Mining-induced desiccation and consequent impact on traditional economic livelihood – an analytical framework

Lekha Mukhopadhyay[1], Bhaskar Ghosh[2]
[1]Department of Economics, Jogamaya Devi College, 92 S.P Mukherjee Road, Calcutta 700026 India. E-mail: lekhamukherjee@msn.com
[2]Department of Geology, Jogamaya Devi College, 92 S.P. Mukherjee Road, Calcutta 700026 India. E-mail: bghosh2006@gmail.com

Abstract

In the context of rising conflict of interest between industry-driven rampant mining and long-term sustainable traditional economy, an analytical model is proposed to derive social optimal mine extraction path and social welfare path. These paths include the sustainability of traditional livelihood and depend upon the shadow cost of mining-induced desiccation. If society sets a maxi-min rule of sacrificing some traditional output for promoting mine production, the model determines the required minimum amount of water and optimal social weight to be given by social planner on traditional welfare following Solow-Hartwick-Brundtland's (SHB) sustainability criteria. Finally to control mining induced desiccation in congruence with SHB criteria, two alternative tax-measures are proposed and compared: on the rate of mine-resource depletion; and on mining-induced loss of traditional output. Both as construed are dynamic in nature irrespective of size of the communities, and significant from policy perspective.

Key words: Base flow, Desiccation dynamics, Dynamic taxation, Extraction of exhaustible resource, Groundwater recharge, Solow-Hartwick-Brundtland sustainability; Shadow cost, Social optimal mine extraction path, Surface runoff

Abbreviations used: MA: Mining activities, MID Mining induced desiccation, PSWB: Particular surface water body, SWB: Surface water bodies

Introduction

Mining induced desiccation of water sources has made significant negative impact on economic livelihood of indigenous communities all over the world (Hilson 2002, Shiva 1991). Liberalization in mine licensing policy due to globalization and increasing demand from industrial economies bring about rampant mining activities (MA), which in some cases are severe threats to the survival of traditional economic livelihood. Lack of irrigation water, coupled with pollution from manganese mining, sharply reduced the rice yields of 60 percent of the farmers in and around Gaodong, China (Jigang and Chuhua, 2008). In Nahi-Kala and Thano villages in Doon Valley, India limestone mining caused siltation and obstruction to some first order stream flows like Bhitarli, Kiarkuli, Arnigad and Baldi in addition to huge soil erosion and land slide (Kumar, 2000), which led to water crisis and consequent fall in agricultural productivity

Resolving the conflict of interests between modern growing industrial economy and long term sustainable traditional economy is becoming a challenge to the government in many developing countries today (Sahu, 2008). MA cannot fully compensate the loss they make to traditional economic livelihood (Singh, 2007). People from traditional communities dependent on the mine lose their livelihood after mining stops, and they cannot go back to their traditional livelihood as MA changes the local biogeochemical environment. Hence there arises a question of trade-off: how much a society should sacrifice the long term loss in traditional production which had been sustaining for hundreds of years to gain benefit from mining operations in the economy? Can one think of development inclusive of the sustainability of traditional economic livelihood?

With this ground reality at the backdrop, this paper contains some theoretical exercises in the arena of natural resource management. It describes the change in the process of natural environment (quantity of water in a particular surface water body (PSWB)) caused by anthropogenic disturbances (mining induced) to exhibit how it may change the economically determined social extraction path of exhaustible resource (mine resources) under different sustainability criteria.

Distribution and quantity of water in a PSWB (streams, lakes, ponds etc.) and groundwater reservoirs (aquifers) depend on various factors like precipitation, evaporation, permeability of surface layers, surface runoffs, capacity of soil to absorb moisture etc. Any natural and anthropogenic change in one or more of these factors may cause disruption in the inflow and outflow of water in the surface water bodies, and recharge, flow and discharge of groundwater, which in turn affects the availability of water in the local water sources. Limnological effects of anthropogenic desiccation and their consequent impact on aquatic species, possible abatement measures have been studied (Beutel, et.al 2001; Yuan, 2004, et.al) with various physical hydrological, hydro geochemical and environmental science perspectives.

Long term control of exhaustible resources in general (Gray 1914; Hotelling 1931; Dasgupta and Heal 1979; Dasgupta 2001) and of mine resource in particular has been separately

addressed in the context of different market structures, discoveries of new reserves, availability of substitutes, different kinds of uncertainty (Stiglitz 1974; Dixit and Pindyck 1994). The sustainable path i.e. path of intergenerational and intertemporal equity in consumption has also been devised for an economy with exhaustible resources (Farzin 2006; Martinet 2008, Martinet and Doyen 2007). But the formulation of long term control of exhaustible resources (which may cause anthropogenic desiccation of PSWB) by internalizing the externally induced cost of desiccation is yet to be done.
The present paper attempts to incorporate the hydrological explanation of desiccation of PSWB in economic methodologies in deriving social extraction path of mine resource thereby narrow down the existing knowledge gaps mentioned above. These theoretical exercises are pre-requisites for any future empirical research and policy formulations in this context.

2. Mining induced changes in quantity of water in surface water bodies

2.1 MID dynamics

Annual change in the quantity of water in the t^{th} year in a PSWB is determined by (i) direct precipitation (P_t), (ii) evaporation from the PSWB (E_t), (iii) water added to PSWB by surface run-off (S_t) and (iv) net addition of water from surface water – ground water interaction in the PSWB (G_t). G_t is the difference between the water flowing into PSWB by base flow (b_t) and that flowing out of the PSWB by groundwater recharge (r_t). Thus water balance equation at t^{th} year becomes:

$$\eta_t = P_t - E_t + S_t \pm G_t \ldots\ldots \text{(2.1)}$$

MA can bring about significant changes in S_t and G_t.

Now as MA occurs, mine extraction not only produces desired material Y but also the wastes (W). Let $W = k_{WY}Y$, where k_{WY} is the quantity of waste material produced per unit production of Y. Then MA at time t generates $Y + W = (1 + k_{WY})Y$. Now In (2.1) above, S_t which is an important contributor in surface water bodies among many factors (assumed to remain the same) depends upon moisture retaining capacity of the soil (μ). After MA starts, precipitated water is not only absorbed and retained by natural soil but also by the waste materials (W) and thus with increase in W water inflow in PSWB by surface run-off reduces.

$$S = S(\mu(W)) = S(\mu(k_{WY}Y)) = k_{WY}S(\mu(Y))$$

Thus at the marginal level of MA, mining induced change in S is $\frac{\partial S}{\partial \mu}\frac{\partial \mu}{\partial Y} = k_{WY} S_Y < 0$

The last component of (2.1) i.e. $G = (b - r)$ depends upon the location of the PSWB (whether existing in recharge area or discharge area) and its water level relative to the water table. As MA starts in most of the cases some groundwater seeps into the excavation site which is evaporated or absorbed in soil after being pumped out. Also, a considerable quantity of groundwater is withdrawn for the consumption of the mining community. In both the cases, the mining-induced loss of groundwater lowers the water table and consequently hydraulic gradient h_B of groundwater discharge (in case a PSWB receives base flow) decreases and hydraulic gradient h_G of groundwater recharge (if PSWB is located in recharge area) increases. As a result b declines and r increases attributing to desiccation in PSWB. Since mining induced change in $G = (b - r)$ occurs altogether by $Y + W = (1 + k_{WY})Y$, at the marginal level of MA, mining induced change in G is $(1 + k_{WY})(b - r)$. In reality in the process of recharge and discharge, water moves much more slowly than direct runoff and thus we with regard to equation. Hence it is quite plausible that negative marginal impact of MA on S at time t dominates over the negative marginal impact of MA on G. Thus we assume that:

$$\left|S_Y\right| > \frac{(1 + k_{WY})}{k_{WY}}\left|\left(b_Y - r_Y\right)\right|$$

Now let the change in water quantity be expressed as:

$$\dot{\eta} = \eta - \eta_Y Y \ \ldots\ldots\ (2.2)$$

i.e. change in quantity of water in a PSWB over time is determined by the water quantity determined in the natural process by P_t, E_t, S_t and G_t less the quantity of water due to mining induced desiccation. From the above discussion it follows that for each marginal unit of mining production Y the change in water quantity in PSWB is given by: $\eta_Y = k_{WY} S_Y + (1 + k_{WY})\left(b_Y - r_Y\right)$.

3. Social optimal mining plan under the circumstance of MID: a model

General outline

There exists a hypothetical community of individuals who share a land with water and mineral resources. A subset of the communities comprises aboriginal people dependent alone on traditional economic activity namely hunting / fishing a species from surface water bodies (a common pool resource). Their economic livelihood and water consumption are assumed to be dependent on availability of water in SWB. The rest of the individuals are settled after launching of mining activities. Their primary interest lies in mining production from the exhaustible mineral resources of the area. Thus there is a conflict of interest between the people dependent on mining production and those eking out their livelihood from traditional production. There is a 'social planner' whose role is to co-ordinate between the interests of private economic agents to maximize the social welfare.

3.1 *Social optimal mining plan (SOMP)*

In our community model, we assume that change in the quantity of water in PSWB is mining induced. MA produces desired material Y along with some mine wastes $W = k_{WY}Y$. In Gray's (1914) framework marginal cost of extraction of an individual mine $C_Y > 0$ and $C_{YY} > 0$. C_Y is assumed to be an increasing function of extraction flow, i.e. $Y(t) + W(t) = (1 + k_{WY})Y_t$ (which of course, is a little deviation from Gray (ibid) as W was not taken into account separately). [N2]. Now, given the competitive price p (assuming that the miner is a price taker, which is quite plausible for a miner in developing country) given the stock of mine resources $R(t)$, extraction of $Y(t)$ creates profit $(= pY(t) - (1 + k_{WY})C(Y(t); R(t))$ to the private miner but at the cost of decrease in traditional production Ω due to mining induced desiccation. The social planner's objective is to control $Y(t)$ i.e. the optimal rate of extraction from a given mine resource stock in such a way that would (i) maximize total discounted social welfare comprising the welfare of the miner and traditional producer, and (ii) ensure a minimum socially acceptable level of $\Omega(t)$, possible by a minimum amount of water quantity say η_b where $\eta \geq \eta_b$

3.2. *Production function of traditional community* (Ω) *and impact of MID on it*

Traditional production $\Omega(a;\eta)$ depends upon a the labour employed and η, the quantity of water, here treated as 'natural capital'. $\Omega(a;\eta)$ declines over time as mining induced desiccation occurs.

The relation between a and η is very important to determine the potential sustainability of Ω. We use the most popular form used in the literature in this context. It is the Cobb-Douglas production function presuming the substitutability of η by a up to a certain extent (say $\eta = \overline{\eta}$). Let,

$$\Omega(a(t);\eta(t)) = \eta^{\theta} a^{\beta}\text{; where, } \theta < \beta < 1\text{; } \eta \geq \overline{\eta} > 0 \ldots\ldots(3.1)$$

$\Omega_a > 0,\ \Omega_\eta > 0;\ \Omega_{aa} < 0,\ \Omega_{\eta\eta} < 0$

As mining induced desiccation occurs for each unit decrease in η, Ω decreases. We assume the possibility that as mining induced desiccation occurs, production starts declining; the traditional community strives to maintain the same level of output by forced substitution (imperfect) of η by a i.e. deploying more a to compensate the loss attributed by decrease in η. This case is similar to the case with η as a 'weak essential' resource in the sense of Dasgupta and Heal (1974). η is needed in production but with an unbounded potential production up to a certain limit of $\eta \geq \overline{\eta}$ This concept will be used in the following sections.

3.3. Social optimal rate of mine resource extraction as MID occurs

The composite welfare of the society that a social planner aims to maximize, is constituted by miner's utility i.e. $U_1(Y(t); p(t))$ plus the utility of traditional community i.e. their production $\Omega(a,\eta)$, weighted by σ, where $0 \leq \sigma \leq 1$. σ is a matter of choice of social planner i.e. how much weight or importance he would give to the loss of traditional production due to MID is the matter of his decision. In dynamic optimal control theory frame work, social planner in her optimization problem has now one control variable Y_t with two state variables: (i) R_t the stock of mined resource and (ii) η_t the state of volume of water available in a PSWB at time t. The trajectory path of η_t that evolves over time is determined by MID dynamics, i.e. $\dot{\eta} = \eta - \eta_Y Y$ as it is modeled in Section (2.1). In addition to those, now the social planner has another constraint, i.e. not allowing η to decrease above η_b, the minimum volume of η that ensure sustainable level of production Ω_b. Her optimization problem therefore is:

$$max_Y \int_0^T U(Y,\Omega,t)e^{-\delta t}dt = max_Y \int_0^T [\bar{p}Y_t - (1+k_{WY})C(Y_t,R_t) + \sigma a_t^{\beta} \eta_t^{\theta}]e^{-\delta t}dt, \text{ .... (3.3) subject}$$

$$\text{to } \dot{R}_t = -(1+k_{WY})Y_t;\ R_{t=0} = R_0, \text{ and } R_t \geq 0\ \forall 0 < t \leq T;\ R_T \geq 0 \quad \text{..... (3.4)}$$

$$\dot{\eta} = \eta - \eta_Y Y \text{ .... (3.5);}$$

$$\eta \geq \eta_b > 0 \text{ ....(3.6)}$$

In the present context, (3.3) taking together (3.4) and (3.5) constitutes the Hamiltonian H, the objective function, which we need to maximize subject to the constraint (3.6). The augmented Hamiltonian in present value term therefore is:

$$H = \mathrm{H} + \varphi(\eta_b - \eta_t) = e^{-\delta t}U(Y,\Omega(\ );t,\sigma) - \lambda_R(1+k_{WY_i})Y + \lambda_\eta(\eta - \eta_Y Y) + \varphi(\eta_b - \eta_t) \text{ .... (3.7)}$$

Assuming that everything is continuously differentiable and those concavity assumptions hold, the first order conditions of this problem are:

$$\frac{\partial \mathrm{H}}{\partial Y} = 0 \Rightarrow \left[\bar{p} - (1+k_{WY})C'_Y + \sigma\theta\, a^{\beta}\, \eta^{\theta-1}\eta_Y\right]e^{-\delta t} = \lambda_R(1+k_{WY}) + \lambda_\eta \eta_Y \quad \text{... (3.7.1)}$$

$$\frac{\partial H}{\partial R} = -\dot{\lambda}_R : \qquad \dot{\lambda}_R = 0 \Rightarrow \lambda_R = \bar{\lambda}_R \text{ Constant...... (3.7.2)}$$

$$\frac{\partial H}{\partial \eta} = -\dot{\lambda}_\eta \Rightarrow \sigma\theta a^{\beta}\eta^{\theta-1}e^{-\delta t} + \lambda_\eta - \varphi = -\dot{\lambda}_\eta$$

$$Or, \dot{\lambda}_\eta + \lambda_\eta = \varphi - \sigma\Omega_\eta e^{-\delta t} \text{ .... (3.7.3) since } \theta a^{\beta} Q^{\theta-1} = \Omega_\eta$$

$\frac{\partial H}{\partial \varphi} \geq 0 \Rightarrow \eta_t \geq \eta_b$ with the complementary slackness condition $\varphi \geq 0$ and

$$\varphi \frac{\partial H}{\partial \varphi} = 0 \Rightarrow \eta = \eta_b \ldots \text{ (3.7.4)}$$

Now re-consider (3.7.1) with (3.7.4). The last term in LHS of (3.7.1) $\lambda_\eta \eta_Y$ is the cost to the discounted social benefit (from Y) of the mining induced desiccation. LHS of the equation (3.7.1) on the other hand, is the discounted value of marginal social benefit from mining production (after taking into account its impact on traditional production), after meeting the sustainability constraint $\eta \geq \eta_b$ equalized with RHS showing the cost to future benefits of a marginal increase in Y. Discounted value of marginal social benefit from mining decreases as the social planner ascribes greater value on σ. This is because after desiccation $\eta_Y < 0$ and the value of the second component of LHS of (3.1.1) is negative.

Again in (3.7.1), let $\overline{p} - \left(1 + \sum k_{WY}\right) b'_Y = \widetilde{p}$, the net price of Y which is assumed to take the functional form of inverse demand with γ as demand parameter $\widetilde{p} = e^{-\gamma Y}$. $\widetilde{p}$ is the social price that internalizes the marginal benefit (or cost) of traditional production Ω. The transversality condition on R i.e. $R_T \geq 0$ (equation 3.4) indicates the possibility that some resource may be left in the ground if at T, $\widetilde{p}$ is equal to minimum average social cost (mine extraction cost + cost on traditional production due to mining induced water pollution). $\lambda_T = 0$. Beyond T, $\widetilde{p} <$ average social cost. (Lasserre 1991). Equation (3.7.1) now becomes:

$$e^{-\gamma Y} = \left[\lambda_R \left(1 + k_{WY}\right) + \lambda_\eta \eta_Y\right] e^{\delta t} - \sigma \theta \Omega_\eta \eta_Y$$

Taking log on both sides, with $\lambda_R \left(1 + k_{WY}\right)$ being a constant, $ln\left[\lambda_R \left(1 + k_{WY}\right)\right] = \overline{\lambda}_R$ we get:

$$-\gamma Y = \delta t + \overline{\lambda}_R + \left(log \lambda_\eta - \sigma \, log \Omega_\eta\right) \eta_Y = \delta t + \overline{\lambda}_R + \eta_Y \, log\left(\lambda_\eta / \sigma \, \Omega_\eta\right) \ldots\ldots \text{ (3.8)}$$

$$Y^{SOC} = -\frac{1}{\gamma}\left[\delta t + \overline{\lambda}_R + \eta_Y \left(\lambda_\eta / \sigma \, \Omega_\eta\right)\right] \ldots\ldots \text{(3.9)}$$

3.3.1 *Shadow cost path of MID and social optimal mine extraction path*

Given the discount factor δ, demand parameter γ, η_Y, $\bar{\lambda}_R$ and, Ω_η equation (3.9) shows that Y_t^{SO} the social optimal rate of extraction at t depends upon λ_η the shadow cost path to social benefit due to MID. Until we solve them we cannot determine Y_t^{SO}. We re-consider equation (3.7.3):

$$\dot{\lambda}_\eta + \lambda_\eta = \varphi - \sigma\Omega_\eta e^{-\delta t}$$

This is a first order differential equation with the variable term $e^{-\delta t}$. Solving it in terms of initial value condition $\lambda_\eta = \lambda_{\eta 0}$ we get the time path of λ_η as:

$$\lambda_\eta = \left(1 - e^{-t}\right)\varphi - \sigma(1-\delta)\Omega_\eta\left(e^{-\delta t} - e^{-t}\right) - \lambda_{\eta_0} e^{-t} \text{ ..... (310)}$$

(Appendix A3). A numerical simulation with $\delta = 0.6$ for $0 \le t \le 20$; $\Omega_\eta = 0.5$; $\varphi = 0.1$, $\lambda_{\eta_0} = 0.3$ and $\sigma = 0.2$ shows that path of shadow cost λ_η is a concave curve which over the time increases at the decreasing rate (Figure 3.1). With increasing the weight σ on the loss of traditional production the λ_η curve will be lower.

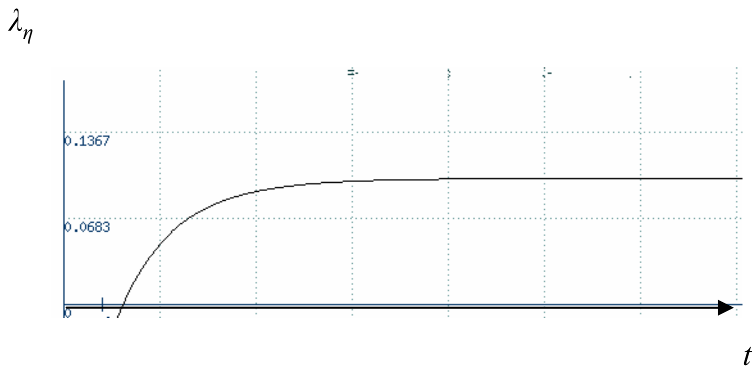

Figure 3.1: Numerical simulation of time path of $\lambda_\eta(t)$

Incorporating the value of λ_η from (3.10) into (3.9) we get the social optimal rate of extraction Y^{SOC}. Numerically simulating with $\delta = 0.6$ $\Omega_\eta = 0.5$, $\varphi = 0.1$, $\lambda_{\eta_0} = 0.2$, $\eta_Y = -0.6$ and $\sigma = 0.2$ we get

$$Y^{SOC} = -\frac{1}{\gamma}\left[\delta t + \bar{\lambda}_R + \eta_Y \log\left(\lambda_\eta / \sigma\,\Omega_\eta\right)\right] = -0.67t - 2.03$$

If the social planner targets to deplete mine reserve up to R_0 and $R_0 = 100$ then

$$R_0 = \int Y^{SOC} dt = -0.67\int t dt - 2.03\int dt$$

Numerically plotting this for $0 \le t \le 20$ the social optimal mine extraction path (Figure 3.2)

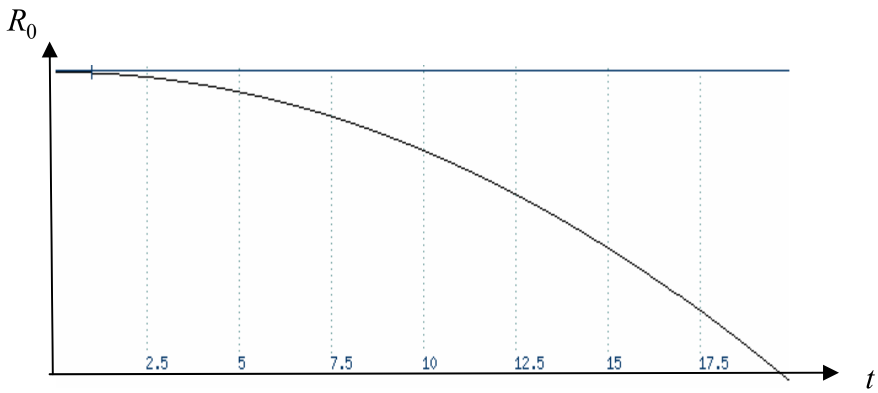

Figure 3.2: Social optimal mine resource extraction path (numerically simulated)

This path is concave downward indicating that social optimal rate of mine resource extraction (which is $\dot{R}_t$, the slope of the curve) should be diminishing at the increasing rate.

3.3.2 Importance of the relative weight given to the loss of traditional production in social welfare

We found above that for both shadow cost path of desiccation λ_η and social optimal mine extraction Y^{SOC}, the relative weight that social planner gives to the mining induced loss in traditional production is important. In the composite welfare $U(Y,\Omega;t,\sigma)$, the first one $\bar{p}Y_t - (1+k_{WY})C(Y_t,R_t)$ is the profit component of the miner say U_M and if we assume that cost of traditional production is nil, in the second component $a_t{}^{\beta}\,\eta_t^{\theta}$ at the normalized price can also be considered as the profit of traditional community say U_e. Then $U = U_M(Y(t);p(t)) + \sigma U_e(\eta)$ along a social indifference curve $U(\pi_M,\pi_e) = \overline{U}$; $\sigma = -dU_M / dU_e$ i.e σ is nothing but how much sacrifice in the benefit (vis-à-vis, profit) from traditional production a social planner will allow for each unit benefit (vis-à-vis, profit) from mine production in the society . In our given construct $U_e = \Omega$. Therefore after rearrangement we can express $\sigma = -\frac{dU_M/dY}{(d\Omega/d\eta)(d\eta/dY)}$. It is already shown in section 2.1 that $d\eta/dY$ i.e.,

$\eta_Y = k_{WY}S_Y + (1+k_{WY})(b_Y - r_Y)$This indicates that for each additional unit of profit from mining production how much loss in traditional production a social planner will allow i.e what value of σ he will assign given $d\Omega/d\eta$ i.e. Ω_η (which is a technology parameter) is to be determined by η_Y i.e. by the hydro-geological parameters k_{WY}, S_Y, b_Y and r_Y. Therefore if society allows to sacrifice some amount of traditional output in order to get more benefit from mine production and set the maximin rule for traditional community (i.e maximum of minimum acceptable quantity of traditional production which is possible at $\eta > \eta_b$ what should be the value of σ is a matter of choice by the social planner.

4 **Sustainable development path of the society as a whole and sustainability of traditional production under mining induced desiccation**

4.1 *Brundtland sustainability*

The sustainability criterion as reported by the Brundtland Commission1987, after the name of its chairperson is the "development that meets the needs of the present without compromising the ability of future generations to meet their own needs." In economic theory analytically, the approach to the sustainability issue has been made by deriving the "stationary equivalent" of the utilitarian optimal welfare path. Some constraints on social objectives i.e. optimization of social utility or welfare are set so that two important components of sustainability (Stavins et.al. 2003), viz., inter generational equity and efficiency - can be assured. As proposed by Solow (1974) it is obtained in terms of non-decreasing utility over time and optimizing the discounted value of utility (economic well-being).

In our present context if MID starts at time $t>0$ and the social planner wants to maximize the social utility: $U(t)=\int_t^{\infty} U(Y(s)+\Omega(a,\eta(Y(s)))e^{-\delta(s-t)}ds$, the Brundtland condition of sustainability requires that, intertemporal social utility U would not decrease over time t, i.e. $U'(t)\geq 0$. This is obtained by differentiating the above expression:

$$U'(t)=-U\left(Y(t)+\Omega\left(a(t),\eta(Y(t))\right)\right)+\delta U(t)\geq 0$$

$$=-\overline{p}Y(t)+(1+k_{WY})b(Y(t))-\sigma\Omega+\delta U(t)$$

This yield, $U(T_1)\geq \dfrac{\overline{p}Y_t-b[(1+k_{WY})Y_t]+\sigma a_t{}^{\beta}\eta_t^{\theta}}{\delta}$

The condition for an optimal sustainable utility path (when it exists) is obtained in two stages: first, by adopting the usual utilitarian approach to optimize a general utilitarian social welfare function in an infinite time horizon: $\int_{T_1}^{\infty}[\overline{p}Y_t-b[(1+k_{WY})Y_t+\sigma a_t{}^{\beta}\eta_t^{\theta}]e^{-\delta t}dt$ and, second, obtaining the condition under which this optimal path is constant over time (Farzin 2006). The optimal control problem and the solution are almost same as that we found in Section 3. The condition (3.6) i.e. $\eta\geq\eta_b$ takes the form $\lim_{t\to T}\lambda_\eta(t)[\eta(t)-\eta_b]e^{-\delta t}=0$. Solving it we obtain $\left(Y^*(t),[R^*(t),\eta^*(t)],\left[\lambda_R^*,\lambda_\eta^*\right]\right)$, the maximized current-value Hamiltonian along the optimal paths. This maximized current-value Hamiltonian, is the "stationary equivalent" of the utilitarian optimal welfare path (Weitzman 1976), i.e.

$$\int_{T_1}^{\infty}e^{-\delta(s-t)}H(t)ds=\int_{T_1}^{\infty}e^{-\delta(s-t)}[\overline{p}Y^*(s)-(1+k_{WY})b(Y^*(s))+\sigma\Omega(\eta(Y^*(s)))]ds \qquad \forall t\geq T_1$$

In this approach, the necessary and sufficient condition for permanently sustaining the highest consumption path (i.e., the maximin path) is that the maximized current-value Hamiltonian remains constant over time, i.e. that $\dfrac{\partial H}{\partial t}=0$ (Farzin, ibid). This implies:

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} + \delta\left[\lambda_R \dot{R} + \lambda_\eta \dot{\eta}\right] = 0 \text{ .... } (4.1)$$

For simplicity if we heuristically assume that direct and exogenous effects of time on the economy i.e. net "pure time effect" is nil, i.e. the economy is time- autonomous, the maximin sustainability cum optimality criterion (which also leads to Rawlsian criterion of intergenerational justice) that follows from equation (4.1) simply leads to:

$$\frac{\partial H}{\partial t} = \left[p - (1 + k_{WY})b'_Y + \sigma \Omega'_Y\right]\frac{\partial Y}{\partial t} = \delta\left[\lambda_R \dot{R} + \lambda_\eta \dot{\eta}\right] = 0 \text{ .... } (4.2)$$

This condition is none but the generalized version of the Solow-Hartwick's sustainability rule. It shows that well being of the society depends not only on Y^* but also on the depletion of the stock of natural capitalı. If the sustainability condition (4.2) is satisfied, from right hand side of this equation we get:

$$-\lambda_R \dot{R} = \lambda_\eta \dot{\eta} \text{ ...... } (4.2.1)$$

In our present context λ_R is the shadow cost of depletion of mine resource stock and ($-\lambda_R$) is the shadow price of keeping the mined resource conserved at time t. λ_η is the shadow cost of MID in PSWB. From (4.2.1), $-\dot{R} = \frac{\lambda_\eta}{\lambda_R}\dot{\eta}$. From this we reach a conclusion that in order to achieve sustainable path of development by reducing desiccation, mined resource R has to be conserved. At what rate R has to be conserved (i.e. mine production has to be sacrificed) depends upon the shadow cost MID and shadow benefit (of mine production) path over time. As it is further envisaged above, $-\dot{R}(t) = Y(t) + W(Y(t)) = (1 + k_{YW})Y(t)$, and therefore, along the sustainable path,

$$Y^* = \frac{\lambda_\eta}{\lambda_R (1 + k_{YW})}\dot{\eta} \text{ ...... } (4.2.2)$$

This (4.2.2) in other words explains that the domain of MID $\dot{\eta}$ along with λ_η and λ_R determines what should be the highest sustainable extraction path $Y^*(t)$ for the economy, which is also the "stationary equivalent" of the utilitarian optimal welfare path $U(Y^*(t), \Omega(Y^*(t)))$. This condition also determines a minimum critical stock of mine resource (threshold) that ought to be preserved to ensure at least Ω_b for the rest of the time concerned. Mining induced change in quantity of water over time $\dot{\eta} = \eta_Y \frac{\partial Y}{\partial t}$. From Section 2 we got $\eta_Y = k_{WY} S_Y + (1 + k_{WY})(b_Y - r_Y)$. Therefore

$$\dot{\eta} = \left[k_{WY} S_Y + (1 + k_{WY})(b_Y - r_Y)\right]\frac{\partial Y}{\partial t}$$

Thus, $-\dot{R} = \frac{\lambda_\eta}{\lambda_R}\left[k_{WY} S_Y + (1 + k_{WY})(b_Y - r_Y)\right]\frac{\partial Y}{\partial t}$. This further indicates that as MID occurs, in order to achieve the sustainable path of development at what rate R has to be conserved (vis-à-vis at what rate mine resource has to be depleted), will have to be determined by at what rate mine production reduces surface runoff S and reduces base flow b after increasing the groundwater recharge g. As we discussed in Section 2, these factors again are determined by the hydro geological parameters in the area concerned.

Whether the society will be able to meet the sustainability condition (4.2) or not also depends on the choice of the value of σ by the social planner. From condition (4.2) we can determine the required value of σ:

$p - (1 + k_{WY})b'_Y - \sigma(-\Omega'_Y) = 0$; since $-\Omega'_Y > 0$, and the net marginal profit $p - (1 + k_{WY})b'_Y = \tilde{p}$, then

$$\sigma^* = \frac{\tilde{p}}{(-\Omega'_Y)} \text{......} \ (4.3)$$

Thus the required σ^* will be the marginal net benefit from the mine production $(\tilde{\pi})$ over the marginal net loss of traditional output due to desiccation. Since both numerator and denominator of (4.3) are functions of time σ^* will also be time-variant.

4.3 *Sustainable development and management of mine production by means of taxation*

For a developing country with no control on market price (determined in the international market), to reach the goal of sustainable development inclusive the sustainability of traditional production social planner can introduce the social management of mine extraction by means of taxation. If her objective is to satisfy the Brundtland sustainability condition (4.2.1) she can set the tax structure principally in two ways. (i) Tax can be imposed on the rate of depletion of mine reserve $\left(-\dot{R}\right)$ that causes desiccation in PSWB, and (ii) it can be imposed on the amount of loss that MA makes in traditional output $\left(-\Omega'_Y\right)$ due to desiccation. In the first case the tax rate τ as a function of time will be derived directly from (4.2.1) i.e.

$\lambda_R\left(-\dot{R}\right)=\lambda_\eta \dot{\eta}$. Replacing $-\dot{R}$ by τ we get:

$$\tau=\frac{\lambda_\eta}{\lambda_R}\dot{\eta}=\frac{\lambda_\eta}{\lambda_R}\left[k_{WY}\,\eta_{_Y}^{SR}+\left(1+k_{WY}\right)\left(b_{_Y}-r_{_Y}\right)\right]\dot{Y}$$

In the second case re-arranging LHS of (4.2),

$\left[p-\left(1+k_{WY}\right)b'_Y-\left(-\Omega'_Y\right)\right]=0$. Replacing $-\Omega'_Y$ by τ we get:

$$Or, \tau = p-\left(1+k_{WY}\right)b'_Y=\widetilde{p}_\tau=e^{-\gamma Y_\tau}$$

In both the cases τ will be increasing exponentially along the sustainable path of development. In the former situation given λ_R, it increases with λ_η and rate of mine production $\left(\dot{Y}_t\right)$, given the hydro geological factors viz., k_{WY}, $S_{_Y}$, $b_{_Y}$ and $r_{_Y}$. In the latter case it increases with the rate of depletion $\left(-Y_\tau\right)$, given the demand parameter γ. In the first case tax rate is constrained by $\eta\geq\eta_b$, where from the production function of traditional output $\eta_b\geq\left(a^{-\beta}\,\Omega_b\right)^{1/\theta}\geq\overline{\eta}$ and $\Omega_b>0$. In the latter case the tax rate is constrained by $R\geq R_b>0$. In the former case evolution of tax rate is directly sensitive to the hydro geological condition of the study area concerned, while in the latter case, tax rate is determined by the degree of sensitivity of Ω to the mining induced desiccation.

5 Summary of results and conclusions

The proposed analytical model shows how optimal mine extraction path depends on the path of shadow cost of mining induced desiccation. The shadow cost path of desiccation increases at the decreasing rate. The social optimal rate of depletion of mine reserves becomes gradually faster over time. The stationary equivalent of the utilitarian optimal welfare path has been derived for a hypothetical society, from which we obtain a generalized version of the Solow-Hartwick's sustainability rule which further corroborates the Brundtland condition of sustainability. The sustainability condition shows that to ensure the minimum quantity of water for survival of production of traditional community, a certain amount of mined resource has to be conserved. The rate of conservation in that context is to be determined by shadow cost path of water desiccation and that of depletion of mine reserve. The sustainable path inclusive of sustainability of traditional production adds some constraint on stationary equivalent of the utilitarian optimal welfare path. If society allows to sacrifice some amount of traditional output in order to get more benefit from mine production and set the maximin rule for traditional community (i.e maximum of minimum acceptable traditional output) then the required target for minimum availability of water in PSWB will be determined by the production technology of the traditional community. Finally, if the social planner wants to introduce taxation as a means of achieving economic development inclusive of sustainability of traditional production, our analytical results suggest that tax rate should evolve dynamically incorporating various hydro geological parameters and the sensitivity of traditional production to MID.

Appendix

A3: The shadow cost path of desiccation can be obtained by $\dot{\lambda}_\eta + \lambda_\eta = \varphi - \sigma\Omega_\eta\, e^{-\delta t}$

Here, the integrating factor is $\int e^{dt} = e^{t}$. Multiplying both sides by e^{t} we get:

$$\lambda'_\eta\, e^{t} + \lambda_\eta\, e^{t} = \varphi e^{t} - \sigma\Omega_\eta\, e^{-\delta t + t}$$

$$or, \left(\lambda_\eta e^t\right)' = \varphi e^t - \sigma\Omega_\eta e^{(1-\delta)t}$$

Integrating both sides we get:

$$\lambda_\eta e^t = \varphi \int e^t dt - \int \sigma\Omega_\eta e^{(1-\delta)t} dt$$

$$= \varphi e^t - \sigma(1-\delta)\Omega_\eta e^{(1-\delta)t} + K$$

At $t = 0$, $\lambda_\eta = \lambda_{\eta_0}$ and thus, $K = \sigma(1-\delta)\Omega_\eta - \varphi - \lambda_{\eta_0}$. Plugging this value into the above equation, we get:

The shadow cost path of desiccation can be obtained by $\dot{\lambda}_\eta + \lambda_\eta = \varphi - \sigma\Omega_\eta e^{-\delta t}$

Here, the integrating factor is $\int e^{dt} = e^t$. Multiplying both sides by e^t we get:

$$\lambda'_\eta e^t + \lambda_\eta e^t = \varphi e^t - \sigma\Omega_\eta e^{-\delta t + t}$$

$$or, \left(\lambda_\eta e^t\right)' = \varphi e^t - \sigma\Omega_\eta e^{(1-\delta)t}$$

Integrating both sides we get:

$$\lambda_\eta e^t = \varphi \int e^t dt - \int \sigma\Omega_\eta e^{(1-\delta)t} dt$$

$$= \varphi e^t - \sigma(1-\delta)\Omega_\eta e^{(1-\delta)t} + K$$

At $t = 0$, $\lambda_\eta = \lambda_{\eta_0}$ and thus, $K = \sigma(1-\delta)\Omega_\eta - \varphi - \lambda_{\eta_0}$. Plugging this value into the above equation, we get:

$$\lambda_\eta = \varphi - e^{-\delta t}\sigma(1-\delta)\Omega_\eta - \left(\varphi - (1-\delta)\sigma\Omega_\eta + \lambda_{\eta_0}\right)e^{-t} = \left(1 - e^{-t}\right)\varphi - \sigma(1-\delta)\Omega_\eta\left(e^{-\delta t} - e^{-t}\right) - \lambda_{\eta_0} e^{-t}$$

References

Dasgupta, P., 2001, Human Well-Being and the Natural Environment, Oxford University Press

Dasgupta, P., Heal, G., 1979. Economic Theory and Exhaustible Resources. Cambridge University Press, Cambridge, UK.

Dixit, A. K., Pindyck, R. S., 1994. Investment under Uncertainty. Princeton University Press, Princeton, N.J.

Farzin, Y. H., 2006. Conditions for sustainable optimal economic development. Review of Development Economics 10 (3), 518 -- 534.

Gray, L., 1914. Rent under the assumption of exhaustibility. Quarterly Journal of Economics 28, 466 – 489.

Grüne, L., Kato, M., Semmler W., 2005. Solving ecological management problems using dynamic programming. Journal of Economic Behaviour and Organization 57, 448 – 473

Hartwick, J. M., 1990. Natural resources, national accounting and economic depreciation. Journal Public Economics 43, 291 – 304

Hilson, G., 2002. An overview of land use conflicts in mining communities. Land Use Policy 19 (1), 65 – 73.

Hotelling, H.,1931. The economics of exhaustible resources. Journal of political economy 39, 137–175

Jigang, Z., Chuhua, Z., 2008. In China's mining region, villagers stand up to pollution. Yale Environment 360, Yale School of Environment Studies, URL: http://e360.yale.edu/author/Zhou_Jiganang_and_Zhu_Chuhua/59/. Last accessed on December 22, 2010

Kumar, P., 2000. Estimation and economic evaluation of soil erosion: a case study of Doon valley in India. Paper presented at the Beijer Institute of ecological economics, Stockholm, Sweden

Lasserre, P., 1991. Long term control of exhaustible resources. Series on Fundamental of Pure and Applied Economics 49, Harwood academic publishers

Martinet, V., 2008. The viability framework: a new way to address the sustainability issue, in: Lopez, R.A. (ed), Progress in sustainable development research. Nova Science Publishers Inc., pp 143 –167.

Martinet, V., Doyen, L., 2007. Sustainability of an economy with an exhaustible resource: A viable control approach. Resource and energy economics 29, 17–39

Perrings, C., 1999. Comment on Ecological and social dynamics in simple models of eco-system management, in: Carpenter, S.R., Brock, W.A., Hanson, P. (eds.), Conservation Ecology 3 (2), pp 10.

Sahu, G., 2008. Mining in the Niyamgiri Hills and tribal rights. Economic and Political Weekly, April12, 19-21

Shiva, B., 1991. Ecology and the Politics of Survival: Conflicts over Natural Resources in India. Sage Publications, Thousand Oaks, California.

Singh, G., 2007. Minenvis: A newsletter of the ENVIS Centre on environmental problems of mining areas 54. Centre of mining environment/ Dept. of Environmental science and engineering. Indian Schools of Mines, Dhanbad, India

Solow, R. M., 1974. Intergenerational equity and exhaustible resources. Review of Economic Studies 41, 29-45

Stavins, R., Wagner, A., Wagner, G., 2003. Interpreting sustainability in economic terms: dynamic efficiency plus intergenerational equity. Economic letters 79, 339 – 343

Stiglitz, J. E., 1974. Growth with exhaustible natural resources: efficient and optimal growth paths. Review of Economic Studies 41, 123 – 137

Web Reference:

Mathematical Equation Plotting Software, URL: http://www.wessa.net/math.wasp, Last accessed: January 5, 2011