REDD and Optimal Carbon Credits Trading

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Abstract

REDD receives a great attention with an effective measure to give an incentive to developing countries to slow the decrease of forests. However, REDD is pointed out to have some problems ranging from the way to set baseline by which the issue of carbon credits is determined, to the uncertainty in estimating degradation accurately. Such problems may be solved if we set a trading ratio of carbon credits between the North and the South based on the forest management level in each country. Given the forest management level of each country, we analyze the trading ratio of carbon credit between the North and the South in terms of the social optimum in the North-South economy. We show that the trading ratio of credits is not one to one due to uncertainty. We also derive a trading ratio that is consistent with social optimum, which can be more or less than unity, depending on the type of uncertainty. Moreover, if we specify some functions, we show that the trading ratio will always be less than unity. Furthermore, we provide with a condition for a country with a greater forest management level in the past to be assigned a higher trading ratio. Finally, we demonstrate that if the forest management level is controllable for each country, then REDD does not in general achieve the social optimum.

1 Introduction

Reducing Emissions from Deforestation and Degradation (REDD) is a financial measure to give an incentive to developing countries to slow the decrease of forests. The measure aims to make the countries release smaller carbon stock by allowing carbon credits to be issued according to the area of forest that each nation saves from deforestation, compared to the baseline level of deforestation (Laurance 2007). Under the measure, the developing countries could financially benefit by trading the credits in the carbon market, and the conservation of forests can pay. In the latest United Nations Framework Convention on Climate Change (UNFCCC) 16th Conference of Parties, REDD was substantially progressed (Phelps et al., 2010).

However, REDD is pointed out to have some problems in implementation. First of all, to set baseline level for each country will face a criticism; baseline level for each country is determined by calculating a multi-year average of historical deforestation rates (Santilli, 2005; Miles and Kapos, 2008). However, this approach may result in an unfair setting of baselines, because the countries that made effort to restore forests in the past will have disadvantageous baselines, in that such countries cannot be admitted to issue enough amount of carbon credits for a given conservation, while on the other hand the countries with less past effort for conservation will have a large amount of credits (Ebeling and Yasue, 2008).

Second problem often pointed out is concerned about the accurate estimation of deforestation and degradation of forest. According to DeFries et al (2005), various methods are available to effectively monitor and verify tropical deforestation. However, the loss of carbon from forest degradation can be larger than from deforestation and the estimation results vary due to the lack of spatially specific data on biomass and the difficulty of identifying and measuring changes in biomass (Houghton, 2005). According to Meridian Institute (2009), degradation is largely caused by illegal and inadequate activities, such as illegal logging, non-mechanized traditional logging, unplanned conventional logging, excessive biomass extraction for fuel, shortening crop-fallow cycle, and forest fragmentation, which as a whole we call "illegal logging" in this paper. In fact, Laurance (1998) reports that 80% of Amazonian logging was illegal. Thus, in order to reduce degradation, it is effective to prevent the illegal logging, which will result in dropping uncertainty of the estimation of carbon stock of forest.

Uncertainty concerning carbon reduction must be very important in particular when the reduction of carbon becomes tradable but the level of reduction is uncertain. If the resulted reduction is lower than expected amount but the credit is issued according to the expected amount, then emission sourced from the credit is more than the amount reduced in the end. Taking this aspect into consideration, the trading system of carbon credit such as REDD might be harmful. The purpose of this paper is to provide with a rule of carbon credits trading between developed countries (the North) and the developing ones (the South) in an economy where carbon credits are issued under REDD with uncertainty regarding illegal logging.

In the literature, various economic viewpoints on REDD+ exist. For example, Phelps et al. (2010) shed a light on financial risk generated by uncertainty over some future variables such as carbon demand under REDD+. Fuss et al. (2011) focus on the impact of low-cost carbon reduction through REDD on investment in energy and on the development of clean technology. Bosetti (2011) also study the effects of RED on energy technology innovation. However, as far as we know, the relationship between uncertainty of estimation of change of carbon stock due to illegal logging and the trading ratio of credits has not yet been studied. If a credit generated in a country with a lower forest management level is discounted more, then such discounting might contribute to mitigating unfair problem of setting the baselines, because the country that has made a greater effort for conservation of forest may have a higher forest management level, and carbon credits issued by the country might face a slight discounting.

We analyze the trading ratio of carbon credit between the North and the South in terms of the social optimum in the North-South economy. We show that the trading ratio of credits is not one to one due to the uncertainty regarding the reduction of carbon emission due to deforestation. We also derive a trading ratio that is consistent with social optimum, which can be more or less than unity, depending on the type of uncertainty. Moreover, if we specify some functions, we show that the trading ratio will always be less than unity. Furthermore, we provide with a condition for a country that launched the conservation of the forest at an earlier date to be assigned a higher trading ratio in the REDD equilibrium. This condition is shown to be always satisfied when the social optimum results in a smaller reduction of deforestation for the country with the disadvantageous conditions with in REDD. Finally, we demonstrate that REDD does not in general achieve the social optimum if each country controls the forest management level. That is, while standard emission trading implements efficiency, REDD cannot be characterized as an instrument for efficiency.

Section 2 introduces the model and section 3 examines a general analysis. In section 4, we specify some functions and drive more detailed results on trading ratio. Section 5 discusses if REDD is compatible with social optimality when the forest management level is controllable for each country. Section 6 remarks our conclusion.

2 The Model

We consider an economy in which developed countries (the North) purchase carbon credits from developed countries (the South) issued under the REDD program. For simplicity, the number of the countries in the North is unity, while we assume that of the South to be n. The North is obliged to reduce carbon emission by $\bar{x}$, though it can purchase emission credits from the South generated through conserving forests under REDD. x expresses the carbon abatement of the North, so that $\bar{x} - x$ represents the amount purchased from the South. The North's abatement cost function, $C_n(x)$, is assumed to be convex, with $C_n(0) = 0$, $C_n' > 0$ and $C_n'' > 0$. $C_n(\cdot)$ is the minimized cost given $\bar{x}$, so emission trading without REDD is reflected in the function if such scheme exists. Without REDD, the price of carbon credits, p, is determined at the level $p = C_n'(\bar{x})$.

Let us incorporate REDD into our model. REDD allows carbon credits to be issued according to the area of forest that each nation saves from deforestation and degradation, compared to the baseline level of deforestation, which is defined as the BAU level. That is, the South can obtain carbon credits if it reduces deforestation from the level of the baseline. The reduction of the deforestation of the forests in the country i, compared to the BAU level, is represented by y_i. We sometimes refer to y_i as the conservation of the country i. This surely contributes to cut carbon emission sourced from forest. However, this contribution is partly lost by the expansion of illegal logging, as the illegal logging will be easier as the area of forest expands, compared with the baseline level. We suppose that illegal logging decreases the density of forest conserved, by which the sequestrated carbon in the forest will decline. This is the degradation of the forest in the context of REDD.

Due to this understanding, illegal logging should be considered to reduce the contribution of the South's conservation of forest to controlling climate change [1]. Let $\bar{M}^i$ be the level of carbon density of the forest without illegal logging per hectare conserved. As we assume, illegal logging reduces the level, which per hectare is expressed by the function $M^i(y_i, e_i, \epsilon) \in (0, \bar{M}^i)$, where e_i expresses the forest management level of the South and $\epsilon \in (-\infty, \infty)$ is the stochastic uncertainty regarding how many poachers succeed to enter into the forest and log the timbers illegally. We suppose $M^i_{y_i} > 0$ and $M^i_{e_i} < 0$. That is, illegal logging is increasing with the conserved area of the forest and decreasing with the level of the authority to manage. Thus, the level of sequestrated carbon is represented by

[1] Illegal logging is one of the major degradation activities (Meridian Institute, 2009).

$\bar{M}^i - M^i(y_i, e_i, \epsilon)$, so that total amount of carbon that will be prevented to be released under REDD is expressed by:

$$G^i(y_i, e_i, \epsilon) \equiv y_i(\bar{M}^i - M^i(y_i, e_i, \epsilon)) \tag{1}$$

We call this "carbon abatement" under REDD. We suppose that, if the South conserves a larger area, total abatement will increase, i.e.,

$$G^i_{y_i} = \bar{M}^i - M^i(y_i, e_i, \epsilon) - y_i M^i_{y_i}(y_i, e_i, \epsilon) > 0 \tag{2}$$

That is, we do not suppose the opposite case such that carbon emission from forest rises when the conserved area is increased.

Let α_i represent the rate for the carbon credit to be issued to the country i in the South against the amount of abatement implemented by the country through the REDD. α_i can be unity, but it is generally more or less than the value as we explain later under optimality. Therefore, the expected total amount of credit that the country i gains is equivalent to $\alpha_i E[G^i(y_i, e_i, \epsilon)]$. Hereafter we call α_i as "trading-ratio".

With respect to the cost of conservation, the country i's conservation cost function, the opportunity cost of the conservation, is expressed by $C_{s_i}(y_i)$ with the property of $C'_{s_i} > 0$ and $C''_{s_i} > 0$.

Finally we define the environmental damage function as $D(Z)$ with $D' > 0$ and $D'' > 0$, where Z is net total emission, expressed by

$$Z = \bar{z} - x - \sum_{k=1}^{n} G^k(y_k, e_k, \epsilon). \tag{3}$$

Here $\bar{z}$ is total emission before abatement. The above is the model we are going to study.

3 Optimal trading ratio of carbon credits

Trading ratio α_i is very closely related with efficiency of emission trading in the situation where uncertainty with respect to the environmental damage is present. To shed light on this aspect, we consider a simple case where the South can control the extent of deforestation but the forest management level is fixed for each country. Therefore, degradation per hectare by illegal logging is expressed by $M^i(y_i, \bar{e}_i, \epsilon)$ given $\bar{e}_i$, so that the relating abatement level is expressed by $G^i(y_i, \bar{e}_i, \epsilon) \equiv y_i(\bar{M}^i - M^i(y_i, \bar{e}_i, \epsilon))$.

The North is assumed to minimize the cost to comply the emission constraint. That is, the objective of the North is represented by:

$$\min_x C_n(x) + p(\bar{x} - x) \tag{4}$$

This leads to

$$C_n'(x) = p \tag{5}$$

On the other hand, the South minimizes the cost of reducing the deforestation, taking into consideration the fact that the reduction generates carbon credits under the REDD. That is, the country i in the South has the objective function as:

$$\min_{y_i} C_{s_i}(y_i) - \alpha_i p E\left(G^i(y_i, \bar{e}_i, \epsilon)\right), i = 1, \cdots, n. \tag{6}$$

This leads to

$$\frac{C_{s_i}'(y_i)}{pE\left(G_{y_i}^i(y_i, \bar{e}_i, \epsilon)\right)} = \alpha_i, i = 1, \cdots, n. \tag{7}$$

On the other hand, carbon credits market clearing condition is represented by:

$$\bar{x} - x = \sum_{k=1}^{n} \alpha_k E\left(G^k(y_k, \bar{e}_k, \epsilon)\right) \tag{8}$$

Let y express $(y_1, \cdots, y_n)$. The number of unknown variables (y, x, p) is $n+2$, which coincides with that of equations (5), (7) and (8). Thus the equilibrium solution can be obtained, which we denote by (y^r, x^r, p^r). (y^r, x^r, p^r) is referred to as REDD equilibrium.

Social optimum is defined as the conservation of the forest in the South and the abatement of emission in the North that minimize the total cost including the environmental damage, which is represented by solving

$$\min_{x, y_i} C_n(x) + \sum_{k}^{n} C_{s_k}(y_k) + E\left(D(Z)\right), i = 1, \cdots, n. \tag{9}$$

This leads to

$$C_n'(x) = E(D') \tag{10}$$

$$C_{s_i}'(y_i) = E\left(D' G_{y_i}^i(y_i, \bar{e}_i, \epsilon)\right), i = 1, \cdots, n. \tag{11}$$

Social optimum is expressed by (y^*, x^*). Let us find the trading ratio $\alpha^* \equiv (\alpha_1^*, \cdots, \alpha_n^*)$ with which REDD equilibrium coincides with the social optimum.

From the conditions (5), (7), (10) and (11), we derive "optimal" trading ratio α_i^*, under which social optimum is achieved. That is, the optimality requires

$$\frac{C_{s_i}'(y_i^r)}{C_n'(x^r)} = \frac{C_{s_i}'(y_i^*)}{C_n'(x^*)} \tag{12}$$

so that it holds

$$\frac{\alpha_i^* p E\left(G_{y_i^*}^i(y_i^*, \bar{e}_i, \epsilon)\right)}{p} = \frac{E\left(D' G_{y_i^*}^i(y_i^*, \bar{e}_i, \epsilon)\right)}{E(D')}, i = 1, \cdots, n. \tag{13}$$

Therefore, we obtain following optimal trading ratio .

$$\begin{aligned}
\alpha_i^* &= \frac{E\left(D' G^i_{y_i}(y_i^*, \bar{e}_i, \epsilon)\right)}{E(D')E\left(G^i_{y_i}(y_i^*, \bar{e}_i, \epsilon)\right)} \qquad (14) \\
&= \frac{E\left(D'\right)E\left(G^i_{y_i}(y_i^*, \bar{e}_i, \epsilon)\right) + Cov(D', G^i_{y_i^*}(y_i^*, \bar{e}_i, \epsilon))}{E(D')E\left(G^i_{y_i}(y_i^*, \bar{e}_i, \epsilon)\right)} \\
&= 1 + \frac{Cov(D', G^i_{y_i}(y_i^*, \bar{e}_i, \epsilon))}{E(D')E\left(G^i_{y_i}(y_i^*, \bar{e}_i, \epsilon)\right)}, i = 1, \cdots, n.
\end{aligned}$$

Then the sign of α_i^* is positive under (2), as $G^i_{y_i}$ is positive so that $D'G^i_{y_i} > 0$. From (14), the sign of $Cov(D', G^i_{y_i}(y_i^*, \bar{e}_i, \epsilon))$ determines whether α_i^* is greater, equal to or less than unity [2]. This is stated in the following proposition.

Proposition 1. *Under (2), the optimal trading-ratio α_i^* is always positive and expressed by (14). Moreover, (a) $\alpha_i^* > 1$ if $Cov(D', G^i_{y_i}(y_i^*, \bar{e}_i, \epsilon)) > 0$. (b) $\alpha_i^* = 1$ if $Cov(D', G^i_{y_i}(y_i^*, \bar{e}_i, \epsilon)) = 0$. (c) $\alpha_i^* < 1$ if $Cov(D', G^i_{y_i}(y_i^*, \bar{e}_i, \epsilon)) < 0$.*

Note that the optimal trading ratio α_i^* depends not only on the reduction of deforestation of the country i, y_i^*, but also on those of other developing countries y_j^* $(j \neq i)$ through D'. We will look how the optimal rates differ across the South countries.

4 Comparison of optimal trading ratios

In this section, we specify our model to clarify further properties of the level of optimal trading ratio and compare those rates of different countries. First of all, let as suppose

[2]In the literature of nonpoint source pollution, non-unity trading ratio between point source and nonpoint source is discussed (see, for example, Horan(2001); Horan and Shortle (2005)). In the literature, uncertainty regarding damage due to natural condition such as weather causes the ratio to be unequal to unity.

that the type of forest is identical and the function regarding the degradation of forest is specified as:

$$G^i(y_i, e_i) = y_i(\bar{M} - m^i(y_i, e_i)f(\epsilon)), \epsilon \in (-\infty, \infty) \tag{15}$$

where $m^i_{y_i} > 0, m^i_{e_i} < 0, m^i_{y_i e_i} < 0, f'(\epsilon) > 0$ and $\lim_{\epsilon \to -\infty} f(\epsilon) = 0$. $\bar{M}^i = \bar{M}$ implies that the carbon density of forest without illegal logging would be equivalent for all developing countries. Secondly, we suppose damage function $D(Z)$ is expressed as a quadratic function

$$D(Z) = \frac{a}{2}Z^2 \tag{16}$$

Then $Cov(D', G^i_{y_i}(y_i, \bar{e}_i, \epsilon))$ is calculated as [3]:

$$Cov(D', G^i_{y_i}(y_i, \bar{e}_i, \epsilon)) = -a\left(m^i(y_i, \bar{e}_i) + y_i m^i_{y_i}(y_i, \bar{e}_i)\right)\sum_{k=1}^{n}\left(\frac{Var(G^k(y_k, \bar{e}_k, \epsilon))}{y_k m_k(y_k, \bar{e}_k)}\right) \tag{17}$$

where $Var(G^k(y_k, \bar{e}_k, \epsilon))$ is variance of carbon reduction by forest conservation for the country k. In view of proposition 1, that the trading ratio is less than unity implies that the expected reduction of carbon by the South will be “discounted” when it is issued as carbon credits. It is also obvious that α^*_i will be smaller if $Var(G^k(y_k, \bar{e}_k, \epsilon))$ increases for some k. That is, if the uncertainty regarding total reduction of carbon under REDD increases, then the discount must be more strengthened. This is because such uncertainty can cause total emission much beyond the constraint $\bar{z}$, so that the damage can be more serious. Smaller trading ratio will play a role to alleviate the damage in such an extreme situation. These results are stated in the following proposition.

[3]See Appendix.

Proposition 2. *Suppose (15) and (16). Then the optimal trading ratio is less than unity, i.e.,* $\alpha_i^* < 1$ *for* $i = 1, \cdots, n$*. Moreover, the ratio declines if the level of uncertainty* $Var(G^k(y_k, \bar{e}_k, \epsilon))$ *for some* k *increases.*

Next we analyze how the optimal trading ratios differ across developing countries. Under the specifications (15) and (16), (14) leads to:

$$\begin{aligned}
\alpha_k^* =& 1 + \frac{-(m^k(y_k, \bar{e}_k) + y_k m_{y_k}^k(y_k, \bar{e}_k))(E\left(D' f(\epsilon)\right) - E\left(D'\right)\mu)}{E(D')(\bar{M} - \mu(m^k(y_k, \bar{e}_k) + y_k m_{y_k}^k(y_k, \bar{e}_k)))} \\
=& 1 - \frac{E\left(D' f(\epsilon)\right) - E\left(D'\right)\mu}{E(D')} \frac{m^k(y_k, \bar{e}_k) + y_k m_{y_k}^k(y_k, \bar{e}_k)}{\bar{M} - \mu(m^k(y_k, \bar{e}_k) + y_k m_{y_k}^k(y_k, \bar{e}_k))} \\
=& 1 - \frac{E\left(D' f(\epsilon)\right) - E\left(D'\right)\mu}{E(D')} \frac{1}{\frac{\bar{M}}{m^k(y_k, \bar{e}_k) + y_k m_{y_k}^k(y_k, \bar{e}_k)} - \mu},
\end{aligned} \tag{18}$$

where $\mu = \int_{-\infty}^{\infty} f(\epsilon) d\epsilon$. From (18), α_k^* is larger, the smaller $m^k(y_k, \bar{e}_k) + y_k m_{y_k}^k(y_k, \bar{e}_k)$ is. Thus, for arbitrary α_i^* and α_j^*, α_i^* is larger than α_j^* if and only if

$$m^i(y_i, \bar{e}_i) + y_i m_{y_i}^i(y_i, \bar{e}_i) < m^j(y_j, \bar{e}_j) + y_j m_{y_j}^j(y_j, \bar{e}_j) \tag{19}$$

which leads to

$$m^i(y_i, \bar{e}_i)(1 + \rho_{m_y}^i) < m^j(y_j, \bar{e}_j)(1 + \rho_{m_y}^j) \tag{20}$$

where $\rho_{m_y}^k \equiv \frac{\partial m^k(y_k, \bar{e}_k)}{\partial y_k} \frac{y_k}{m^k(y_k, \bar{e}_k)} > 0$ is elasticity of $m^k(y_k, \bar{e}_k)$ with respect to y_k. This is stated in the following proposition.

Proposition 3. *Suppose (15) and (16). Then the sign of difference between optimal trading ratios for the countries* i *and* j *is expressed as follows:*

$$\mathrm{sgn}(\alpha_i^* - \alpha_j^*) = \mathrm{sgn}[m^i(y_i, \bar{e}_i)(1 + \rho_{m_y}^i) - m^j(y_j, \bar{e}_j)(1 + \rho_{m_y}^j)]$$

That is, smaller the carbon emission per hectare due to illegal logging and lower the elasticity of the marginal carbon emission for the country i, compared to those of the other country j, higher the possibility that α_i^* exceeds α_j^* will be.

It is possible that a country that has launched the reduction of carbon emission due to deforestation and degradation at an earlier date before REDD starts, currently cannot achieve a significant reduction, because the country is assigned a disadvantageous baseline ignoring its past reduction effort so that it faces a higher opportunity cost of reducing the emission. This disadvantage is pointed out as a problem of unfairness concerning the baseline (Eberling and Yasue, 2008). Yet, if such the country is assigned a higher trading ratio than other countries, the unfairness might be mitigated, because the country faces an advantageous trading ratio. In what follows, we examine whether this is true or not.

To see this issue, we suppose that all the South countries are identical except the two variables that can be related with the past reduction of deforestation and degradation. One variable is supposed be the cost to reduce the emission. We assume that larger the past reduction, the higher the opportunity cost of current reduction is. The second one is the forest management level. We suppose that the forest management level of a country will be more efficient, larger the forest it has conserved before.

To state in a formal way, the function m^i is identical across the South countries, that is,

$$m^k(\cdot,\cdot) = m(\cdot,\cdot),\ k = 1, \cdots, n. \tag{21}$$

Moreover, let us assume that the cost function of reducing deforestation is identical for each country expressed as:

$$C_{s_k}(y) = A_k C(y), A_k > 0, C' > 0, C'' > 0. \tag{22}$$

A_k is referred to as "cost coefficient" for the country k, only which reflects the difference among countries. A higher A_k can be interpreted that the reduction of deforestation is costly.

Let the accumulated reduction of carbon emission by deforestation and degradation in the past of the country k be expressed by v_k. We say that the country i is more *sustainable* than the country j if $v_i > v_j$, because this may mean that the country i has saved the larger area of forest than the country j. We suppose that

$$A_k = A(v_k), A' > 0 \tag{23}$$

and

$$\bar{e}_k = e(v_k), e' \geq 0 \tag{24}$$

Under our suppositions, if the country i has made a larger effort before to reduce carbon emission from the deforestation and degradation, compared to the country j, which means $v_i > v_k$, it holds

$$\begin{aligned} \bar{e}_i &\geq \bar{e}_j \\ A_i &> A_j. \end{aligned} \tag{25}$$

In this case, will REDD result in $\alpha_i^* > \alpha_j^*$? If so, we say that the optimal trading ratios in the REDD *advantage* more sustainable country.

To see this, let us focus on the country j and differentiate totally the second condition of (11) with respect to A_j, y_j and $\bar{e}_j$ at the social optimum. Note that we are interested in how y_i^* is different from y_j^* by the difference of A_i and $\bar{e}_i$, so Z is fixed. With (17), this leads to

$$dy_j = -\frac{(E(D'(Z))\mu + aV)(m_{e_j} + y_j m_{y_j e_j})de_j + C'dA_j}{A_j C'' + E(D'(Z))\mu + aV} \tag{26}$$

where $V = \sum_{k=1}^{n} \left(\frac{Var(G^k(y_k, \bar{e}_k, \epsilon))}{y_k m_k(y_k, \bar{e}_k)} \right)$.

In view of (19) and (26), $\alpha_i^* > \alpha_j^*$ holds if and only if dA_j and $d\bar{e}_j$ satisfy

$$\frac{dy_j}{de_j} < -\frac{m_{e_j} + y_j m_{y_j e_j}}{2m_{y_j} + y_j m_{y_j y_j}} \tag{27}$$

where dy_j is given by (26). Rearranging (27) with respect to $dA_j(> 0)$ and $de_j(> 0)$ leads to

$$dA_j > \left(\frac{A_j C'' + E(D'(Z))\mu + aV)(m_{e_j} + y_j m_{y_j e_j})}{(2m_{y_j} + y_j m_{y_j y_j})C'} - \frac{(E(D'(Z))\mu + aV)(m_{e_j} + y_j m_{y_j e_j})}{C'} \right) de_j \tag{28}$$

where m_e, m_{yy}, m_{ye}, C' and C'' are all evaluated at $(y_j^*, \bar{e}_j)$. That is, $\alpha_i^* > \alpha_j^*$ can hold under $dA_j > 0$ and $de_j > 0$. This is claimed in the following proposition.

Proposition 4. *Suppose (15), (16), (21), (22), (23) and (24). Moreover, suppose that the countries i and j satisfy (25) because of $v_i > v_j$. Then if it holds in the REDD equilibrium that*

$$A_i - A_j >$$
$$\left(\frac{(A_j C'' + E(D'(Z))\mu + aV)(m_{e_j} + y_j m_{y_j e_j})}{(2m_{y_j} + y_j m_{y_j y_j})C'} - \frac{(E(D'(Z))\mu + aV)(m_{e_j} + y_j m_{y_j e_j})}{C'} \right) (\bar{e}_i - \bar{e}_j)$$

then the optimal trading ratios advantage more sustainable country, i.e., it holds that $\alpha_i^* > \alpha_j^*$.

From this proposition, we immediately obtain the following property.

Corollary 1. *Under the suppositions of proposition 4 with* $\bar{e}_i = \bar{e}_j$*, then* $A_i > A_j$ *always leads to* $\alpha_i^* > \alpha_j^*$.

From (26), the social optimum requires y_i^* to be equal to or smaller than y_j^* if and only if the numerator of the r.h.s. of (26) is positive, i.e., dA_j and de_j satisfy

$$dA_j \geq -\frac{(E(D'(Z))\mu + aV)(m_{e_j} + y_j m_{y_j e_j})}{C'(y_j)} de_j. \tag{29}$$

If (29) is true, it is easy to see that the sufficient condition of proposition 4 is satisfied, because

$$\frac{(A_j C'' + E(D'(Z))\mu + aV)(m_{e_j} + y_j m_{y_j e_j})}{(2m_{y_j} + y_j m_{y_j y_j})C'} < 0. \tag{30}$$

Thus, we also have the following result.

Proposition 5. *Under the suppositions of proposition 4, if it holds that* $y_i^* \leq y_j^*$ *in the social optimum, which is equivalent to that (29) is satisfied, then the optimal trading ratios always advantage more sustainable country, i.e., it holds that* $\alpha_i^* > \alpha_j^*$.

As a consequence of determination of the optimum trading ratio in REDD, this proposition says that if the reduction of deforestation of a country is smaller because the country launched the conservation of the forest at an earlier date, it must be assigned a higher trading ratio. This might mitigate the disadvantageous conditions of the country in REDD.

5 Control for the forest management level and the implementation of social optimum under REDD

So far, we have supposed that the South countries control deforestation, while the forest management level is assumed to be given. Now we relax this constraint and suppose that the forest management level is controllable as well as the reduction of deforestation. However, this will give a problem concerning the possibility of the social optimum under REDD.

Let the developing countries control the forest management level e_i given the management cost w_i. Then the objective of the country i in the South is expressed in this case by:

$$\min_{y_i, e_i} C_{s_i}(y_i) + w_i e_i - \alpha_i p E\left(G^i(y_i, e_i, \epsilon)\right), i = 1, \cdots, n. \tag{31}$$

This leads to

$$\begin{aligned} \frac{C'_{s_i}(y_i)}{pE\left(G^i_{y_i}(y_i, e_i, \epsilon)\right)} &= \alpha_i, i = 1, \cdots, n. & (32) \\ \frac{w_i}{pE\left(G^i_{e_i}(y_i, e_i, \epsilon)\right)} &= \alpha_i, i = 1, \cdots, n. & (33) \end{aligned}$$

The objective of the North remains unchanged, leading to (5). The carbon market equilibrium this case is represented by $(y^{rr}, e^{rr}, x^{rr}, p^{rr})$.

Social optimization problem is expressed by:

$$\min_{x, y_i, e_i} C_n(x) + \sum_{k}^{n} (C_{s_k}(y_k) + w_i e_i) + E\left(D(Z)\right), i = 1, \cdots, n. \tag{34}$$

This leads to

$$
\begin{aligned}
C_n'(x) &= E(D') && (35)\\
C_{s_i}'(y_i) &= E\left(D'G^i_{y_i}(y_i,e_i,\epsilon)\right), i=1,\cdots,n. && (36)\\
w_i &= E\left(D'G^i_{e_i}(y_i,e_i,\epsilon)\right), i=1,\cdots,n. && (37)
\end{aligned}
$$

The social optimum is expressed by (y^{**}, e^{**}, x^{**}).

It is obvious that $(y^{rr}, e^{rr}, x^{rr}, p^{rr}) = (y^{**}, e^{**}, x^{**})$ if and only if it holds

$$E\left(D'G^i_{e_i}(y_i^{rr}, e_i^{rr}, \epsilon)\right) E\left(G^i_{y_i}(y_i^{rr}, e_i^{rr}, \epsilon)\right) = E\left(G^i_{e_i}(y_i^{rr}, e_i^{rr}, \epsilon)\right) E\left(D'G^i_{y_i}(y_i^{rr}, e_i^{rr}, \epsilon)\right). \quad (38)$$

However, we obtain

$$
\begin{aligned}
E(D'G^i_{e_i})E(G^i_{y_i}) &= E(D'G^i_{e_i}G^i_{y_i}) - Cov(D'G^i_{e_i}, G^i_{y_i}) && (39)\\
E(D'G^i_{y_i})E(G^i_{e_i}) &= E(D'G^i_{y_i}G^i_{e_i}) - Cov(D'G^i_{y_i}, G^i_{e_i}).
\end{aligned}
$$

Therefore, (38) can be achieved under REDD if and only if

$$Cov(D'G^i_{e_i}, G^i_{y_i}) = Cov(D'G^i_{y_i}, G^i_{e_i}) \quad (40)$$

This result is claimed in the following proposition.

Proposition 6. *Suppose that the forest management level is controllable for each country of the South under (2). Then REDD does not achieve the social optimum unless it holds that* $Cov(D'G^i_{e_i}, G^i_{y_i}) = Cov(D'G^i_{y_i}, G^i_{e_i})$.

(40) does not in general hold. Under REDD, each South country might control the the forest management level as well as deforestation, so we may conclude that it is hard to characterize REDD as a measure compatible with the social optimality.

6 Some concluding remarks

In this paper, we examined the optimal rule of carbon credits trading between the North and the South under the REDD, where the South can reduce the carbon emission from the forest through decreasing the deforestation and increasing the forest management level. In addition, we take into consideration that the conserved forests face uncertainty due to illegal logging activities.

If we include this aspect of reality given the forest management level, we show that the optimal trading-ratio is not in general unity, depending on the stochastic relationships among marginal damage, marginal abatement of carbon through the reduction of deforestation and marginal illegal logging of conservation.

Under some specifications, we also demonstrate that the optimal trading ratio is always less than unity and that the ratio becomes smaller depending on the size of uncertainty. Moreover, we examine if a country with having a greater forest management level in the past and a higher opportunity cost of conservation can be attached a higher trading-ratio, that is, whether the optimal trading ratios advantage more sustainable country. We derive a condition that this will be true. In particular, we demonstrate that if the reduction of deforestation of a country is smaller, it must always be assigned a higher trading ratio. In this sense, this paper demonstrates the possibility that the determination of the trading ratio in the optimal way mitigates the problem of fairness arising under REDD that is pointed out in the literature (Ebeling and Yasue, 2008).

However, we also demonstrate that if the forest management level is controllable for the South, then REDD does not in general lead to the social optimum. That is, REDD cannot be characterized as a measure to implement efficiency as the standard emission trading. This drawback should be remedied somehow. For example, suppose that REDD targets only the areas that satisfy a certain forest management level and that the carbon credits are issued based on the determined forest management level. Under this scheme, each participating country might set the forest management level higher than the required one unless setting higher generates some other benefits, so that the each country's forest management level must be equivalent, i.e., $e_i = e_j = \bar{e}$ where $\bar{e}$ is the determined level. Then each participating country controls the level of deforestation and our analysis will apply.

Needless to say, this scheme also has some problems. First of all, the resulting equilibrium will be the second best, not the first best one, so that the problem of REDD in terms of achieving the social optimum is still unsolved. Another problem is concerning the incentive to enhance the forest management level; the scheme does not give the incentive to set the level beyond the required minimum one for the participating countries. In addition, it will leave non-participating countries to extend deforestation and degradation of the forest owned by such countries. Therefore, it must be important to study comprehensively how to solve the problem of REDD with the social optimality, posed in this paper.

Finally, two limitations of the paper are stated. To begin with, our study assumes that the carbon credit price is given and not affected by the introduction of REDD in the global carbon market. However, as Bossetti et al. (2011) stress, the amount of credits issued by REDD is not negligible and should have an influence on the carbon market. This aspect is important; at least Bossetti et al. link the effect of REDD to innovation of energy sector, via declining the price of the carbon credit. Secondly, REDD would be developed in the long run, so the decision of conservation of forest is ranging overtime, so a dynamic aspect might give us some key insights into the nature and function of REDD that are not dealt with in a static model as our paper supposes. These interesting analyses will also be left for future studies.

7 Appendix

We show that the expressions of (17) is derived under (15) and (16). Variance of carbon reduction is expressed as:

$$
\begin{aligned}
&Var(G^i(y_i, \bar{e}_i, \epsilon)) \\
=&E\left(\left(G^i(y_i, \bar{e}_i, \epsilon)\right)^2\right) - \left(E\left(G^i(y_i, \bar{e}_i, \epsilon)\right)\right)^2 \\
=&E\left(\left(y_i(\bar{M}_i - m^i(y_i, \bar{e}_i)f(\epsilon))\right)^2\right) - \left(E\left(y_i(\bar{M}_i - m^i(y_i, \bar{e}_i)f(\epsilon))\right)\right)^2 \\
=&y_i^2\left(\bar{M}_i^2 - 2\bar{M}m^i(y_i, \bar{e}_i)\mu + (m^i(y_i, \bar{e}_i))^2 E((f(\epsilon))^2)\right) - y_i^2\left(\bar{M}_i^2 - 2\bar{M}_i m^i(y_i, \bar{e}_i)\mu + (m^i(y_i, \bar{e}_i))^2\mu^2\right) \\
=&(y_i m^i(y_i, \bar{e}_i))^2\left(E((f(\epsilon))^2 - m^i(y_i, \bar{e}_i))^2\mu^2\right) \\
=&(y_i m^i(y_i, \bar{e}_i))^2 Var(f(\epsilon))
\end{aligned} \tag{41}
$$

We demonstrate that (17) is derived as follows.

$$
\begin{aligned}
&Cov(D', G^i_{y_i}(y_i, \bar{e}_i, \epsilon)) \\
=&E\left(D' G^i_{y_i}(y_i, \bar{e}_i, \epsilon)\right) - E\left(D'\right)E\left(G^i_{y_i}(y_i, \bar{e}_i, \epsilon)\right) \\
=&E\left(D'(\bar{M} - f(\epsilon)(m^i(y_i, \bar{e}_i) + y_i m^i_{y_i}(y_i, \bar{e}_i)))\right) - E\left(D'\right)E\left(\bar{M} - f(\epsilon)(m^i(y_i, \bar{e}_i) + y_i m^i_{y_i}(y_i, \bar{e}_i))\right) \\
=&-E\left(D' f(\epsilon)(m^i(y_i, \bar{e}_i) + y_i m^i_{y_i}(y_i, \bar{e}_i))\right) + E\left(D'\right)E\left(f(\epsilon)(m^i(y_i, \bar{e}_i) + y_i m^i_{y_i}(y_i, \bar{e}_i))\right) \\
=&-E\left(D' f(\epsilon)\right)\left(m^i(y_i, \bar{e}_i) + y_i m^i_{y_i}(y_i, \bar{e}_i)\right) + E\left(D'\right)\mu(m^i(y_i, \bar{e}_i) + y_i m^i_{y_i}(y_i, \bar{e}_i)) \\
=&-(m^i(y_i, \bar{e}_i) + y_i m^i_{y_i}(y_i, \bar{e}_i))(E\left(D' f(\epsilon)\right) - E\left(D'\right)\mu)
\end{aligned} \tag{42}
$$

From the specified form of $D(\cdot)$, $E\left(D'f(\epsilon)\right) - E\left(D'\right)\mu$ becomes as follows:

$$
\begin{aligned}
& E\left(D'f(\epsilon)\right) - E\left(D'\right)\mu \\
=& E\left(aZf(\epsilon)\right) - E\left(aZ\right)\mu \\
=& aE\left(\bar{z} - x - \sum_{k=1}^{n} G^k(y_k, \bar{e}_k, \epsilon)f(\epsilon)\right) - aE\left(\bar{z} - x - \sum_{k=1}^{n} G^k(y_k, \bar{e}_k, \epsilon)\right)\mu \\
=& -aE\left(\sum_{k=1}^{n} G^k(y_k, \bar{e}_k, \epsilon)f(\epsilon)\right) + aE\left(\sum_{k=1}^{n} G^k(y_k, \bar{e}_k, \epsilon)\right)\mu \\
=& -aE\left(\sum_{i=1}^{n} (y_i(\bar{M}_k - m^k(y_k, \bar{e}_k)f(\epsilon)))f(\epsilon)\right) + aE\left(\sum_{k=1}^{n} (y_k(\bar{M}_k - m^k(y_k, \bar{e}_k)f(\epsilon)))\right)\mu \\
=& aE\left(\sum_{k=1}^{n} y_k m^k(y_k, \bar{e}_k)f(\epsilon)f(\epsilon)\right) - aE\left(\sum_{k=1}^{n} y_k m^k(y_k, \bar{e}_k)f(\epsilon)\right)\mu \\
=& a\sum_{k=1}^{n} y_k m^k(y_k, \bar{e}_k)E\left((f(\epsilon))^2\right) - a\sum_{k=1}^{n} y_k m^k(y_k, \bar{e}_k)f(\epsilon)\mu^2 \\
=& a\sum_{k=1}^{n} \left(y_k m^k(y_k, \bar{e}_k)(E((f(\epsilon))^2) - \mu^2)\right) \\
=& a\sum_{k=1}^{n} \left(y_k m^k(y_k, \bar{e}_k)Var(f(\epsilon))\right)
\end{aligned}
\tag{43}
$$

Rearranging (43) by (41), we obtain

$$
a\sum_{k=1}^{n} \left(y_k m^k(y_k, \bar{e}_k)Var(f(\epsilon))\right) = a\sum_{k=1}^{n} \left(\frac{Var(G^k(y_k, \bar{e}_k, \epsilon))}{y_k m_k(y_k, \bar{e}_k)}\right) \tag{44}
$$

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