Resources rent collection and tradable permit programs: market efficiency and firm dynamics.

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Abstract

We develop a general equilibrium model of firm dynamics in a regulated natural resource industry where the total production is given by the authority and in which there is a system of Individual Transferable Quotas. Although there is uncertainty regarding productivity at firm level, we are able to develop a highly tractable model. Then, we introduce taxes on output trading permits to show that these taxes are distortionary, opposed to previous results in the literature, and there is no simple mechanism to offset this distortion. We then show how taxes on profits are also distortionary, but with the appropriate policy on redeemable investment, an efficiency-neutral system of rent collection can be implemented in such industry.

1 Introduction

Weninger and Just [7] analysis of firm dynamics with tradable output permit extended the previous static Montgomory [6] analysis. They study the effect

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of idiosyncratic firm uncertainty on the value of a firm, firm entry and exit behavior, the price of tradable permits and industry-level production efficiency. They show that the value of capital used in the production process has nonseparable effects on equilibrium permit prices and market efficiency.

The importance of these findings is concerned with understanding the role of rent-collection instruments on entry decisions for a given distribution of firms. Namely, if the productivity of the marginal entering plant is affected by fiscal instruments and if the rent-collection method has nonseparable effects on market efficiency. Therefore not all rent collecting instruments will have the property of market efficiency neutrality. Weninger and Just [7] show that in their framework a tax on the permit price is neutral while tax on the operating profits is non neutral. As they pointed out, this claim has important policy implications. In the U.S. fishing profits are taxed as personal or business income by the Internal Revenue Service. Hence, their finding is that this scheme is harmful for the economy.

In this paper we analyze the market efficiency neutrality of permit output and operating profit taxes in industries with firm dynamics and tradable output permits, providing a general equilibrium extension of the model in Weninger and Just [7]. As they do, we consider a model where firms face idiosyncratic productivity uncertainty, also similar to Hopenhayn [3] and Hopenhayn and Rogerson [4]. As Luttmer [5], we use the forward Kolmogorov equation to analytically characterize the endogenous distribution of entering plants. Rather than assuming exogenous permit output prices we characterize it as a function of the firm productivity distribution. Our analysis of these policy distortions is similar to Da-Rocha and Pujolàs [1].

We prove that a tax on the output permit price is not market efficiency neutral. If prices are distorted, conditions for entry and remaining in the industry are distorted as well. Hence the productivity of the marginal entering plant and the productivity of the marginal exiting plant are affected. Furthermore, the non arbitrage condition for owning quota and renting quota is also distorted, and there is no simple manner to offset this distortion.

Moreover, we prove that taxes on operating profits can be market efficiency neutral if the cost of entry to the industry can be redeemable at an equivalent rate. In this case, the distortion caused on operating profits is offset by the distortion on the entry cost. Thus, the marginal incentive for remaining in the industry stays the same as in the market efficient case. We also show that if this system is stablished, then pure rents are collected in this economy.

The paper is organized in the following manner. We start out by describing the economy in section 2. In section 3 we characterize the steady state. Section 4 shows how market efficiency is affected by taxes on operating profits and taxes on permit output prices. The final section, 5, concludes.

2 Model

Consider a natural resource industry where there is a continuum with measure N of heterogeneous active firms who faces an operating cost,

$$c(h, \varphi) = h^2/\varphi$$

to produce h units of output. As Weninger and Just [7], firms face idiosyncratic (firm specific) productivity uncertainty. Therefore, we allow φ to vary across firms and over time. To approximate the above assumption, we determine that productivity φ of a given firm evolves according to the following a geometric Brownian motion stochastic process

$$\frac{d\varphi}{\varphi} = \alpha dt + \sigma dz, \tag{1}$$

where $\alpha < 0$ is the expected growth rate, σ is the standard deviation (per unit volatility), and dz is the random increment to a Wiener process. We assume that these parameters satisfy $\frac{1}{2}\sigma^2 < -\alpha < 2\sigma^2$ in order to guarantee stationarity.

We assume that a permit lease market develop. Furthermore, a firm needs to pay a fixed cost of operation, c_f, measured in units of consumption good if she wants to remain in the industry. Therefore active firm with q permits and productivity φ solves

$$\max_h (p-r)h - \frac{h^2}{\varphi} + rq - c_f$$

where p is the constant unit output price, r is the single period lease rate per unit of permit. Active's optimal output supply is

$$h(\varphi) = \frac{(p-r)}{2}\varphi,$$

and its operating profits are equal to

$$\pi(\varphi, q) = \frac{(p-r)^2}{4}\varphi + rq - c_f$$

We also assume that there is a fixed entry cost, c_{entry}, measured in consumption good units to enter the economy. A firm collects the operating profits and retain the option to reassess the exit decision in the following period.

We assume that it is more profitable for a firm to remain in the market than to shut down and start a new one by assuming that $\rho c_{entry} > c_f$. In words, the cost of remaining one more period is smaller than the discounted cost of entering the market with a new firm.

If a firm exits the industry, she sells the output permit at price p_q. The firm chooses to remain in the economy by solving

$$W(\varphi, q) \quad = \max_{exit} \left\{ p_q q, \pi(\varphi, q) + (1+\rho dt)^{-1} EW(\varphi + d\varphi, q + dq) \right\},$$

$$s.t. \begin{cases} \pi(\varphi, q) = \dfrac{(p-r)^2}{4}\varphi + rq - c_f \\ \\ \frac{d\varphi}{\varphi} = -\alpha dt + \sigma dz \end{cases}$$

As in Weninger and Just [7], the mechanism that triggers exit from the industry is the fixed operating cost (the economic capital cost) and the permit holding cost, $p_q q$. Both costs imply a minimum firm productivity level, a threshold φ^*, that separates the continuation and abandonment region of the state space. Following proposition characterizes the firm productivity level in a stationary equilibrium where $q = q'$.

Proposition 1. *The minimum plant productivity level, φ^*, and the discounted present value of an active firm, $W(\varphi, q)$, is given by*

$$W(\varphi, q) = \frac{(p-r)^2}{4(\rho-\alpha)}\left[\varphi - \underline{\varphi}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] - \left[\frac{c_f - rq}{\rho}\right]\left[1 - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] + p_q q\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} \tag{2}$$

where

$$\underline{\varphi}(q) = \frac{4\beta(\rho-\alpha)}{(\beta-1)(p-r)^2}\left[\frac{(c_f - qr)}{\rho} + p_q q\right] \tag{3}$$

and

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} \tag{4}$$

Proof See appendix A.1.

Equations (2) and (3) highlight the distorting role operating cost plays in this economy. An increase in the operating cost, all other things equal, decreases the option value of remaining in the economy. Therefore, higher operating cost implies a higher minimum productivity level. This is a selection effect. The higher the operating cost, the higher the productivity of the marginal firm that remains active in the industry.

Finally, we assume that potential entering firms make their entry decision taking the productivity distribution, $G(\varphi)$ as given.[1] That is, we assume that the potential entrant optimally decides whether to engage in production and how much invest in q permit output before observing their realized draw φ. Therefore, entrants invest in q permit output by solving

$$\frac{\partial \int_{\underline{\varphi}}^{\infty} W(\varphi, q) f(\varphi) d\varphi}{\partial q} = p_q,$$

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[1] Throughout the paper, we assume the entry distribution is given by $g(\varphi)$ and the final distribution is given by $f(\varphi)$. Since we assume imitation and the distribution $f(\varphi)$ is normalized to 1, when we need to normalize the distribution $g(\varphi)$ to 1, as it is the case, we use the distribution $f(\varphi)$.

where the permit price is determined as in Weninger and Just [7] by a competitive entry condition

$$\int_{\underline{\varphi}}^{\infty} \left[W(\varphi, q) - p_q q \right] f(\varphi) d\varphi = c_{entry}. \tag{5}$$

3 Steady state

Operating and entry cost affect firm operating profits and entry and delay-exit decisions. Therefore the stationary distribution of plants productivity, obtained by using forward Kolmogorov equations subject to boundary conditions determined by the optimal delay-exit decisions, is endogenous and it may depend on both costs. That is, associated with the Brownian process, the measure of plant follows the following Kolmogorov forward equation

$$\frac{\partial f(\varphi, q, t)}{\partial t} = -\alpha \frac{\partial f(\varphi, q, t)}{\partial \varphi} + \frac{\sigma^2}{2} \frac{\partial^2 f(\varphi, q, t)}{\partial \varphi^2}$$

with boundary condition $f(\underline{\varphi}, q, t) = 0 \ \forall t$.

We restrict ourselves to symmetric equilibrium, where all firms hold the same number of permits, q. As Luttmer [5] we assume that the potential entering firm imitate existing firm $dG(\varphi) = \epsilon f(\varphi)$. Therefore,

$$\alpha \frac{\partial f(\varphi|\varphi^*)}{\partial \varphi} + \frac{\sigma^2}{2} \frac{\partial^2 f(\varphi|\varphi^*)}{\partial \varphi^2} + \epsilon f(\varphi|\varphi^*) = 0 \tag{6}$$

determines the distribution of new entrants

$$dG(\varphi|\varphi^*) = \frac{1}{2} \frac{\alpha^2}{\sigma^2} f(\varphi|\varphi^*), \tag{7}$$

where [2]

$$f(\varphi|\varphi^*) = \left(\alpha/\sigma^2\right)^2 (\varphi - \varphi^*) e^{-\alpha/\sigma^2(\varphi - \varphi^*)}. \tag{8}$$

It is now possible to define a steady state equilibrium in this economy. In an steady state equilibrium prices r and p_q will be constant. As we will see, operating firms problem will determine the steady state rental price of quotas,

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[2] See Lemma 2 in [5].

r. Given this price, the zero profit condition for entry of firm will determine the steady-state quota price p_q. Therefore, we can define the steady state equilibrium as follows.

Definition. *Given* c_{entry} *and* c_f*, a steady state equilibrium is a cut-off* $\underline{\varphi}$ *a lease rate,* r*, an output permit price,* p_q *and a quantity of quota per firm,* q*, such that:*

a) *Individual delay-exit decision is optimal*

$$\underline{\varphi}(q) = \frac{4\beta(\rho-\alpha)}{(\beta-1)(p-r)^2}\left[\frac{(c_f-qr)}{\rho}+p_q q\right]$$

b) *the lease rate per unit of quota clears the market*

$$\int_{\underline{\varphi}}^{\infty}(h(\varphi)-q)f(\varphi|\underline{\varphi})d\varphi = 0.$$

c) *individual demand for quota is optimal*

$$\frac{\partial\int_{\underline{\varphi}}^{\infty}W(\varphi,q)f(\varphi)d\varphi}{\partial q} = p_q.$$

d) *and free entry is guaranteed free entry*

$$\int_{\underline{\varphi}}^{\infty}\left[W(\varphi,q)-p_q q\right]f(\varphi)d\varphi = c_{entry}.$$

In an partial equilibrium analysis, the cutoff productivity level is found by solving the firm delay exit problem. However, when endogenous firm productivity distribution is taken into account, threshold productivity value depends on permit output prices. In the following propositions we characterize the four variables of this economy.

The following proposition states that there exists a relationship between lease per period and property permit output price in the form of a non-arbitrage condition.

Proposition 2 *The permit output price is equal to the net present values of the lease permit price. That is* $p_q = r/\rho$.

Proof See appendix A.2.

Therefore, the former arbitrage condition reduces the equilibrium to three variables. Next proposition characterizes the symmetric steady state firm productivity distribution as a function of the Brownian process, the discount rate and the entry and operating cost.

Proposition 3. *In a steady state equilibrium, the minimum firm productivity level, permit output prices and quota per firm is given by:*

$$\underline{\varphi} = \frac{-\beta \frac{2\sigma^2}{-\alpha}}{\frac{\rho c_{entry}}{c_f}(1-\beta) + 1 - \int_{\underline{\varphi}}^{\infty} \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} f(\varphi) d\varphi}$$

$$r = p - 2\sqrt{\frac{\rho-\alpha}{2\sigma^2}\frac{-\alpha}{\rho}\left(\rho c_{entry} + \frac{c_f}{1-\beta}\left(1 - \int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} f(\varphi) d\varphi\right)\right)}. \tag{9}$$

and quota per firm, q,

$$q = \frac{(p-r)}{2}\left(\underline{\varphi} - \frac{2\sigma^2}{\alpha}\right). \tag{10}$$

Proof See appendix A.3.

4 Market efficiency

Market efficiency depends on the abandonment threshold productivity value, $\underline{\varphi}$. This threshold productivity value separates the continuation and the abandonment value of the productivity state.

We are interested on the effects that two different policies have on the productivity outcome, which is measured with the cutoff in the economy, $\underline{\varphi}$.

The two policies are, respectively, a tax on the output price, τ_q, which makes firms to pay an extra cost when they want to enter the economy to produce, and a tax on profits, τ_π, which collects taxes from the operational profits of firms.

In the setup studied by Weninger and Just [7], they propose that taxes on the traded quota do not distort the equilibrium allocation, because on the one hand they reduce the value of active firms, so it forces inefficient firms to exit, while on the other hand it decreases the value of operating a firm, which lowers the amount of firms and, in turn, both effects cancel each other out. Different than their analysis, in our model the quota price is determined in equilibrium by the demand of entering firms and the supply of those that exit the economy, and it is precisely in their demand for quota that the equilibrium allocation is distorted, since they have to pay a higher tax, which makes the previous non arbitrage condition to be distorted.

Since this tax is paid by firms entering the economy, Proposition 1 from the previous part stays the same, and the only difference is on parts (c) and (d) of the equilibrium definition. This tax makes these two parts to be

c) *individual demand for quota is optimal*

$$\frac{\partial \int_{\underline{\varphi}}^{\infty} W(\varphi, q) f(\varphi) d\varphi}{\partial q} = p_q \left(1 + \tau_q\right)$$

d) *and free entry is guaranteed free entry*

$$\int_{\underline{\varphi}}^{\infty} \left[W(\varphi, q) - p_q q \left(1 + \tau_q\right)\right] f(\varphi) d\varphi = c_{entry}.$$

Next proposition shows that taxes on quota are not neutral on the cutoff.

Proposition 4. *In a steady state equilibrium with a tax on traded quota, the permit output price is smaller than the net present value of the lease permit price,*[3] *that is*

$$\frac{r}{\rho} \frac{1}{1 + \frac{\tau_q}{1 - \int_{\underline{\varphi}}^{\infty} \left(\frac{\underline{\varphi}}{\varphi}\right)^{\beta} f(\varphi) d\varphi}} = p_q$$

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[3] Note that $1 - \int_{\underline{\varphi}}^{\infty} \left(\frac{\underline{\varphi}}{\varphi}\right)^{\beta} f(\varphi) d\varphi > 0$ because the term $\left(\frac{\underline{\varphi}}{\varphi}\right)^{\beta} < 1, \forall \varphi > \underline{\varphi}$

and the cutoff level $\underline{\varphi}$ is smaller than it would be without the tax

$$\underline{\varphi} = \frac{-\beta\frac{2\sigma^2}{-\alpha}}{\frac{c_{entry}}{\frac{c_f}{\rho} - \frac{\tau_q q \frac{r}{\rho}}{1-\int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} f(\varphi)d\varphi + \tau_q}}(1-\beta) + 1 - \int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} f(\varphi)d\varphi}$$

while q and r have the same expressions.

Proof See appendix A.4.

Thus, we have seen that in this more general setup, taxes on the transactions for quota are distortive in equilibrium, and thus they are not neutral to the market efficient allocation. Furthermore, it seems difficult to obtain a policy that offsets this negative effect, since it is not possible to make q, or r, to be 0.

Next, we show how a tax on operational profits distort the equilibrium allocation as well, and we propose a simple mechanism that can offset this distortion. Obviously, a tax on operational profits distort the value of entering the economy with respect to its entry cost. The mechanism to acheive tax collection without distortion is, precisely, to use the revenues form tax collection to pay a subsidy on the entry cost.

Proposition 5. *In a steady state equilibrium, with a tax on operating profits τ_π and capital subsidy s the minimum firm productivity level is given by:*

$$\underline{\varphi} = \frac{-\beta\frac{2\sigma^2}{-\alpha}}{\frac{\rho c_{entry}(1-s)}{c_f(1-\tau_\pi)}(1-\beta) + 1 - \int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} f(\varphi)d\varphi}$$

Corollary. *A system whose tax rate equals the redeemable investment rate collects pure rents and does not distort.*

Proof See appendix A.5.

Thus, we have shown that taxes on opearting profits are, in general, distortionary in equlibrium as well. However, we have provided an easy mechanism by which efficiency is restored in this case, and which produces pure rent-collection.

5 Conclusion

In this paper we have extended the benchmark model of a dynamic industry with Individual Transferable Quotas first proposed by Weninger and Just [7]. Our extension can be seen as a general equilibrium version which accounts for the market of quotas, in which firms decide to buy and sell their rights for harvesting.

Contrary to the stablished results in which a tax on the quota transaction are market efficiency neutral, we have shown that it does distort and that there is no simple manner to offset this result.

Furthermore, we have also shown that, although in the general case a tax on profits is also distorting - which is already pointed out in Weninger and Just - we have shown that a mechanism of redeemable investment is a non-distortive instrument for rent colleciton.

References

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A Appendix

A.1 Proof proposition 1

We follow Dixit and Pindyck [2] to solve this exit-delay problem. We know that the firm chooses to stay in the market as long as the firm has a positive option value. Let profits be

$$\pi(\varphi) = a_1\varphi - a_0$$

where

$$a_1 = \frac{(p-r)^2}{4},$$

and

$$a_0 = c_f - rq.$$

Hence,

$$W(\varphi) = B_1\varphi^{\beta} + A_1\varphi - A_0$$

where $\beta < 0$.Given that

$$\rho W(\varphi) = \pi(\varphi) - \alpha W'(\varphi)\varphi + \frac{\sigma^2}{2}W''^2$$

then

$$A_1 = \frac{a_1}{\rho - \alpha},$$

and

$$A_0 = \frac{a_0}{\rho}.$$

To find B, and $\underline{\varphi}$, note that Given that

$$W(\underline{\varphi}, q) = p_q q$$

$$W'(\underline{\varphi}, q) = 0$$

Then

$$B_1\underline{\varphi}^{\beta} + A_1\underline{\varphi} - A_0 = p_q q$$

$$\beta B_1\underline{\varphi}^{\beta-1} + A_1 = 0$$

Then

$$\underline{\varphi} = \frac{\beta}{(\beta-1)}\left[\frac{A_0 + p_q q}{A_1}\right]$$

and

$$W(\varphi, q) = A_1\left[\varphi - \underline{\varphi}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] - \left[\frac{c_f - rq}{\rho}\right]\left[1 - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] + p_q q\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}$$

which gives the desired result.■

A.2 Proof proposition 2

$$\begin{aligned}\frac{\partial}{\partial q}\int_{\underline{\varphi}}^{\infty} W(\varphi, q) f(\varphi) d\varphi &= \frac{r}{\rho}\left[1 - \int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} f(\varphi)d\varphi\right] + p_q\int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} f(\varphi)d\varphi = p_q \\ &\Rightarrow \frac{r}{\rho} = p_q\end{aligned}$$

A.3 Proof proposition 3

if $p_q = r/\rho$, then

$$W(\varphi, q) = A_1 \left[\varphi - \underline{\varphi}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] - \left[\frac{c_f}{\rho}\right]\left[1 - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] + p_q q$$

and

$$\begin{aligned} \underline{\varphi} &= \frac{\beta}{(\beta-1)}\left[\frac{A_0 + p_q q}{A_1}\right] = \frac{\beta}{(\beta-1)}\left[\frac{c_f}{\rho}\frac{1}{A_1}\right] \\ &\Rightarrow \quad A_1 = \frac{\beta}{(\beta-1)}\left[\frac{A_0 + p_q q}{\underline{\varphi}}\right] = \frac{\beta}{(\beta-1)}\frac{c_f}{\rho}\frac{1}{\underline{\varphi}} \end{aligned}$$

Therefore

$$\begin{aligned} & \int_{\underline{\varphi}}^{\infty} \left[W(\varphi, q) - p_q q\right] f(\varphi) d\varphi \\ = & \int_{\underline{\varphi}}^{\infty} \left\{A_1 \left[\varphi - \underline{\varphi}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] - \left[\frac{c_f}{\rho}\right]\left[1 - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right]\right\} f(\varphi) d\varphi = c_{entry} \end{aligned}$$

implies that

$$\begin{aligned} \frac{\rho c_{entry}}{c_f} &= \int_{\underline{\varphi}}^{\infty} \left\{\frac{\beta}{(\beta-1)}\left[\frac{\varphi}{\underline{\varphi}} - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] - \left[1 - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right]\right\} f(\varphi) d\varphi \\ \underline{\varphi} &= \frac{-\beta \frac{2\sigma^2}{-\alpha}}{\frac{\rho c_{entry}}{c_f}(1-\beta) + 1 - \int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} f(\varphi) d\varphi} \end{aligned}$$

From the cutoff expression we get an equation for the rental price, r

$$r = p - \sqrt{\frac{4\beta(\rho - \alpha)}{(\beta - 1)\underline{\varphi}}\frac{c_f}{\rho}}$$

replacing the value for the cutoff

$$r = p - 2\sqrt{\frac{\rho - \alpha}{2\sigma^2}\frac{-\alpha}{\rho}\left(\rho c_{entry} + \frac{c_f}{1-\beta}\left(1 - \int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} f(\varphi) d\varphi\right)\right)}$$

Finally, for the quota, we have that

$$\int_{\underline{\varphi}}^{\infty} (h(\varphi) - q) f(\varphi | \underline{\varphi}) d\varphi = 0$$

which implies that

$$q = \frac{(p-r)}{2} \left(\underline{\varphi} - \frac{2\sigma^2}{\alpha} \right)$$

A.4 Proof proposition 4

$$\begin{aligned}
\frac{\partial}{\partial q} \int_{\underline{\varphi}}^{\infty} W(\varphi, q) f(\varphi) d\varphi &= \frac{r}{\rho} \left[1 - \int_{\underline{\varphi}}^{\infty} \left(\frac{\varphi}{\underline{\varphi}} \right)^{\beta} f(\varphi) d\varphi \right] + p_q \int_{\underline{\varphi}}^{\infty} \left(\frac{\varphi}{\underline{\varphi}} \right)^{\beta} f(\varphi) d\varphi = p_q \left(1 + \tau_q \right) \\
&\Rightarrow \frac{r}{\rho} = p_q \left(1 + \frac{\tau_q}{1 - \int_{\underline{\varphi}}^{\infty} \left(\frac{\varphi}{\underline{\varphi}} \right)^{\beta} f(\varphi) d\varphi} \right)
\end{aligned}$$

Hence, in this case,

$$W(\varphi, q) = A_1 \left[\varphi - \underline{\varphi} \left(\frac{\varphi}{\underline{\varphi}} \right)^{\beta} \right] - \left[\frac{c_f}{\rho} \right] \left[1 - \left(\frac{\varphi}{\underline{\varphi}} \right)^{\beta} \right] + p_q q$$

then

$$A_1 = \frac{\frac{(p-r)^2}{4}}{\rho - \alpha} \varphi,$$

and

$$A_0 = \frac{c_f - rq}{\rho}.$$

$$\begin{aligned}
\underline{\varphi} &= \frac{\beta}{(\beta - 1)} \left[\frac{\frac{c_f - rq}{\rho} + p_q q}{\frac{\frac{(p-r)^2}{4}}{\rho - \alpha}} \right] = \frac{\beta}{(\beta - 1)} \left[\frac{\frac{c_f}{\rho} - \frac{p_q \tau_q}{1 - \int_{\underline{\varphi}}^{\infty} \left(\frac{\varphi}{\underline{\varphi}} \right)^{\beta} f(\varphi) d\varphi}}{\frac{\frac{(p-r)^2}{4}}{\rho - \alpha}} \right] \\
\Rightarrow \frac{\frac{(p-r)^2}{4}}{\rho - \alpha} &= \frac{\beta}{(\beta - 1)} \left[\frac{\frac{c_f}{\rho} - \frac{p_q \tau_q}{1 - \int_{\underline{\varphi}}^{\infty} \left(\frac{\varphi}{\underline{\varphi}} \right)^{\beta} f(\varphi) d\varphi}}{\underline{\varphi}} \right]
\end{aligned}$$

Therefore

$$\begin{aligned}
&\int_{\underline{\varphi}}^{\infty} \left[W(\varphi, q) - p_q q\right] f(\varphi) d\varphi \\
&= \int_{\underline{\varphi}}^{\infty} \left\{ \begin{array}{c} \frac{\beta}{(\beta-1)} \left[\frac{c_f}{\rho} - \frac{\tau_q p_q q}{1-\int_{\underline{\varphi}}^{\infty} \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} f(\varphi) d\varphi}\right] \left[\frac{\varphi}{\underline{\varphi}} - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] \\ - \left[\frac{c_f}{\rho} - \frac{\tau_q p_q q}{1-\int_{\underline{\varphi}}^{\infty} \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} f(\varphi) d\varphi}\right] \left[1 - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] \end{array} \right\} f(\varphi) d\varphi = c_{entry}
\end{aligned}$$

implies that

$$\begin{aligned}
\frac{c_{entry}}{\frac{c_f}{\rho} - \frac{\tau_q p_q q}{1-\int_{\underline{\varphi}}^{\infty} \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} f(\varphi) d\varphi}} &= \int_{\underline{\varphi}}^{\infty} \left\{ \frac{\beta}{(\beta-1)} \left[\frac{\varphi}{\underline{\varphi}} - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] - \left[1 - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] \right\} f(\varphi) d\varphi \\
\underline{\varphi} &= \frac{-\beta \frac{2\sigma^2}{-\alpha}}{\frac{c_{entry}}{\frac{c_f}{\rho} - \frac{\tau_q p_q q}{1-\int_{\underline{\varphi}}^{\infty} \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} f(\varphi) d\varphi}} (1-\beta) + 1 - \int_{\underline{\varphi}}^{\infty} \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} f(\varphi) d\varphi}
\end{aligned}$$

Sbustituing the expression of the quota price delivers the desired result.

The expression for r, and the expression for q do not change since they are developed from equations that are not affected by this tax.

A.5 Proof Proposition 5 and Corollary

We follow Dixit and Pindyck [2] to solve this exit-delay problem. We know that the firm chooses to stay in the market as long as the firm has a positive option value. Let profits be

$$\pi(\varphi) = (1 - \tau_{\pi}) \left[a_1 \varphi - a_0\right]$$

where

$$a_1 = \frac{(p-r)^2}{4},$$

and

$$a_0 = c_f - rq.$$

Hence,

$$W(\varphi) = B_1\varphi^{\beta} + A_1\varphi - A_0$$

where $\beta < 0$.Given that

$$\rho W(\varphi) = \pi(\varphi) - \alpha W'(\varphi)\varphi + \frac{\sigma^2}{2}W''^{2}$$

then

$$A_1 = (1-\tau_{\pi})\frac{a_1}{\rho - \alpha},$$

and

$$A_0 = (1-\tau_{\pi})\frac{a_0}{\rho}.$$

To find B, and $\underline{\varphi}$, note that Given that

$$W(\underline{\varphi}, q) = p_q q$$

$$W'(\underline{\varphi}, q) = 0$$

Then

$$B_1\underline{\varphi}^{\beta} + A_1\underline{\varphi} - A_0 = p_q q$$

$$\beta B_1\underline{\varphi}^{\beta-1} + A_1 = 0$$

Then

$$\underline{\varphi} = \frac{\beta}{(\beta-1)}\left[\frac{A_0 + p_q q}{A_1}\right]$$

and

$$W(\varphi, q) = A_1\left[\varphi - \underline{\varphi}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] - (1-\tau_{\pi})\left[\frac{c_f - rq}{\rho}\right]\left[1 - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] + p_q q\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}$$

Then

$$\frac{\partial}{\partial q}W(\varphi, q) = \frac{(1-\tau_{\pi})r}{\rho}\left[1 - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] + p_q\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta} = p_q \Rightarrow \frac{(1-\tau_{\pi})r}{\rho} = p_q$$

Therefore if $\frac{(1-\tau_{\pi})r}{\rho} = p_q$, then

$$W(\varphi, q) = A_1\left[\varphi - \underline{\varphi}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] - \left[\frac{(1-\tau_{\pi})c_f}{\rho}\right]\left[1 - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] + p_q q$$

and

$$\underline{\varphi} = \frac{\beta}{(\beta-1)}\left[\frac{A_0 + p_q q}{A_1}\right] = \frac{\beta}{(\beta-1)}\left[\frac{(1-\tau_\pi)c_f}{\rho}\frac{1}{A_1}\right] \Rightarrow A_1 = \frac{\beta}{(\beta-1)}\frac{(1-\tau_\pi)c_f}{\rho}\frac{1}{\underline{\varphi}}$$

Therefore

$$\int_{\underline{\varphi}}^{\infty}\left[W(\varphi,q) - p_q q\right]dG(\varphi) = \int_{\underline{\varphi}}^{\infty}\left\{A_1\left[\varphi - \underline{\varphi}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] - \left[\frac{c_f}{\rho}\right]\left[1 - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right]\right\}dG(\varphi) = (1-s)c_{entry}$$

implies that

$$\int_{\underline{\varphi}}^{\infty}\left\{\frac{\beta}{(\beta-1)}\left[\frac{\varphi}{\underline{\varphi}} - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right] - \left[1 - \left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\right]\right\}dG(\varphi) = \frac{\rho(1-s)c_{entry}}{(1-\tau_\pi)c_f}$$

$$\underline{\varphi} = \frac{-\beta\frac{2\sigma^2}{-\alpha}}{\frac{\rho c_{entry}(1-s)}{c_f(1-\tau_\pi)}(1-\beta) + 1 - \int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}f(\varphi)d\varphi}$$

Then if $s = \tau_k$, it is clear that the outcome is the efficient allocation one.

The value function can be rewritten as (replacing the cutoff inside the value funciton) and using the non arbitrage condition

$$W = (1-\tau_\pi)\frac{(p-r)^2}{4}\varphi\frac{1}{\rho-\alpha} - (1-\tau_\pi)\frac{c_f - rq}{\rho} + (1-\tau_\pi)\frac{c_f}{\rho}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\frac{-1}{\beta-1}$$

There is a mass $\frac{\alpha^2}{2\sigma^2} < 1$ of entrants with respect to the incumbents. Hence, normalizing the size of the distribution of incumbents to one, using the free entry condition, we have that the total cost of this policy, S, is equal to

$$
\begin{aligned}
S &= s\frac{\alpha^2}{2\sigma^2}\ c_{entry} = s\frac{\alpha^2}{2\sigma^2}\frac{1}{1-s}\int_{\underline{\varphi}}^{\infty}\left[W(\varphi,q)-p_q q\right]f(\varphi)d\varphi \\
&= s\frac{\alpha^2}{2\sigma^2}\frac{1}{1-s}\int_{\underline{\varphi}}^{\infty}\left(\begin{array}{c} (1-\tau_\pi)\frac{(p-r)^2}{4}\varphi\frac{1}{\rho-\alpha} \\ -(1-\tau_\pi)\frac{c_f}{\rho}\left(1-\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\frac{-1}{\beta-1}\right)\end{array}\right)f(\varphi)d\varphi \\
&= s\frac{\alpha^2}{2\sigma^2}\left(\begin{array}{c} \int_{\underline{\varphi}}^{\infty}\frac{(p-r)^2}{4}\varphi\frac{1}{\rho-\alpha}f(\varphi)d\varphi \\ -\frac{c_f}{\rho}\left(1-\int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\frac{1}{1-\beta}f(\varphi)d\varphi\right)\end{array}\right) \\
\int_{\underline{\varphi}}^{\infty}\frac{(p-r)^2}{4}\varphi f(\varphi)d\varphi &= \left(\begin{array}{c} \frac{S}{s\frac{\alpha^2}{2\sigma^2}} \\ +\frac{c_f}{\rho}\left(1-\int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\frac{1}{1-\beta}f(\varphi)d\varphi\right)\end{array}\right)(\rho-\alpha)
\end{aligned}
$$

The total amount of taxes collected with this policy, P, are equal to

$$
\begin{aligned}
P &= \tau_\pi\int_{\underline{\varphi}}^{\infty}\left(\frac{(p-r)^2}{4}\varphi+rq-c_f\right)f(\varphi)d\varphi \\
&= \tau_\pi\int_{\underline{\varphi}}^{\infty}\varphi\frac{(p-r)^2}{4}f(\varphi)d\varphi+\tau_\pi rq-\tau_\pi c_f \\
\int_{\underline{\varphi}}^{\infty}\varphi\frac{(p-r)^2}{4}f(\varphi)d\varphi &= \frac{P}{\tau_\pi}+c_f-rq
\end{aligned}
$$

Hence, we can combine both expressions to get that $S < P$ if

$$
\begin{aligned}
s\frac{\alpha^2}{2\sigma^2}\ c_{entry} &< \tau_\pi\int_{\underline{\varphi}}^{\infty}\varphi\frac{(p-r)^2}{4}f(\varphi)d\varphi+\tau_\pi rq-\tau_\pi c_f \\
\frac{\alpha^2}{2\sigma^2}\ c_{entry}+c_f &< \left(\underline{\varphi}-\frac{2\sigma^2}{\alpha}\right)\left(\frac{(p-r)^2}{4}+r\frac{(p-r)}{2}\right) \\
\frac{\frac{\alpha^2}{2\sigma^2}\ c_{entry}+c_f}{\frac{p-r}{2}\left(\frac{p-r}{2}+r\right)} &< \left(\underline{\varphi}-\frac{2\sigma^2}{\alpha}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{\sqrt{\frac{\rho-\alpha}{2\sigma^2}\frac{-\alpha}{\rho}\left(\rho c_{entry}+\frac{c_f}{1-\beta}\left(1-\int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}f(\varphi)d\varphi\right)\right)}} \\
&\frac{\frac{\alpha^2}{2\sigma^2}\ c_{entry}+c_f}{\sqrt{\frac{\rho-\alpha}{2\sigma^2}\frac{-\alpha}{\rho}\left(\rho c_{entry}+\frac{c_f}{1-\beta}\left(1-\int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}f(\varphi)d\varphi\right)\right)}+r} \\
<\ &\left(\underline{\varphi}-\frac{2\sigma^2}{\alpha}\right)
\end{aligned}
$$

Recall that the cutoff is

$$
\underline{\varphi}=\frac{-\beta\frac{2\sigma^2}{-\alpha}}{\frac{\rho c_{entry}}{c_f}(1-\beta)+1-\int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}f(\varphi)d\varphi}
$$

$$
\frac{\frac{\rho}{\rho-\alpha}\left(\frac{\alpha^2}{2\sigma^2}\ \frac{c_{entry}}{c_f}+1\right)}{1+\frac{r\sqrt{\frac{\rho}{\rho-\alpha}\frac{2\sigma^2}{-\alpha}}}{\sqrt{\left(\frac{\rho c_{entry}}{c_f}+\frac{1}{1-\beta}\left(1-\int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}f(\varphi)d\varphi\right)\right)}}}<\frac{\rho c_{entry}}{c_f}+1-\int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\frac{1}{1-\beta}f(\varphi)d\varphi
$$

This inequality holds if

$$
\begin{aligned}
\frac{\rho}{\rho-\alpha}\left(\frac{\alpha^2}{2\sigma^2}\ \frac{c_{entry}}{c_f}+1\right) &< \frac{\rho c_{entry}}{c_f} \\
\frac{\alpha^2}{2\sigma^2}\ \frac{c_{entry}}{c_f}+1 &< \frac{(\rho-\alpha)\,c_{entry}}{c_f}
\end{aligned}
$$

because the denominator of the first term is larger than one and $1-\int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}}\right)^{\beta}\frac{1}{1-\beta}f(\varphi)d\varphi>0$ because $\left(\frac{\underline{\varphi}}{\varphi}\right)^{\beta}<1$ for all φ.

The inequality holds if

$$
\frac{\alpha^2}{2\sigma^2}+\frac{c_f}{c_{entry}}<\rho-\alpha
$$

This inequality can be rewritten in the following form

$$
c_f<c_{entry}\left(\rho-\alpha\left(1+\frac{\alpha}{2\sigma^2}\right)\right)
$$

Given our assumptions on parameters, this restriction holds.