,0000
0,175	298022	21978	0,0121	217964	102036	0,0000	103284	216716	0,0000
0,2	279866	40134	0,0195	203845	116155	0,0000	96822	223178	0,0000
0,225	262808	57192	0,0249	190485	129515	0,0003	90501	229499	0,0000
0,25	246806	73194	0,0309	177925	142075	0,0064	84417	235583	0,0000
0,275	231812	88188	0,0377	166169	153831	0,0132	78632	241368	0,0000
0,3	217777	102223	0,0452	155201	164799	0,0207	73179	246821	0,0000
0,325	204654	115346	0,0534	144992	175008	0,0290	68076	251924	0,0030

Table 10: current used land, optimal stock and long-run average rate of deforestation in scenario 2.

	α=0.00			α=0.025			α=0.05		
σ	A0	Abar-A0	Def rate	A0	Abar-A0	Def rate	A0	Abar-A0	Def rate
0	233143	86857	0,0000	148609	171391	0,0000	62595	257405	0,0000
0,025	218122	101878	0,0003	146740	173260	0,0000	62173	257827	0,0000
0,05	203931	116069	0,0013	141762	178238	0,0000	60952	259048	0,0000
0,075	190603	129397	0,0029	134916	185084	0,0000	59047	260953	0,0000
0,1	178109	141891	0,0051	127222	192778	0,0000	56617	263383	0,0000
0,125	166415	153585	0,0081	119305	200695	0,0000	53827	266173	0,0000
0,15	155487	164513	0,0117	111508	208492	0,0000	50831	269169	0,0000
0,175	145288	174712	0,0143	104010	215990	0,0000	47750	272250	0,0000
0,2	135780	184220	0,0209	96902	223098	0,0000	44680	275320	0,0000
0,225	126926	193074	0,0266	90223	229777	0,0003	41687	278313	0,0000
0,25	118689	201311	0,0330	83986	236014	0,0067	38817	281183	0,0000
0,275	111031	208969	0,0401	78184	241816	0,0138	36095	283905	0,0000
0,3	103916	216084	0,0479	72804	247196	0,0216	33539	286461	0,0000
0,325	97308	222692	0,0564	67823	252177	0,0301	31153	288847	0,0031

Table 11: current used land, optimal stock and long-run average rate of deforestation in scenario 3.

	α=0.00			α=0.025			α=0.05		
σ	A0	Abar-A0	Def rate	A0	Abar-A0	Def rate	A0	Abar-A0	Def rate
0	320000	0	0,0000	272057	47943	0,0000	118863	201137	0,0000
0,025	320000	0	0,0000	268838	51162	0,0000	118085	201915	0,0000
0,05	320000	0	0,0000	260240	59760	0,0000	115830	204170	0,0000
0,075	320000	0	0,0000	248360	71640	0,0000	112308	207692	0,0000
0,1	320000	0	0,0000	234936	85064	0,0000	107807	212193	0,0000
0,125	302507	17493	0,0075	221039	98961	0,0000	102629	217371	0,0000
0,15	283867	36133	0,0109	207267	112733	0,0000	97051	222949	0,0000
0,175	266332	53668	0,0125	193944	126056	0,0000	91302	228698	0,0000
0,2	249863	70137	0,0198	181239	138761	0,0000	85557	234443	0,0000
0,225	234419	85581	0,0252	169235	150765	0,0003	79941	240059	0,0000
0,25	219954	100046	0,0314	157964	162036	0,0069	74539	245461	0,0000
0,275	206422	113578	0,0382	147429	172571	0,0142	69406	250594	0,0000
0,3	193776	126224	0,0458	137612	182388	0,0222	64572	255428	0,0000
0,325	181966	138034	0,0540	128485	191515	0,0309	60050	259950	0,0031

Table 12: current used land, optimal stock and long-run average rate of deforestation in scenario 4.

	α=0.00			α=0.025			α=0.05		
σ	A0	Abar-A0		A0	Abar-A0		A0	Abar-A0	
0	320000	0	0,0000	320000	0	0,0000	165742	154258	0,0000
0,025	320000	0	0,0000	320000	0	0,0000	164683	155317	0,0000
0,05	320000	0	0,0000	320000	0	0,0000	161610	158390	0,0000
0,075	320000	0	0,0000	320000	0	0,0000	156805	163195	0,0000
0,1	320000	0	0,0000	320000	0	0,0000	150654	169346	0,0000
0,125	320000	0	0,0000	302388	17612	0,0000	143566	176434	0,0000
0,15	320000	0	0,0000	284252	35748	0,0000	135916	184084	0,0000
0,175	320000	0	0,0000	266626	53374	0,0000	128013	191987	0,0000
0,2	320000	0	0,0000	249743	70257	0,0000	120098	199902	0,0000
0,225	319926	74	0,0242	233722	86278	0,0003	112345	207655	0,0000
0,25	300962	19038	0,0301	218619	101381	0,0063	104871	215129	0,0000
0,275	283137	36863	0,0368	204445	115555	0,0130	97754	222246	0,0000
0,3	266404	53596	0,0441	191190	128810	0,0204	91038	228962	0,0000
0,325	250712	69288	0,0522	178825	141175	0,0285	84743	235257	0,0030

Table 13: current used land, optimal stock and long-run average rate of deforestation in scenario 5.

The main insights are that a higher conversion cost c induces a lower long-run average rate of deforestation and a smaller initial converted plot (i.e. $A(\tilde{B})$) , as we expect. If we compare the five scenarios, we get the following inequalities for the long-run deforestation rates: scenario 3 > scenario 1 > scenario 4 >scenario 2 > scenario 5. As far as the initial converted area ($A(\tilde{B})$) is concerned, the comments are in line with the previous five scenarios. First of all, we observe a reverse relation between initial converted plot and deforestation rate. That is, the higher the converted parcel for a given forest value $\tilde{B}$, the lower the deforestation rate. For the remaining parcel the reduction of the available land (λ) seems to be an important push factor for deforestation rate (scenario 3 and 4), especially if combined with η_1. This last parameter is surely important, more than η_2.

In the last three tables, we show the long-run deforestation rates and the optimal timing for the complete deforestation, taking fixed the initial point $\tilde{A} = 25000$ and for varying c value between 0 to 400.

	α=0.00	α=0.00	α=0.025	α=0.025	α=0.05	α=0.05
σ	DEF RATE	TIMING	DEF RATE	TIMING	DEF RATE	TIMING
0	0	+∞				
0,025	0,00035	35755				
0,05	0,00141	8944				
0,075	0,00317	3978				
0,1	0,00564	2241				
0,125	0,00881	1436				
0,15	0,01268	999				
0,175	0,01726	736				
0,2	0,02255	565				
0,225	0,02854	448	0,00035	35755		
0,25	0,03523	364	0,00705	1794		
0,275	0,04263	302	0,01444	878		
0,3	0,05073	255	0,02255	565		
0,325	0,05954	218	0,03136	408	0,00317	3978

Table 14: Long-run defoestation rates and timing with c=0

	α=0.00	α=0.00	α=0.025	α=0.025	α=0.05	α=0.05
σ	DEF RATE	TIMING	DEF RATE	TIMING	DEF RATE	TIMING
0	0	+∞				
0,025	0,00035	36334				
0,05	0,00139	9089				
0,075	0,00312	4043				
0,1	0,00555	2277				
0,125	0,00867	1459				
0,15	0,01248	1015				
0,175	0,01699	748				
0,2	0,02219	574				
0,225	0,02808	455	0,00035	36334		
0,25	0,03467	370	0,00693	1823		
0,275	0,04195	306	0,01421	892		
0,3	0,04992	259	0,02219	574		
0,325	0,05859	221	0,03086	414	0,0031	4043

Table 15: Long-run defoestation rates and timing with c=200

	α=0.00	α=0.00	α=0.025	α=0.025	α=0.05	α=0.05
σ	DEF RATE	TIMING	DEF RATE	TIMING	DEF RATE	TIMING
0	0	+∞				
0,025	0,00034	36933				
0,05	0,00136	9238				
0,075	0,00307	4109				
0,1	0,00546	2314				
0,125	0,00853	1483				
0,15	0,01228	1032				
0,175	0,01671	760				
0,2	0,02183	583				
0,225	0,02763	462	0,00034	36933		
0,25	0,03411	376	0,00682	1853		
0,275	0,04127	311	0,01398	907		
0,3	0,04912	263	0,02183	583		
0,325	0,05764	225	0,03036	421	0,0031	4109

Table 16: Long-run defoestation rates and timing with c=400

Our main comments that confirm our previour results:

a) the rate is increasing in the volatility of future payments as we expect from equation (17);

b) an increase of α leads ς downward far from $\hat{\varsigma}$, so it reduces the average rate and increases the time necessary to clear the area;

c) a higher conversion cost c induces a lower long-run average rate of deforestation and higher clearing timing.

7 Conclusions

In this paper we contribute to the vast literature on optimal land allocation under uncertainty and irreversible habitat conversion. We extend previous work in three respects. First, departing from the standard central planner perspective we investigate the role that competing farming may have on conversion dynamics. Profits from conversion decreasing in the number of developers may discourage conversion in particular if society is willing to reward habitat conservation as land-use. Second, in this decentralized frame, we look at the conservation effort that Government land policy, through a combination of voluntary and command approaches, may stimulate. In this regard, we show that even partially compensating ES the Government may induce considerable amount of conservation. In addition, we show that the existence of a ceiling for the stock of developable land may produce perverse effects on conversion dynamic by activating a run which instantaneously exhausts the stock. Third, we believe that time matters when dynamic land allocation is analysed. Hence, we suggest the use of the optimal long-run average expected rate of conversion to assess the temporal performance of conservation policy and we show its utility by running several numerical simulations based on realistic policy scenarios.

A Appendix

A.1 Equilibrium under $\eta_1 > \lambda\eta_2$

The value function of a farmer is given by:

$$V(A,B;\bar{A}) = Z_2(A)B^{\beta_2} + \frac{(1-\lambda)\delta}{r}A^{-\gamma} + (\lambda\eta_2 - \eta_1)\frac{B}{r-\alpha} \tag{A.1.1}$$

Since each agent rationally forecasts the future dynamics of the market for agricultural goods at $B^*(A)$ s/he must be indifferent between conserving and converting. That is:

$$Z_2(A)B^*(A)^{\beta_2} + \frac{(1-\lambda)\delta}{r}A^{-\gamma} + (\lambda\eta_2 - \eta_1)\frac{B^*(A)}{r-\alpha} = (1-\lambda)c \tag{A.1.2}$$

In addition, the following conditions must hold (See e.g. proposition 1 in Bartolini (1993) and Grenadier (2002, p. 699):

$$V_A(A,B^*(A);\bar{A}) = Z_2'(A)B^*(A)^{\beta_2} - (1-\lambda)\frac{\delta\gamma A^{-(\gamma+1)}}{r} = 0 \tag{A.1.3}$$

and

$$\begin{aligned}\frac{\partial V(A,B^*(A);\bar{A})}{\partial A} &= V_A(A,B^*(A);\bar{A}) + V_B(A,B^*(A);\bar{A})\frac{dB^*(A)}{dA} \\ &= \left[\beta_2 Z_2(A)\,B^*(A)^{\beta_2-1} + \frac{\lambda\eta_2-\eta_1}{r-\alpha}\right]\frac{dB^*(A)}{dA} = 0\end{aligned} \tag{A.1.4}$$

Finally, considering the limit $\bar{A}$ imposed by the Government on the land that can be converted, it must be:

$$Z_2(\bar{A}) = 0 \tag{A.1.5}$$

Condition (A.1.4) illustrates two scenarios. On the first one, each landholder exercises the option to convert at the level of $B^*(A)$ where the value, $V(A,B^*(A);\bar{A})$ is tangent to the conversion cost, $(1-\lambda)\,c$.[38] That is, $V_B(A,B^*(A);\bar{A}) = \beta_2 Z_2(A)B^*(A)^{\beta_2-1} + \frac{\lambda\eta_2-\eta_1}{r-\alpha} = 0$. It easy to verify that as conjectured $Z_2(A) < 0$. In the case $V(A,B^*(A);\bar{A})$ is smooth at the conversion threshold and $B^*(A)$ is a continuous function of A. On the second scenario, the optimal threshold $B^*(A)$ does not vary with A, i.e. $V_B(A,B^*(A);\bar{A}) \neq 0$ and $\frac{dB^*(A)}{dA} = 0$. This implies that the landholder may benefit from marginally anticipating or delaying the conversion decision. In particular, if $V_B(A,B^*(A);\bar{A}) < 0$ then the value of conversion is expected to increase as B drops. On the contrary, if $V_B(A,B^*(A);\bar{A}) > 0$ then losses must be expected as B drops. However, in both cases (A.1.4) holds by imposing $\frac{dB^*(A)}{dA} = 0$.

By (A.1.4) we can split $[A_0,\bar{A}]$ into two intervals where one of the following two conditions must hold:

$$\beta_2 Z_2(A)\,B^*(A)^{\beta_2-1} + \frac{\lambda\eta_2-\eta_1}{r-\alpha} = 0 \tag{A.1.6}$$

$$\frac{dB^*(A)}{dA} = 0 \tag{A.1.7}$$

Since $Z_2\left(\bar{A}\right) = 0$ and $\frac{\lambda\eta_2-\eta_1}{r-\alpha} < 0$, then (A.1.6) cannot hold at $A = \bar{A}$. Therefore, (A.1.7) must hold at $A = \bar{A}$ and by (A.1.2) follows:

$$B^*(\bar{A}) = \frac{1-\lambda}{\eta_1 - \lambda\eta_2}(r-\alpha)\left[(\frac{\hat{A}}{\bar{A}})^{\gamma} - 1\right]c \quad \text{for } A^* \leq A \leq \bar{A} \tag{A.1.8}$$

[38] This condition holds at any reflecting barrier without any optimization being involved (Dixit, 1993).

where $\hat{A} = (\frac{\delta}{rc})^{1/\gamma}$ represents the last parcel conversion which makes economic sense. In fact, note that since $(\lambda\eta_2 - \eta_1)\frac{B}{r-\alpha} < 0$ then $\frac{\delta}{r}A^{-\gamma} \leq c$ for $A \geq \hat{A}$.

Now, let us define A^* as the largest $A \leq \bar{A}$ that satisfies (A.1.6). This implies that for all the landholders in the range $A^* \leq A \leq \bar{A}$, we have $\frac{dB^*(A)}{dA} = 0$ and conversion takes place at $B^*(\bar{A})$. Over the range $A < A^*$ (A.1.3) holds by definition. Hence, plugging (A.1.6) into (A.1.4) we obtain:

$$B^*(A) = \frac{\beta_2}{\beta_2 - 1}(r-\alpha)\frac{1-\lambda}{\eta_1 - \lambda\eta_2}\left[(\frac{\hat{A}}{A})^{\gamma} - 1\right]c \quad \text{for } A < A^* \tag{A.1.9}$$

Finally, by continuity of $B^*(A)$ it must be $B^*(A^*) = B^*(\bar{A})$. Substituting:

$$\frac{\beta_2}{\beta_2 - 1}(r-\alpha)\frac{1-\lambda}{\eta_1 - \lambda\eta_2}\left[(\frac{\hat{A}}{A^*})^{\gamma} - 1\right]c = (r-\alpha)\frac{1-\lambda}{\eta_1 - \lambda\eta_2}\left[(\frac{\hat{A}}{\bar{A}})^{\gamma} - 1\right]c \tag{A.1.10}$$

where

$$A^* = [\frac{(\beta_2 - 1)\bar{A}^{-\gamma} + \hat{A}^{-\gamma}}{\beta_2}]^{-\frac{1}{\gamma}}$$

The conversion policy is summarized by (A.1.8) and (A.1.9). The conversion policy should be smooth until the surface $A^* < \bar{A}$ has been converted. At A^* landholders rush and a run takes place to convert the residual land before the limit imposed by the Government is met. By (A.1.9), $B^*(A)$ is decreasing with respect to A. This makes sense since further land conversion reduces the profit from agriculture and a landholder would convert land only if s/he expects a future reduction in B.

We must investigate two different scenarios, i.e. $\hat{A} \leq \bar{A}$ and $\hat{A} > \bar{A}$. From (A.1.10) follows:

$$\frac{\beta_2}{\beta_2 - 1}\left[(\frac{\hat{A}}{A^*})^{\gamma} - 1\right] = (\frac{\hat{A}}{\bar{A}})^{\gamma} - 1 \tag{A.1.10 bis}$$

Studying (A.1.10 bis) we can state that since $\frac{\beta_2}{\beta_2 - 1} > 0$:

- if $\hat{A} \leq \bar{A}$ then it must be $\bar{A} \leq A^*$. This implies that there is no run taking place. Land will be converted smoothly according to (A.1.8) up to $\hat{A}$ since $\frac{\delta}{r}A^{-\gamma} \leq c$ for $A \geq \hat{A}$.

- if $\hat{A} > \bar{A}$ then it must be $A^* < \bar{A}$. In this case, land is converted smootly up to A^* where landholders start a run to convert land up to $\bar{A}$.

A.2 Value of the option to convert

In this appendix we show that, by competition, the value of the opportunity to develop the plot by the single farmer is null at the conversion threshold. The value of the option to convert, $F(A,B;\bar{A})$, is the solution of the following differential equation (Dixit and Pindyck, 1994, ch. 8):

$$\frac{1}{2}\sigma^2 B^2 F_{BB}(A,B;\bar{A}) + \alpha B F_B(A,B;\bar{A}) - rF(A,B;\bar{A}) = 0 \quad \text{for } B > B^C(A) \tag{A.2.1}$$

where $B^C(A)$ is the optimal threshold for conversion. Note that this is an ordinary differential equation which general solution can be written as:

$$F(A,B;\bar{A}) = C_1(A)B^{\beta_1} + C_2(A)B^{\beta_2} \tag{A.2.2}$$

where $1 < \beta_1 < r/\alpha$, $\beta_2 < 0$ are the positive and the negative root of the characteristic equation $\Psi(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r = 0$, and C_1, C_2 are two constants to be determined.

Suppose for instance that $\eta_1 > \lambda\eta_2$.[39] This implies that as B increases, the value of the option to

[39] It is easy to show that the results in this section would hold also for the case $\eta_1 < \eta_2\lambda$.

convert should vanish as ($\lim_{B\to\infty} F(A,B;\bar{A}) = 0$) then we set $C_1 = 0$. Now, let determine the optimal conversion threshold $\overline{B}$ and the constant $C_1(A)$. We attach to the differential equation above the following value matching and the smooth pasting conditions:

$$F(A, B^C(A); \bar{A}) = V(A, B^C(A); \bar{A}) - (1-\lambda)\,c \tag{A.2.3}$$

$$C_2(A)B^C(A)^{\beta_2} = Z_2(A)B^C(A)^{\beta_2} + \frac{(1-\lambda)\,\delta A^{-\gamma}}{r} + \frac{(\lambda\eta_2 - \eta_1)B^C(A)}{r-\alpha} - (1-\lambda)\,c$$

$$\begin{aligned} F_B(A, B^C(A); \bar{A}) &= V_B(A, B^C(A); \bar{A}) \\ \beta_2 C_2(A)B^C(A)^{\beta_2 - 1} &= \beta_2 Z_2(A)B^C(A)^{\beta_2 - 1} + \frac{\lambda\eta_2 - \eta_1}{r-\alpha} \end{aligned} \tag{A.2.4}$$

It follows:

$$B^C(A) = B^*(A) = \frac{\beta_2}{\beta_2 - 1}(r-\alpha)\frac{1-\lambda}{\eta_1 - \lambda\eta_2}[\frac{\delta}{r}A^{-\gamma} - c]$$

From (A.1.4) it must be $V_B(A,B^*(A);\bar{A}\ ) = \beta_2 Z_2(A)B^*(A)^{\beta_2 - 1} + \frac{\lambda\eta_2-\eta_1}{r-\alpha} = 0$ which in turn implies $Z_2(A) = -\frac{\lambda\eta_2-\eta_1}{r-\alpha}\frac{B^*(A)^{1-\beta_2}}{\beta_2}$.

Rearranging (A.2.4) and substituting we have:

$$\begin{aligned} C(A) &= Z_2(A) + \frac{\lambda\eta_2 - \eta_1}{r-\alpha}\frac{B^C(A)^{1-\beta_2}}{\beta_2} \\ &= \frac{\lambda\eta_2 - \eta_1}{(r-\alpha)\beta_2}(B^C(A)^{1-\beta_2} - B^*(A)^{1-\beta_2}) = 0 \end{aligned}$$

As expected the value of the option to convert is null at $B^C(A) = B^*(A)$.

A.3 Equilibrium under $\eta_1 < \lambda\eta_2$

The value function of a farmer is given by:

$$V(A, B; \bar{A}) = Z_1(A)B^{\beta_1} + \frac{(1-\lambda)\delta}{r}A^{-\gamma} + (\lambda\eta_2 - \eta_1)\frac{B}{r-\alpha} \tag{A.3.1}$$

As in section A.1 the following conditions must hold at:

$$B^{**}(A) : Z_1(A)B^{**}(A)^{\beta_1} + \frac{(1-\lambda)\delta}{r}A^{-\gamma} + (\lambda\eta_2 - \eta_1)\frac{B^{**}(A)}{r-\alpha} = (1-\lambda)c \tag{A.3.2}$$

$$V_A(A, B^{**}(A); \bar{A}) = Z_1'(A)B^{**}(A)^{\beta_1} - \gamma\frac{(1-\lambda)\delta A^{-(\gamma+1)}}{r} = 0 \tag{A.3.3}$$

$$\begin{aligned} \frac{\partial V(A, B^{**}(A); \bar{A})}{\partial A} &= V_A(A, B^{**}(A); \bar{A}) + V_B(A, B^{**}(A); \bar{A})\frac{dB^{**}(A)}{dA} \\ &= \left[\beta_1 Z_1(A)\,B^{**}(A)^{\beta_1 - 1} + \frac{\lambda\eta_2 - \eta_1}{r-\alpha}\right]\frac{dB^{**}(A)}{dA} = 0 \end{aligned} \tag{A.3.4}$$

In addition, since land can be converted up to $\bar{A}$, it must be:

$$Z_1(\bar{A}) = 0 \tag{A.3.5}$$

Condition (A.3.4) illustrates two scenarios. On the first one, each landholder exercises the option to convert at the level of $B^{**}(A)$ where the value, $V(A,B^{**}(A);\bar{A}\ )$ is tangent to the conversion cost, $(1-\lambda)\,c$. That is, $V_B(A,B^{**}(A);\bar{A}\ ) = \beta_1 Z_1(A)B^{*}(A)^{\beta_1-1} + \frac{\lambda\eta_2-\eta_1}{r-\alpha} = 0$. Since $\frac{\lambda\eta_2-\eta_1}{r-\alpha} > 0$ one can easily verify that as conjectured $Z_1(A) < 0$. In the case $V(A,B^{**}(A);\bar{A}\ )$ is smooth at the conversion threshold and $B^{**}(A)$ is a continuous function of A. On the second scenario, the optimal threshold $B^{**}(A)$ does not change with A, i.e. $V_B(A,B^{**}(A);\bar{A}\ ) \neq 0$ and $\frac{dB^{**}(A)}{dA} = 0$. As discussed in section A.1, this implies that at the margin the landholder may benefit from anticipating or delaying the conversion decision.

By (A.3.4) the set $[A_0,\bar{A}]$ can be split into two subsets where one of the following two conditions must hold:

$$\beta_1 Z_1\left(A\right) B^{**}(A)^{\beta_1-1} + \frac{\lambda\eta_2-\eta_1}{r-\alpha} = 0 \tag{A.3.6}$$

$$\frac{dB^{**}(A)}{dA} = 0 \tag{A.3.7}$$

Note that as $Z_1\left(\bar{A}\right) = 0$ and $\frac{\lambda\eta_2-\eta_1}{r-\alpha} > 0$, then (A.3.6) cannot hold at $A = \bar{A}$. Therefore, at $A = \bar{A}$ it must be (A.3.7). From (A.3.2) it follows:

$$B^{**}(\bar{A}) = (r-\alpha)\frac{1-\lambda}{\lambda\eta_2-\eta_1}\left[1-(\frac{\hat{A}}{\bar{A}})^{\gamma}\right]c \quad \text{for } A^{**} < A \leq\ \bar{A} \tag{A.3.8}$$

Now, define A^{**} as the largest $A \leq \bar{A}$ for which (A.3.6) holds. This means that for all the landholders in the range $A^{**} \leq A \leq \bar{A}$, we must have $\frac{dB^{**}(A)}{dA} = 0$ and conversion occurs at $B^{**}(\bar{A})$. Over the range $A < A^{**}$ (A.3.3) holds by definition. Then, substituting (A.3.6) into (A.3.4) we get:

$$B^{**}(A) = \frac{\beta_1}{\beta_1-1}(r-\alpha)\frac{1-\lambda}{\lambda\eta_2-\eta_1}\left[1-(\frac{\hat{A}}{A})^{\gamma}\right]c \quad \text{for } A_0 < A \leq\ A^{**} \tag{A.3.9}$$

However, note that $B^{**}(A) < 0$ for $A < \hat{A}$. Since by (2) B cannot be negative then it must be:

$$B^{**}(A) = \begin{cases} 0, & \text{for } A_0 < A \leq\ \hat{A} \\ \frac{\beta_1}{\beta_1-1}(r-\alpha)\frac{1-\lambda}{\lambda\eta_2-\eta_1}\left[1-(\frac{\hat{A}}{A})^{\gamma}\right]c, & \text{for } \hat{A} < A \leq\ A^{**} \end{cases} \tag{A.3.9 bis}$$

By continuity of $B^{**}(A)$ it follows that $B^{**}(A^{**}) = B^{**}(\bar{A})$. Substituting:

$$\frac{\beta_1}{\beta_1-1}(r-\alpha)\frac{1-\lambda}{\lambda\eta_2-\eta_1}\left[1-(\frac{\hat{A}}{A^{**}})^{\gamma}\right]c\ = (r-\alpha)\frac{1-\lambda}{\lambda\eta_2-\eta_1}\left[1-(\frac{\hat{A}}{\bar{A}})^{\gamma}\right]c \tag{A.3.10}$$

where

$$A^{*} = [\frac{(\beta_1-1)\bar{A}^{-\gamma}+\hat{A}^{-\gamma}}{\beta_1}]^{-\frac{1}{\gamma}}$$

From (A.3.10) it follows:

$$\frac{\beta_1}{\beta_1-1}[1-(\frac{\hat{A}}{A^{**}})^{\gamma}] = 1-(\frac{\hat{A}}{\bar{A}})^{\gamma} \tag{A.3.10 bis}$$

As in section (A.1), two different scenarios may arise. In fact, studying (A.3.10 bis) we observe that since $\frac{\beta_1}{\beta_1-1} > 0$:

- if $\hat{A} \leq \bar{A}$ then it must be $\hat{A} \leq A^{**} \leq \bar{A}$. Land will be converted according to (A.3.9 bis) up to A^{**}. At A^{**} landholders will rush to convert land up to the limit, $\bar{A}$, fixed by the Government (A.3.8).
- if $\hat{A} > \bar{A}$ then it must be $\hat{A} > A^{**} > \bar{A}$. This implies that the conversion process follows (A.3.9 bis) and land is istantaneously developed up to $\bar{A}$.

A.4 Long-run distributions

Let h be a linear Brownian motion with parameters μ and σ that evolves according to $dh = \mu dt + \sigma dw$. Following Harrison (1985, pp. 90-91; see also Dixit 1993, pp. 58-68) the long-run density function for h fluctuating between a lower reflecting barrier, $a \in (-\infty, \infty)$, and an upper reflecting barrier, $b \in (-\infty, \infty)$, is represented by the following truncated exponential distribution:

$$f(h) = \begin{cases} \frac{2\mu}{\sigma^2} \frac{e^{\frac{2\mu}{\sigma^2} h}}{e^{\frac{2\mu}{\sigma^2} b} - e^{\frac{2\mu}{\sigma^2} a}} & \mu \neq 0 \\ \frac{1}{b-a} & \mu = 0 \end{cases} \tag{A.4.1}$$

The case of interest for us is obtained letting $a \to -\infty$ in (A.4.1). That is:

$$f(h) = \begin{cases} \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2}(b-h)} & \mu > 0 \\ 0 & \mu \leq 0 \end{cases} \qquad \text{for } -\infty < h < b \tag{A.4.2}$$

Hence, the long-run average of h can be evaluated as $E[h] = \int_\Phi hf(h)\,dh$, where Φ depends on the distribution assumed. In the steady-state this yields:

$$E[h] = \int_{-\infty}^{b} hf(h)\,dh = \int_{-\infty}^{b} h\frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2}(b-h)} dh = \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2} b} \int_{-\infty}^{b} he^{\frac{2\mu}{\sigma^2} h} dh = b - \frac{2\mu}{\sigma^2} \tag{A.4.3}$$

A.5 Long-run average growth rate of deforestation

Taking the logarithm of (16) we get:[40]

$$\begin{aligned} \ln \xi &= \ln \left[\frac{\beta_2}{\beta_2 - 1} (1-\lambda) \frac{P_A(A)}{r} - \frac{\eta_1 - \lambda\eta_2}{r - \alpha} B \right] \\ &= \ln \left[\frac{\eta_1 - \lambda\eta_2}{r - \alpha} \right] + \ln[J - B] \end{aligned} \tag{A.5.1}$$

where $J = \frac{\beta_2}{\beta_2 - 1} (r-\alpha) \frac{1-\lambda}{\eta_1 - \lambda\eta_2} \frac{P_A(A)}{r}$ and $J > B$. Rewriting $\ln[J-B]$ as $\ln\left[e^{\ln J} - e^{\ln B}\right]$ and expanding it by Taylor's theorem around the point $(\widetilde{\ln J}\ , \widetilde{\ln B})$ yields:

$$\ln[J - B] \simeq v_0 + v_1 \ln J + v_2 \ln B$$

where

$$\begin{aligned} v_0 &= \ln\left[e^{\widetilde{\ln J}} - e^{\widetilde{\ln B}}\right] - \left[\frac{\widetilde{\ln J}}{1 - e^{\widetilde{\ln B} - \widetilde{\ln J}}} + \frac{\widetilde{\ln B}}{1 - e^{-(\widetilde{\ln B} - \widetilde{\ln J})}} \right] \\ v_1 &= \frac{1}{1 - e^{\widetilde{\ln B} - \widetilde{\ln J}}},\ v_2 = \frac{1}{1 - e^{-(\widetilde{\ln B} - \widetilde{\ln J})}},\ \frac{v_2}{v_1} = \frac{1 - v_1}{v_1} < 0 \end{aligned}$$

[40] See Hartman and Hendrickson (2002) for a calculation of the long-run average growth rate of capital.

By substituting the approximation into (A.5.1) it follows that:

$$\ln \xi \simeq \ln \frac{\eta_1 - \lambda\eta_2}{r - \alpha} + v_0 + v_1 \ln J + v_2 \ln B \tag{A.5.2}$$

Now, by Ito's lemma and the considerations discussed in the paper on the competitive equilibrium, $\ln \xi$ evolves according to $d \ln \xi = v_2 d \ln B = v_2[(\alpha - \frac{1}{2}\sigma^2)dt + \sigma dw]$ with $\ln \hat{\xi}$ as upper reflecting barrier. Setting $h = \ln \xi$, the random variable $\ln \xi$ follows a linear Brownian motion with parameter $\mu = v_2(\alpha - \frac{1}{2}\sigma^2) > 0$ and has a long-run distribution with (A.4.2) as density function.

Solving (A.5.2) with respect to $\ln A$ we obtain the long-run optimal stock of deforested land. That is:

$$\ln A \simeq \frac{\ln\left[\frac{\eta_1-\lambda\eta_2}{r-\alpha}\right] + v_0 + v_1 \ln\left[\frac{\beta_2}{\beta_2-1}(r-\alpha)\frac{1-\lambda}{\eta_1-\lambda\eta_2}\frac{\delta}{r}\right] + v_2 \ln B - h}{\gamma v_1} \tag{A.5.3}$$

Taking the expected value on both sides of (A.5.3) brings to:

$$E[\ln A] \simeq \frac{\ln\left[\frac{\eta_1-\lambda\eta_2}{r-\alpha}\right] + v_0 + v_1 \ln\left[\frac{\beta_2}{\beta_2-1}(r-\alpha)\frac{1-\lambda}{\eta_1-\lambda\eta_2}\frac{\delta}{r}\right] + v_2\left[B_0 + (\alpha - \frac{1}{2}\sigma^2)t\right] - E[h]}{\gamma v_1}$$

Since by (A.4.3) $E(h)$ is indipendent on t, differentiating with respect t, we obtain the expected long-run rate of deforestation:

$$\begin{aligned}\frac{1}{dt}E[d\ln A] &\simeq \frac{v_2}{v_1}\frac{\alpha - \frac{1}{2}\sigma^2}{\gamma}\\ &= -\frac{\alpha - \frac{1}{2}\sigma^2}{\gamma}e^{\widetilde{\ln B} - \widetilde{\ln J}} \quad \text{for } \alpha < \frac{1}{2}\sigma^2\end{aligned}$$

By the monotonicity property of the logarithm it must exists $\widetilde{B}$ such that $\ln \widetilde{B} = \widetilde{\ln B}$. Further, by plugging $\widetilde{B}$ into (10), we can always find a surface $\tilde{A}$ and $\widetilde{J} = \frac{\beta_2}{\beta_2-1}(r-\alpha)\frac{1-\lambda}{\eta_1-\lambda\eta_2}\frac{P_A(\tilde{A})}{r}$ such that a linearization along $(\widetilde{\ln B}, \widetilde{\ln J})$ is equivalent to a linearization along $(\ln \widetilde{B}, \ln \widetilde{J})$, where and $\widetilde{\ln J} = \ln \widetilde{J}$. This implies that by setting $(\widetilde{B}, \tilde{A})$, the long-run average rate of deforestation can be written as:

$$\begin{aligned}\frac{1}{dt}E[d\ln A] &= -\frac{\alpha - \frac{1}{2}\sigma^2}{\gamma}\frac{\widetilde{B}}{\widetilde{J}}\\ &= -\frac{\alpha - \frac{1}{2}\sigma^2}{\gamma}\frac{1}{1 + \frac{\beta_2}{\beta_2-1}(r-\alpha)\frac{1-\lambda}{\eta_1-\lambda\eta_2}\frac{c}{\widetilde{B}}}\\ &= -\frac{\alpha - \frac{1}{2}\sigma^2}{\gamma}\frac{\frac{P_A(\tilde{A})}{r} - c}{\frac{P_A(\tilde{A})}{r}}\\ &= -\frac{\alpha - \frac{1}{2}\sigma^2}{\gamma}\left(1 - \frac{c}{\frac{\delta}{r}\tilde{A}^{-\gamma}}\right)\end{aligned}$$

where $\frac{P_A(\tilde{A})}{r} = \frac{\widetilde{B}}{\frac{\beta_2}{\beta_2-1}(r-\alpha)\frac{1-\lambda}{\eta_1-\lambda\eta_2}} + c$ and $\tilde{A} < \hat{A}$.

Similarly, from the logarithm of (18) we derive:

$$\begin{aligned}\ln \varsigma &= \ln\left[\frac{\beta_1}{\beta_1 - 1}(1-\lambda)\frac{P_A}{r} + \frac{\lambda\eta_2 - \eta_1}{r-\alpha}B\right]\\ &= \ln\left[\frac{\lambda\eta_2 - \eta_1}{r - \alpha}\right] + \ln[K + B]\end{aligned} \tag{A.5.4}$$

where $K{=}\frac{\beta_1}{\beta_1-1}\left(r-\alpha\right)\frac{1-\lambda}{\lambda\eta_2-\eta_1}\frac{P_A(A)}{r}$ and $K>-B$. Rewriting $\ln\left[K+B\right]$ as $\ln\left[e^{\ln K}+e^{\ln B}\right]$ and expanding by Taylor's theorem around the point $(\widetilde{\ln K}\ ,\widetilde{\ln B})$ leads to:

$$
\begin{aligned}
\ln\left[K+B\right] &\simeq \ln\left[e^{\widetilde{\ln K}}+e^{\widetilde{\ln B}}\right]+\frac{\ln K-\widetilde{\ln K}}{1+e^{\widetilde{\ln B}-\widetilde{\ln K}}}+\frac{\ln B-\widetilde{\ln B}}{1+e^{-(\widetilde{\ln B}-\widetilde{\ln K})}}\\
&= w_0+w_1\ln K+w_2\ln B
\end{aligned}
$$

where

$$
\begin{aligned}
w_0 &= \ln\left[e^{\widetilde{\ln K}}+e^{\widetilde{\ln B}}\right]-\left[\frac{\widetilde{\ln K}}{1+e^{\widetilde{\ln B}-\widetilde{\ln K}}}+\frac{\widetilde{\ln B}}{1+e^{-(\widetilde{\ln B}-\widetilde{\ln K})}}\right]\\
w_1 &= \frac{1}{1+e^{\widetilde{\ln B}-\widetilde{\ln K}}},\ w_2=\frac{1}{1+e^{-(\widetilde{\ln B}-\widetilde{\ln K})}},\ \frac{w_2}{w_1}=\frac{1-w_1}{w_1}>0
\end{aligned}
$$

By plugging the approximated value for $\ln\left[K+B\right]$ into (A.5.4) we obtain:

$$
\ln\varsigma\simeq\ln\left[\frac{\lambda\eta_2-\eta_1}{r-\alpha}\right]+w_0+w_1\ln K+w_2\ln B \tag{A.5.5}
$$

Also in this case, $\ln\varsigma$ evolves according to the motion $d\ln\varsigma=w_2 d\ln B=w_2[(\alpha-\frac{1}{2}\sigma^2)dt+\sigma dw]$ and $\ln\hat{\varsigma}$ is a reflecting barrier. By setting $h=\ln\varsigma$, $\ln\varsigma$ evolves as a linear Brownian motion with parameter $\mu=w_2(\alpha-\frac{1}{2}\sigma^2)>0$ and (A.4.2) as density function. Then, solving (A.5.5) with respect to $\ln A$ yields the long-run optimal stock of cleared land:

$$
\ln A\simeq\frac{\ln\left[\frac{\lambda\eta_2-\eta_1}{(r-\alpha)}\right]+w_0+w_1\ln\left[\frac{\beta_1}{\beta_1-1}\left(r-\alpha\right)\frac{1-\lambda}{\lambda\eta_2-\eta_1}\frac{\delta}{r}\right]+w_2\ln B-h}{\gamma w_1} \tag{A.5.6}
$$

The expected value of $\ln A$ is given by:

$$
E\left[\ln A\right]\simeq\frac{\ln\left[\frac{\lambda\eta_2-\eta_1}{(r-\alpha)}\right]+w_0+w_1\ln\left[\frac{\beta_1}{\beta_1-1}\left(r-\alpha\right)\frac{1-\lambda}{\lambda\eta_2-\eta_1}\frac{\delta}{r}\right]+w_2\left[B_0+(\alpha-\frac{1}{2}\sigma^2)t\right]-E\left[h\right]}{\gamma w_1}
$$

By the usual steps:

$$
\begin{aligned}
\frac{1}{dt}E\left[d\ln A\right] &\simeq \frac{w_2}{w_1}\frac{\alpha-\frac{1}{2}\sigma^2}{\gamma}\\
&= \frac{\alpha-\frac{1}{2}\sigma^2}{\gamma}e^{\widetilde{\ln B}-\widetilde{\ln K}}\quad\text{for}\ \ \alpha>\frac{1}{2}\sigma^2
\end{aligned}
$$

As shown above, fixed a pair $(\widetilde{B},\tilde{A})$ we can conveniently linearize along $(\ln\widetilde{B},\ln\widetilde{J})$ where $\ln\widetilde{B}=\widetilde{\ln B}$ and $\widetilde{\ln K}=\ln\widetilde{K}$ with $\widetilde{K}{=}\frac{\beta_1}{\beta_1-1}\left(r-\alpha\right)\frac{1-\lambda}{\eta_2\lambda-\eta_1}\frac{P_A(\tilde{A})}{r}$. This leads to the following expression for the long-run average rate of deforestation:

$$\begin{aligned}
\frac{1}{dt}E\left[d\ln A\right] &= \frac{\alpha-\frac{1}{2}\sigma^2}{\gamma}\frac{\widetilde{B}}{\widetilde{K}} \\
&= \frac{\alpha-\frac{1}{2}\sigma^2}{\gamma}\frac{1}{\frac{\beta_1}{\beta_1-1}\left(r-\alpha\right)\frac{1-\lambda}{\lambda\eta_2-\eta_1}\frac{c}{\widetilde{B}}-1} \\
&= \frac{\alpha-\frac{1}{2}\sigma^2}{\gamma}\frac{c-\frac{P_A\left(\tilde{A}\right)}{r}}{\frac{P_A\left(\tilde{A}\right)}{r}} \\
&= \frac{\alpha-\frac{1}{2}\sigma^2}{\gamma}\left(\frac{c}{\frac{\delta}{r}\tilde{A}^{-\gamma}}-1\right)
\end{aligned}$$

where $\frac{P_A\left(\tilde{A}\right)}{r}=c-\frac{\widetilde{B}}{\frac{\beta_1}{\beta_1-1}(r-\alpha)\frac{1-\lambda}{\lambda\eta_2-\eta_1}}$ and $\tilde{A}>\hat{A}$.

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