The economics of sheep farming at northern latitudes. Potential effects of climate change

Abstract

The paper studies the economy and ecology of sheep farming when climate change and uncertainty may affect vegetation quantity. The analysis is at the farm level and includes two different categories of the animals, ewes (adult females) and lambs. The model is formulated in a Nordic economic and biological setting with the crucial distinction between the outdoors grazing season and the winter indoors season. During the outdoors grazing season, animals may face limited grazing resources so that the animal density in addition to temperature, precipitation and other weather conditions determine the weight and hence the value of lambs. The model is analyzed in two steps, without taking uncertainty into account and with uncertainty. Because empirical evidence suggests that climate changes, e.g. increased spring temperature, would have contrasting effects on lamb weights, we compare the impact of such changes across specific sites.

Keywords: sheep farming, weather conditions, climate change, vegetation growth, stage model, uncertainty

1. Introduction

IPCC projections indicate that temperatures will increase and the increase will be strongest at higher latitudes (IPCC 2007). These projections indicate that Nordic sheep farmers will face novel climate conditions in the future. Nielsen *et al.* (2012) showed that e.g. increased spring temperature would have contrasting effects on lamb autumn body mass, depending on the area under study. This indicates that any attempt to include weather conditions and climate change in optimization models for individual farmers has to be site specific. To illustrate the effect of the spatially inconsistency in climate effect we include in our reasoning and models two areas where the effect of increased spring temperature has contrasting effect. Our aim is to illustrate how potential climatic effects may alter the optimal stocking rate, the per animal slaughter values and profitability of the farmers.

Our sheep farming study is conducted with a crucial distinction made between the outdoors grazing season (spring, summer and fall) and the indoor winter feeding period, and different categories of animals (lambs and ewes) are included. Lambs are born in early spring, just before the outdoor grazing season starts which is the typical situation found in many strongly seasonal environments at northern latitudes such as in the Nordic countries and at high altitudes in continental Europe (e.g., mountainous areas in France and Spain). The analysis essentially relates to the economic and biological setting found in Norway, but should also have relevance for sheep framers in Iceland and Greenland. The problem analyzed here is to find the optimal number of animals to be fed and kept indoors during the winter season for a given farm capacity (i.e., farm size). A corollary of this problem is to find the effect that summer grazing sheep density has on vegetation productivity and hence on per-animal meat production. The problem is analyzed under the assumption that the farmer aims to do it 'as well as possible,' represented by expected present-value profit maximization.

The animal growth model presented in this paper builds on Skonhoft (2008), but is extended to include the effect on animal weight gain from outdoors grazing conditions. Balancing the number of animals and weight of animals is indeed seen as a crucial management problem in the Nordic countries (e.g., Olafsdottir and Juliusson 2000, Mysterud and Austrheim 2005, Thomson et al. 2005). However, there is considerable variation among years in the weight gain experienced by the animals during the outdoors grazing season, suggesting that weather conditions are affecting the quality and quantity of the vegetation as well as the behavior and physiology of the animals. Skonhoft et al. (2010) extended the Skonhoft's model (2008) to

include a relationship between vegetation availability and lamb weight. Here we aim to develop this relationship further to include weather conditions and uncertainty affecting the vegetation. The uncertainty cannot be controlled by the farmer, but increased knowledge on the relationship between particular weather variables and weight gain might enable the farmers to adapt to changing climate conditions.

In the natural resource and agricultural economics literature, there is an increased focus on the potential effects of climate changes. Diekert et al. (2010) analyzing the Barents Sea cod fishery assume that climate changes are channeled through a temperature variable affecting the recruitment of the cod stock, and where a higher temperature improves the recruitment. There is no uncertainty included in this analysis, and the recruitment function simply shifts for a given value of the temperature. Hannesson (2007) also studies a situation where climate changes are materialized through sea temperature. His analysis is dealing with potential effects on the migration pattern of fish between different jurisdictions. Quaas and Baumgärtner (2012) study optimal livestock management in semi-arid rangelands with uncertain rainfall. They solve for the optimal stocking rate (ratio of livestock population to vegetation biomass) under rainfall uncertainty when assuming that increased rainfall increases vegetation biomass.

The present study differs from the above contributions in two ways. First, we consider climatic factors (i.e., temperature) as having no direct impact on animal recruitment but as detrimental to lamb weights. Furthermore, we present an age-specific model consisting of adult animals and lams. Second, along with empirical findings, we consider increased spring temperature as having a positive or negative effect on lamb weights depending on the specific site of consideration. We study how temperature changes effects may alter the optimal slaughtering composition (lamb and adults), the stocking rate, and profitability of the farmers.

This paper is organized as follows. Section 2 describes briefly the Nordic sheep farming system. Section 3 provides information about sheep animal growth and presents the biological model. While animal population growth is unaffected by potential climate effects and uncertainty, weight growth per animal is affected and this relationship is discussed in Section 4. The revenue and cost functions follow in Section 5. The stocking problem of the farmer is solved under the assumption of no uncertainty in Section 6 while Section 7 provides a

numerical illustration. The stocking problem in presence of uncertainty is analyzed in Section 8. Section 9 finally summarizes our findings.

2. The Nordic sheep farming system

There are approximately 16,000 sheep farms in Norway, all family farms. Because there are around 2.1 million animals during the outdoors grazing season, the average farm size only accounts for some 130 animals during the summer. Norwegian farms are located either close to mountain areas and other sparsely populated areas or along the coast, with a means to transport sheep to more distant alpine areas for summer grazing. The main product is meat, which accounts for about 80% of the average farmer's income. The remainder comes from wool, because sheep milk production is virtually nonexistent today (Nersten et al. 2003). In Iceland, there are about 450,000 winterfed animals today. Meat is also the most important product from sheep farming here. In Greenland, the land for sheep grazing is much more restricted, and the population of ewes in 2007 was estimated at 25,000 (Austrheim et al. 2008a).

Housing and indoor feeding is required throughout winter because of snow and harsh weather conditions (Figure 1). In Norway, winter feeding typically consists of hay grown on pastures close to farms (80%), with the addition of concentrate pellets provided by the industry (20%).. The spring lambing scheme is controlled by the farmers because of the In Vitro Fertilization protocol used to time the lambing to fit current climatic conditions (REF??). In late spring and early summer, the animals usually graze on fenced land close to the farm at low elevations, typically in the areas where winter food for the sheep is harvested during summer. When weather conditions permit sheep are released into rough grazing areas in the valleys and mountains. Plant spring phenology is driven by the amount of snow the previous winter, but also temperature and precipitation in spring (Nielsen *et al.* 2012). In Norway, most sheep (about 75% of the total metabolic biomass) graze in the northern boreal and alpine region (Austrheim et al. 2008b).

The outdoors grazing season ends between late August and the middle of September. The length of outdoor the grazing season is relatively fixed, partly because of local climatic conditions but also, at least in certain areas, local traditions and historical reasons play a role in the timing. In general the outdoor grazing season does not exceed 130 days. During the rough grazing period, flocks may be vulnerable to accidents and disease and in some regions

also to large predators (Aunsmo et al. 1998, Nersten et al. 2003). At least in theory the sheep are taken down from the mountains when the animals' growth curves are flattening out (Svalheim *et al.* 2004). Consequently, among years variation in local weather induces additional uncertainty for both the extent of the grazing season and the nature of the animals' growth curve on the outfield pastures. After the grazing season, the animals are mustered and the wool is shorn. Slaughtering takes place immediately or after a period of grazing on the farmland (more details are provided in Austrheim et al. 2008a). The seasonal subdivision is similar in Iceland and Greenland.

Figure 1 about here

3. Biological model

The sheep animal growth model is formulated at a discrete time with a seasonal subdivision between the outdoors grazing period (spring, summer and fall) and indoors winter-feeding period. The sheep population is structured (e.g., Caswell 2001) as ewes and lambs. The farmers are in full control of the stocking rate, as fertility and the number of animals released in spring are unaffected by weather conditions. All natural mortality is supposed to occur during the grazing season and is also assumed to be independent of summer grazing and weather conditions. Accordingly, the growth in the number of animals is independent of grazing conditions and not subject to uncertainty. Natural mortality differs between adults and lambs, and is considered fixed and density independent. Lambs not slaughtered, enters the adult population after the slaughtering period (i.e., September–October). All male lambs are assumed to be slaughtered because very few (or none when artificial insemination is practiced) are kept for breeding. Therefore, only female adults are considered. Demographic data on sheep are available in Mysterud et al. (2002).

The number of adult females in year $(t+1)$ after the slaughter, consists of the previous year's adults and female lambs that have survived natural mortality and have not been slaughtered. This is written as $X_{t+1} = Y_t s^Y (1-h_t^Y) + X_t s^X (1-h_t^X)$, where Y_t is the number of female lambs, s^X and s^Y are the natural survival fractions of adult females and lambs, respectively, and h_t^X and h_t^Y are the fractions slaughtered. With the fecundity rate b (lambs per adult female) and ψ as the fraction of female lambs recruited (ψ is usually close to 0.5), $Y_t = \psi b X_t$ yields the

number of female lambs. Therefore, when ignoring the possibility of additional animals from outside, the ewe population growth is governed by:

(1) $$X_{t+1} = \psi b X_t s^Y (1 - h_t^Y) + X_t s^X (1 - h_t^X).$$

Because the population growth equation (1) is linear for number of animals, there are infinite combinations of harvesting fractions that sustain a stable population. Therefore, for a constant number of animals $X_{t+1} = X_t = X$, we have:

(1') $$X = \psi b X s^Y (1 - h^Y) + X s^X (1 - h^X),$$

or simply $1 = \psi b s^Y (1 - h^Y) + s^X (1 - h^X)$ when $X > 0$ (see Figure 2). This isocline intersects with the h^X axis at $[1 - (1 - \psi b s^Y) / s^X]$, which may be above or below 1. Therefore, the highest adult slaughter rate compatible with zero animal growth is $\min\{1, [1 - (1 - \psi b s^Y) / s^X]\}$. For all realistic parameter values, it is below 1 (see numerical section), and this is assumed to hold in the subsequent analysis. The isocline intersects with the h_Y axis at $[1 - (1 - s^X) / \psi b s^Y] < 1$ and is hence the highest lamb-slaughtering rate compatible with equilibrium.

Figure 2 about here

4. Weather conditions, herbivore performance, and weight gain

High grazing pressure may cause a reduction in plant quality and/or quantity (Mysterud and Austrheim 2005). However, moderate grazing is expected to facilitate plant biomass production in productive habitats and thus the fodder availability for moderate grazing as compared with no grazing (McNaughton 1979). In low productivity ecosystems in particular, such as the one considered here, heavy grazing may favor heavily defended, nonpalatable plant species to the detriment of palatable species (Austrheim et al. 2007). Increased dominance of such species affects the strength of density-dependent effects on the weight growth of sheep in the long term. The farmer may thus increase current stock numbers at the expense of growth rate in subsequent years. Indeed, with increasing density of sheep on pasture, a higher proportion of low-quality plant species (Kausrud et al. 2006) and vegetation types (Mobæk et al. 2008) may result.

In the Norwegian sheep farming system, the major growth season of the animals is when they roam freely in the mountains and hence, the *per animal* productivity is affected by environmental conditions (e.g. temperature). It has previously been shown that local weather conditions during winter, in spring (before the animals are released to the mountains) and during summer (the outfield grazing season) affect lamb weights (Nielsen et al. 2012). However, which weather variables (snow depth, precipitation or temperature) that is of importance varies among Norwegian mountain ranges; not only in strength but also in direction. Increased precipitation in spring and summer on the west side of the mountains of Southern Norway (high precipitation) is negative for lamb growth, while the effect of precipitation is positive on the drier east side of the mountains. In the north, increased temperature in spring and summer implies increased lamb growth while the effect of temperature is negative in the south and southwest (Nielsen et al. 2012).

Since the effect of certain changes in weather conditions are site specific we model two particular areas where the effect of change differ. We focus here on temperature, but the exercise could be done on any measure of local weather conditions where its relationship to lamb growth is known. The relationship between lamb weight gain during the grazing season year t w_t^Y, the spring/summer stocking rate $(1+b)X_t$ and the effect of a certain change in spring temperature ΔT is formulated as:

(2) $$w_t^Y = w^Y((1+b)X_t, \Delta T)$$

$\Delta T = 0$ represents the situation as it is today while $\Delta T > 0$ indicate a positive and $\Delta T < 0$ a negative effect of a certain future climate change. We focus on two mountain ranges; Forollhogna (north) and Hardangervidda (southwest), where increased spring temperature has been shown to have a positive and negative effect, respectively, on lamb growth. Nielsen et al. (2012) found that for an increase in average spring temperature of 1° C the average lamb weight would increase with 0.37kg ($\Delta T = 0.37$) in the north and decrease with 0.69kg ($\Delta T = -0.69$) in southwest. Though they modeled lamb autumn body mass, we use the same estimates to illustrate the weight gain through the summer.

A negative relationship between the stocking rate and lamb autumn weight is well-established (Mysterud et al. 2011), also in our focal areas (Nielsen et al. 2012); that is, we typically have $\partial w^Y / \partial((1+b)X_t) < 0$. However, this functional form may also be concave and increasing for a small stocking rate, reach a peak value and then decline. This might be the case if a certain

level of grazing pressure facilitates plant growth and increases the quality and/or quantity of the vegetation available to the animals (e.g. Mysterud et al. 2011). The effect of ΔT is defined such that a higher value ($\Delta T > 0$) shifts the weight function upwards, while a negative value ($\Delta T < 0$) shifts it downwards irrespectively of the form the function takes (more details numerical section 7). The weight gain of lambs during the grazing season coincides with the weight at the end of the season; that is the slaughter weight. Therefore, equation (2) represents the lamb slaughter weight (kg per animal).

In the weight growth model (2), ΔT is assumed to be constant across time. In the next version of the model, uncertainty is included, and we then have:

(2') $\quad w_t^Y = w^Y((1+b)X_t, \Delta T, \varepsilon_t)$

where ε_t is a stochastic variable, assumed to be independent and identically distributed (i.i.d.) over time with mean zero and variance σ^2.

For the adults, there is generally no weight change during the grazing season on productive pastures while there may be some loss in low productivity areas (Mysterud and Austrheim 2005). However, as a reasonably good approximation, we simply neglect any possible connection between the amount of vegetation and weight, and therefore also weather factors and weight. The adult slaughter weight is therefore simply fixed and determined outside the model and given as:

(3) $\quad w_t^X = w^X$.

5. Revenue and costs

We disregard income from wool production, and meat sales are then the only revenue component for the farmer. Because slaughtering takes place after natural mortality, the number of ewes and female lambs removed are $X_t s^X h_t^X$ and $\psi b X_t s^Y h_t^Y$, respectively. As mentioned above, the entire male lamb subpopulation $(1-\psi)bX_t s_Y$ is slaughtered. The number of animals removed year t is then $H_t = bX_t s^Y(\psi h_t^Y + 1 - \psi) + X_t s^X h_t^X$. With p^X as the net (of slaughtering costs) ewe slaughtering price (NOK per kg) and p^Y the lamb net slaughtering price, both assumed to be fixed and independent of the number of animals supplied at the farm level, the current meat income of the farmer is given by

$R_t = [p^Y w_t^Y b X_t s^Y(\psi h_t^Y + 1 - \psi) + p^X w^X X_t s^X h_t^X]$.

The cost structure differs sharply between the outdoor grazing season and the indoor feeding season, and the indoor costs are substantial higher. Throughout this analysis, we assume a given farm capacity. Therefore, the costs of buildings, machinery and so forth are fixed (see also below). The indoor season variable costs include labor (typically an opportunity cost), electricity, and veterinary costs in addition to fodder. It depends on the indoors stock size and is given as $C_t = C(X_t)$. The cost function is assumed go be increasing and convex; that is, $C' > 0$ and $C'' \geq 0$.

As indicated, during the grazing period the sheep may graze on communally owned lands ('commons') or private land. Within the Nordic sheep farming system, such land may be available cost free, or the farmer may pay a *fixed* yearly rental (Austrheim et al. 2008a). There may be some transportation and maintenance costs, but such costs are neglected because they are generally rather low. The total yearly variable cost is hence simply assumed to be the indoor season cost. Therefore, when ignoring discounting within the year, the current profit of the farmer is described by:

$$(4) \qquad \pi_t = R_t - C_t = \left[p^Y w_t^Y b X_t s^Y (\psi h_t^Y + 1 - \psi) + p^X w^X X_t s^X h_t^X \right] - C(X_t).$$

6. The optimal program when no uncertainty

Optimality conditions

We start to analyze the situation without uncertainty, and hence equation (2) represents the per lamb weight relationship. We assume that the farmer is well informed and rational, and aims to maximize the present value of profit over an infinite time horizon, $\sum_{t=0}^{\infty} \rho^t \pi_t$ under the biological constraint in (1). $\rho = 1/(1+\delta)$ is the discount factor with $\delta \geq 0$ as the (yearly) fixed discount rate. Inserted from (1), (2), and (4), the Lagrange function may be written as

$$L = \sum_{t=0}^{\infty} \left\{ \rho^t \left[p^Y w^Y ((1+b)X_t, \Delta T) b X_t s^Y (\psi h_t^Y + 1 - \psi) + p^X w^X X_t s^X h_t^X - C(X_t) \right] \right.$$
$$\left. - \rho^{t+1} \lambda_{t+1} \left[X_{t+1} - X_t s^X (1 - h_t^X) - \psi b X_t s^Y (1 - h_t^Y) \right] \right\}$$

where $\lambda_{t+1} > 0$ is the animal shadow price. Following the Kuhn-Tucker theorem the first-order necessary conditions of this problem (assuming $X_t > 0$) are:

$$(5) \qquad \partial L / \partial h_t^X = X_t (p^X w^X - \rho \lambda_{t+1}) \underset{>}{\overset{\leq}{-}} 0; \qquad 0 \leq h_t^X \leq 1,$$

(6) $$\partial L / \partial h_t^Y = X_t[p^Y w^Y((1+b)X_t, \Delta T) - \rho\lambda_{t+1}] \begin{matrix} \leq \\ > \end{matrix} 0; \qquad 0 \leq h_t^Y \leq 1$$

and

(7) $$\begin{aligned} \partial L / \partial X_t = {} & p^Y b s^Y(\psi h_t^Y + 1 - \psi)\left[w^Y((1+b)X_t, \Delta T) + (\partial w^Y / \partial X_t)(1+b)X_t\right] \\ & + p^X w^X s^X h_t^X - C'(X_t) + \rho\lambda_{t+1}\left[s^X(1-h_t^X) + \psi b s^Y(1-h_t^Y)\right] - \lambda_t = 0 \end{aligned}.$$

The control condition (5) states that the slaughtering of the adults should take place up to the point where the per animal value is below, equal or above the cost of reduced growth in animal numbers, evaluated at the shadow price. The lamb control condition (6) is analogous. Equation (7) is the portfolio condition, which states that the number of female adults is determined so that the immediate net return on adult females equals the shadow price of natural growth. The first term in the first bracket reflects that increased animal numbers increases the total meat weight, whereas the second term accounts for the marginal cost of increased animal numbers due to reduced weight per lamb.

From the control conditions (5) and (6) it is evident that the per animal slaughter value steers the slaughter composition and no other factors, e.g., fertility, play a *direct* role. If the demand conditions are in favor of lambs, then $p^Y > p^X$ (see the numerical analysis). If, in addition, the climatic conditions are favorable so that weight of the lambs $w^Y((1+b)X_t, \Delta T)$ is 'high', then the per animal slaughter value of the lambs exceeds that of the ewes. That is $p^Y w^Y((1+b)X_t, \Delta T) > p^X w^X$. The control conditions then indicate a higher harvesting fraction of the lambs than the ewes. This can be satisfied in three ways: i) $h_t^Y = 1$ and $0 < h_t^X < 1$, ii) $h_t^Y = 1$ and $h_t^X = 0$ and iii) $0 < h_t^Y < 1$ and $h_t^X = 0$. On the contrary, with unfavourable climatic conditions and 'low' lamb weight, such that $p^X w^X > p^Y w^Y((1+b)X_t, \Delta T)$ and hence a more aggressively harvesting of the adults, we find that the control conditions can be satisfied either as iv) $h_t^X = 1$ and $0 < h_t^Y < 1$, v) $h_t^X = 1$ and $h_t^Y = 0$, or as vi) $0 < h_t^X < 1$ and $h_t^Y = 0$. We may also find that the lamb weight is such that the value per lamb equals the value per ewe, $p^Y w^Y((1+b)X_t, \Delta T) = p^X w^X$. This is possibility vii) and can be satisfied by a certain combination of slaughter rates in the domain $0 < h_t^Y < 1$ and $0 < h_t^X < 1$ (more details below). When having determined the slaughtering rates from (5)

and (6), the control condition in (7) determines the number of animals which then must be constant over time.

Steady state analysis

In a possible (interior) steady state where all variables are constant over time with a 'high' lamb weight and $p^Y w^Y((1+b)X,\Delta T) > p^X w^X$ (the time subscript is dropped when considering steady state) , we find that the above control conditions can be satisfied only as possibility iii), $0 < h^Y < 1$ and $h^X = 0$ because slaughtering all the lambs is not an option in a possible steady state. See equation (1') and Figure 2. A corollary of $h^{X*} = 0$ is that (*female*) lamb slaughtering should take place at the highest level compatible with the sheep population equilibrium; that is, $h^{Y*} = 1-(1-s^x)/\psi bs^y < 1$. The optimal slaughtering rate depends therefore on biological conditions only (natural mortality and fertility). Single stage slaughtering was the optimal harvesting strategy derived by Skonhoft et al. (2010). They argued that this result occurred because they considered harvest benefit as linear in both harvest controls and because of the lack of any density-dependent effects in the animal growth function. In the present, the additional condition is that the slaughter value per lamb is above that of the slaughter value per ewe slaughter (see above).

Lambs not slaughtered enter the ewe population next spring. When inserting $h^{X*} = 0$, $h^{Y*} = 1-(1-s^X)/(\psi bs^Y)$, and for $\lambda^* = p^Y w^Y((1+b)X,\Delta T)/\rho$ from (6) into (7) and rearranging, the optimal number of animals to keep during the indoor season is determined by the equality

$p^Y(bs^Y + s^X - 1 - \delta)w^Y((1+b)X,\Delta T) = C'(X) - p^Y(bs^Y + s^X - 1)(1+b)X(\partial w^Y/\partial X)$. The left hand side is the marginal benefit of saving animals for next season lamb slaughtering net of the discount rate, and reflects that saving an additional animal increases the total number of lambs available for slaughtering next year. The right hand side is the marginal cost of keeping animals for the next season, and equals the sum of the cost of keeping an additional animal indoor plus the weight loss an additional animal impose on all lambs. Note that economic parameters, in addition to biological parameters, influence the optimal number of adult animals.

In the opposite case of a 'low' lamb weight and more valuable ewes than lambs, the control conditions in a possible steady state can generally be satisfied either as case iv) with $h^x = 1$ and $0 < h^Y < 1$, case v) with $h^x = 1$ and $h^y = 0$, or case vi) with $0 < h^x < 1$ and $h^y = 0$. However, as already indicated, steady state slaughtering of all adults is not a possible option due to the actual demographic parameter values (again, see numerical section). Therefore case vi) with $h^{x*} = 1-(1-\psi bs^y)/s^x < 1$ and $h^{y*} = 0$ will be the only steady state possibility when adults are more valuable than lambs. That is, *female* lamb slaughtering equals zero whereas adult slaughtering should take place at the highest level compatible with the sustainable sheep population equilibrium, cf. equation (1') and figure 2.

When inserting for the optimal steady state slaughtering values into (7) and rearranging, the optimal animal population is determined by the equality

$p^Y bs^Y(1-\psi)w^Y((1+b)X,\Delta T) + p^X w^X(s^X + \psi bs^Y - \delta)$.

$= C'(X) - p^Y bs^Y(1-\psi)(1+b)X(\partial w^Y/\partial X)$. The left hand side equals the marginal benefit of saving animals for next season male lamb and adult slaughtering, while the right hand side equals the marginal cost of keeping animals for the next season. The second term on the right hand side indicates that keeping an additional animal imposes a weight loss on male lambs. Compared to above, the first term on the left hand side reflects that saving an additional adult increases the total number of *male* lambs available for slaughtering next year. The second term reflects that saving an additional adult increases the adult population available for slaughtering next year. On the right hand side, the second term is the weight loss imposed on future male lambs of keeping an additional animal for the next season.

As already mentioned option vii) is met when $p^Y w^Y((1+b)X,\Delta T) = p^X w^X$, meaning that the slaughter value per lamb equals the slaughter value per ewe. Then, this condition alone determines the optimal number of animals. In case of a peak valued concave weight function (see numerical section 8) there are generally two stocking rates giving the lamb weight that fulfills this equation. However, at this weight the higher stocking rate is for obvious reasons the only candidate for representing a maximum value. The animal shadow price is then given by $\lambda = p^X w^X/\rho = p^Y w^Y((1+b)X,\Delta T)/\rho$. When inserting this into (7) and rearranging, we find $p^Y(bs^Y - \delta)w^Y((1+b)X,\Delta T) = C'(X) - p^Y bs^Y(\psi h^Y + 1 - \psi)(1+b)X(\partial w^Y/\partial X)$. That is, once the optimal number of animals is determined by the condition above, then the optimal

slaughtering rate of lambs is determined such that the marginal benefit of saving animals for next season lamb slaughtering equalizes the marginal cost of keeping animals for the next season. The second term on the right hand side is the weight loss imposed on future *female* and *male* lambs of keeping an additional animal for the next season. When having determined the optimal animal population and lamb slaughtering, the optimal adult slaughter follows from equation (1').

To sum up, the optimal steady state slaughter composition depends only on the per animal slaughter values. If the demand and climatic conditions drive the slaughtering value per lamb above that of adults, then slaughtering lambs only is optimal. Slaughtering adults only is, however, optimal if the slaughtering value per adult exceeds that of lambs. Finally, a combination of lamb and adult slaughtering is optimal if the prevailing demand and climatic conditions result in equalized per animal slaughtering values.

In our example from two mountain ranges in Norway an increase in temperature would imply more favourable conditions in the north, but unfavourable conditions in southwest. Assume initially that all farmers are faced with equal prices and climatic conditions favouring lamb slaughtering only. Increased temperature will then have no impact on the slaughtering composition among farmers. However, the marginal benefit of saving animals for next season lamb slaughtering increases for these farmers, and hence, the sheep population increases in north. In south, on the other hand, farmers less likely to slaughter lambs only when faced with a temperature increase. Furthermore, the marginal benefit of saving animals for next season lamb slaughtering reduces, which stimulates southern farmers to reduce the sheep population.

Dynamic analysis

Above some properties of a possible steady state with a constant number of animals through time was studied. We now ask if a dynamic solution may be stable. From the animal growth equation (1) $X_{t+1} = \psi b X_t s^Y (1-h_t^Y) + X_t s^X (1-h_t^X)$, or $X_{t+1} = Q(X_t)$, we find $\partial Q / \partial X_t = \psi b s^Y (1-h_t^Y) + s^X (1-h_t^X)$ which equalizes 1 in any steady state. Therefore, from this we cannot conclude whether any steady state of our system is stable or not. The dynamics is further considered in the numerical section 8.

7. Numerical illustration

Data and specific functional forms

To shed some further light on the above analysis, the model is illustrated numerically, applying the above-specified vegetation natural growth function and animal consumption curve. The sheep biological data are based on a large set of observations from Norwegian sheep farming, and the baseline parameter values are shown in Table 1. The ewe weight is 30 (kg/animal) with a meat market slaughter value of 35 (NOK/kg). Therefore, the fixed ewe slaughter value is $p^x w^x = 35 \cdot 30 = 1,050$ (NOK/animal) (Table 1). The lamb meat value is $p^Y = 60$ (NOK/kg). We assume further a strictly concave maintenance cost function, $C(X_t) = (c/2)X_t^2$, with $c = 12$ (NOK/animal^2).

Table 1 about here

In the baseline calculations, the lamb weight function is specified downward sloping as $w_t^Y = w^Y((1+b)X_t, \Delta T) = k^0 - k^1(1+b)X_t + \Delta T$ with $k^0 = 22$ and $k^1 = 0.1$. As mentioned in section 4 above, evidence show that local weather conditions affect lamb weights, but the effect varies both in strength and direction depending on the area under study (Nielsen et al. 2012). Therefore, the following numerical analysis is based on estimated effects of increased spring temperature in two particular Norwegian mountain ranges, North and South: In the North case increased temperature in spring increases the lamb weights, while in the South case the relationship is negative. As mentioned in section 4, a temperature increase of 1° C increases the average lamb weight with 0.37kg in the north and decreases the lamb weight with 0.69kg in the south (Nielsen et al. 2012). These values are used for the North and South case, respectively, in the numerical analysis.

Hence, the two cases illustrate how the possible biological and economic impact of increased temperature on sheep farming varies between the northern and southern parts of Norway. In the following, the optimal management policy is found for the baseline parameter values. Then we study the effects of temperature changes, as well as changes in the discount rate and the cost parameter. If slaughtering lambs only is optimal prior to a temperature change, then farmers experiencing increased lamb weight will not alter the optimal slaughtering combination in steady state. However, the dynamics towards steady state will be affected. On the other hand, farmers experiencing reduced lamb weight may, eventually, change the

optimal slaughtering combination. Also in this case, we will illustrate the effect on the dynamics towards steady state.

Results

We first present the basic dynamic results. As already indicated, because of the strong degree of linearity in the model together with density dependent regulation of the value gain of the animals through the weight – stocking relationship, the model approaches a stable equilibrium. As the profit function is linear in the harvest when there is no uncertainty, economic theory suggests that harvest should be adjusted such as to lead the population the steady state as fast as possible (for proof of the Most Rapid Approach Path Theorem (MRAP), see Spence and Starrett 1975). Hence, if the number of ewes is below the optimal steady state it will be no harvest until it has grown to the steady state. On the other, if it is above the steady state will be slaughtered down to the optimal level the first year.

Figure 3 demonstrates first the stocking rate dynamics without with the baseline parameter values (see Table 1) but when allowing for changes in the discount rate. The result here seems partly to confirm MRAP where the steady state stocking rate in baseline ($\delta = 0.03$) of 123 animals is reached in year 3. Because the value per lamb exceeds the value per adult, it is optimal to slaughter only lambs in all years and the steady state optimal lamb harvesting rate is 0.93. See Table 2.

Not surprisingly, increasing the discount rate results in progressively smaller populations with corresponding higher harvesting rates of lambs, while the dynamics do not change qualitatively[1]. It is hence beneficial for the farmer, to invest less in the animal stock, and instead put the process in 'the bank'. We have also studied the effects of changing initial stock size, and all the time we find that the stock size and harvest approach the same steady state (ergodic dynamics).

Figure 3 about here

[1] Because the steady state harvesting composition is independent of the discount rate, adult harvesting in steady state is zero also in all three scenarios.

Table 2 reports steady state stock size, slaughtering rates, lamb weight, and the present value profit over all periods for different parameter values. In the baseline scenario meat prices and climatic conditions are in favour of lamb slaughtering only (see also Figure 3). An increase in the price of lamb meat ($P^Y = 70$ NOK) will therefore have no impact on the slaughtering composition and hence, no impact on the slaughtering rates. The latter is because the optimal slaughtering rates depend on biological conditions only. See the steady-state analysis in section 6. However, a higher price of lamb meat increases the marginal benefit of saving animals for next season lamb slaughtering and hence, the animal stock increases and the number of lambs slaughtered increases compared to the baseline scenario. Therefore, although lamb weights reduce, an increase in the lamb meat price increases the present value profits.

If, on the other hand, the price of lamb meat is reduced ($P^Y = 50$ NOK) so that the meat value per lamb falls below the meat value per ewe, then slaughtering ewes only is optimal. Now the marginal benefit of saving animals for next season lamb slaughtering reduces compared to the baseline scenario and hence, the steady-state animal stock reduces. The latter has a negative effect on the number of animals slaughtered and therefore reinforces the direct negative effect of reduced price of lamb meat on present value profits.

A sufficient increase in the price of ewes works in the direction of slaughtering ewes only ($P^X = 45$ NOK). Then, a higher price of ewes stimulates farmers to keep more animal for future ewe slaughtering compared to the baseline scenario. The present value profits increases but less compared to the case of a higher lamb meat price ($P^Y = 70$ NOK). The reason is that when P^X increases adults are slaughtered at a higher price while male lambs are slaughtered at the baseline price, whereas when P^Y increases all slaughtered animals (male and female lambs) are sold at a higher price.

An increase in the cost of keeping animals indoors over the winter season (increased c) stimulates farmers to save less animals for next season slaughtering, and hence, the sheep population reduces. The increase in lamb weight cannot compensate for a reduced animal stock and hence, the present value profits reduces.

Table 2 about here

Next we study the effect of increased temperature. See the Table 3. Consider first the North case where a higher temperature increases the lamb weights. Increased weights increase the net income of lamb slaughtering for a fixed stock size. Furthermore, higher temperatures stimulate farmers to keep more animals over the winter for next season lambing and slaughtering, which in turn increases the number of lambs and adds an additional positive effect on the profit. This means that increased temperature involves a double dividend for sheep farmers in the sense that it increases the slaughtering value per lamb and increases the number of lambs slaughtered. At a temperature increase of 1°C, the direct effect increased weights adds 3 663 NOK to the slaughtering income, whereas the indirect effect working through a changing stock size is stronger and increases the slaughtering income by 4 444 NOK.

Finally we study the South case where increased temperatures affect lamb weights negatively. A small temperature increase (e.g., 1°C) reduces the lamb weight but not enough to alter the optimal slaughtering composition. Reduced lamb weight reduces the net income of lamb slaughtering for a fixed stock size. Further, farmers will keep fewer animals over the winter for next season lambing and slaughtering, which in turn reduces the number of lambs and shrinks the profit even more. At a temperature increase of 1°C, the direct effect reduces the slaughtering income by 6 832 NOK, whereas the indirect effect working through a changing stock size reduces the slaughtering income by 4 373 NOK.

A further increase in the temperature may drive the slaughtering value per lamb below the slaughtering value per adult animal and hence, shift the optimal slaughtering composition from lamb slaughtering only to adult slaughtering only. At first this stimulates farmers to increase the winter stock. The reason is that the marginal cost of keeping a larger stock in terms of reduced lamb weights is of less importance when (female) lamb slaughtering is zero. However, all male lambs are still slaughtered and hence, further temperature increases stimulate reductions in the winter stock because the marginal benefit of saving animals for future male slaughtering reduces as their weight reduces.

Table 3 about here

8. Introducing uncertainty

Equation (2) says that the lamb weight can be predicted exactly from the current stocking rate and weather conditions. These changes are in fact partly random. Equation (2') says that the current stocking rate and current weather conditions are known to the farmer with certainty, but future weather conditions and hence, future lamb weight is (in part) random. The next step is to analyze how such uncertainty affects the optimal slaughtering composition and the animal stock. In the next version of the model, uncertainty is included, and we then have:

(8) $\quad w_t^Y = w^Y((1+b)X_t, \Delta T, \varepsilon_t)$

where ε_t is a stochastic variable, assumed to be independent and identically distributed (i.i.d.) over time with mean zero and variance σ^2.

Equation (8) says that climatic conditions and their impact on weights are random to the farmer. In what follows we solve for the optimal slaughtering program in presence of climatic uncertainty. We now depart from section 6 above by assuming that farmers are risk averse[2]. That is, we assume that farmer utility increases with the profit at a decreasing rate i.e., $U'(\pi_t) > 0$ and $U''(\pi_t) < 0$. Therefore, the farmer now aims to maximize the expected present value utility over an infinite time horizon, $E_0\left\{\sum_{t=0}^{\infty}\rho^t U(\pi_t)\right\}$, under the biological constraint (1). E_0 is expectation given information at time 0. Inserted from (1), (2'), and (4), the Lagrange function of this problem may be written as

$$L = E_0\left[\begin{array}{l}\sum_{t=0}^{\infty}\Big\{\rho^t U\Big[p^Y\Big[w_t^Y((1+b)X_t, \Delta T, \varepsilon_t)\Big]bX_t s^Y(\psi h_t^Y + 1 - \psi) + p^X w^X X_t s^X h_t^X - C(X_t)\Big] \\ \qquad -\rho^{t+1}\lambda_{t+1}\Big[X_{t+1} - X_t s^X(1-h_t^X) - \psi b X_t s^Y(1-h_t^Y)\Big]\Big\}\end{array}\right]$$

The first order conditions are now given by:

(9) $\quad \partial L / \partial h_t^X = X_t\Big[E_t[U'(\pi_t)]p^X w^X - \rho\lambda_{t+1}\Big] \underset{>}{\overset{\leq}{=}} 0;\ 0 \leq h_t^X \leq 1$,

(10) $\quad \partial L / \partial h_t^Y = X_t\Big[E_t[U'(\pi_t)w^Y((1+b)X_t, \Delta T, \varepsilon_t)]p^Y - \rho\lambda_{t+1}\Big] \underset{>}{\overset{\leq}{=}} 0;\ 0 \leq h_t^Y \leq 1$

and

[2] It can be verified that uncertainty and risk neutrality yields the same solution as in section 6 where climatic uncertainty is neglected.

$$\partial L/\partial X_t = p^Y bs^Y(\psi h_t^Y + 1-\psi)\Big\{E_t[U'(\pi_t)w^Y((1+b)X_t,\Delta T,\varepsilon_t)]$$

(11) $$+(1+b)X_t(\partial w^Y/\partial X_t)E_t[U'(\pi_t)]\Big\} + p^X w^X s^X h_t^X E_t[U'(\pi_t)] - C'(X_t)E_t[U'(\pi_t)]$$

$$+\rho\lambda_{t+1}\left[s^X(1-h_t^X)+\psi bs^Y(1-h_t^Y)\right]-\lambda_t = 0$$

Because the weather conditions at time t are known when h_t^Y and h_t^X are to be determined, the expectation operator in (9) - (11) at period t is E_t. The structure of this problem is hence parallel to that of price uncertainty in the fisheries economic literature (see, e.g., Andersen 1982). The control conditions (9) and (10) can be given similar interpretations as the control conditions (5) and 6), except that the marginal gain now is given as expected values. Equation (11) states that the stocking rate is determined such that the immediate expected marginal utility of ewes equals the shadow price of natural growth.

Steady state analysis

The first term in the bracket in (10) can be rewritten as $p^Y\left[E[U'(\pi)]\cdot E[w^Y((1+b)X,\Delta T,\varepsilon)]+\mathrm{cov}(U'(\pi),w^Y((1+b)X,\Delta T,\varepsilon))\right]$. The covariance term is negative as higher lamb weight, and hence higher profit, yields reduced marginal utility for a risk averse farmer. The expected marginal utility of lamb slaughtering is therefore smaller the larger is the covariance term (in absolute value). When combining this expression with (9), we find that the farmer in presence of uncertainty will slaughter a higher fraction of lambs than adults only if $p^Y E[w^Y((1+b)X,\Delta T,\varepsilon)]-p^X w^X > -\mathrm{cov}(U'(\pi),w^Y((1+b)X,\Delta T,\varepsilon))/E[U'(\pi)] > 0$. That is, with the risk attached to lamb weight, the expected slaughtering value per lamb should exceed the slaughtering value per adult by more than required in the deterministic case for a higher fraction of lamb slaughtering to be optimal. More precisely, the difference in the expected slaughtering values should exceed the absolute value of the covariance term divided by the expected marginal utility of income, i.e. the sensitivity rate of the marginal utility to lamb weight changes. If this condition is fulfilled, then a higher harvesting fraction of lambs than adults can only be satisfied with $0 < h^Y < 1$ and $h^X = 0$ because, as already seen, slaughtering all the lambs is not a possible option at steady-state. Hence, as in the deterministic case, optimal slaughtering rate then equals $h^Y = 1-(1-s^X)/\psi bs^Y < 1$. However, with uncertainty, the likelihood for lamb slaughtering only to be optimal is smaller.

When inserting $h^{X*}=0$, $h^{Y*}=1-(1-s^X)/\psi bs^Y$, and $\lambda^*=E[U'(\pi)w^Y((1+b)X,\Delta T,\varepsilon)]/\rho$ from (9) into (10), inserting the covariance, and rearranging, the optimal number of animals is determined by the equality

$p^Y(bs^Y+s^X-1-\delta)\left[E[w^Y((1+b)X,\Delta T,\varepsilon)]+\text{cov}(U'(\pi),w^Y((1+b)X,\Delta T,\varepsilon))/E[U'(\pi)]\right]$
$=C'(X)-p^Y(bs^Y+s^X-1)(1+b)X(\partial w^Y/\partial X)$. The left hand side is the expected marginal benefit of keeping lambs for next season slaughtering net of the discount rate. The right hand side is the marginal cost of saving animals for the next season when taking the loss weight of lambs into account. Consequently, a larger covariance (in absolute value) reduces the expected marginal benefit of keeping animals for the next season relatively to the marginal cost, and hence, reduces the optimal number of animals. That is, the more sensitive the marginal utility of income is to lamb weight changes, the smaller is the optimal sheep stock.

If instead the expected slaughtering value per lamb exceeds the slaughtering value per adult by less than the covariance term, then slaughtering a larger fraction of adult animals is optimal. As in the deterministic case, this slaughtering strategy can only be satisfied as $0<h^X<1$ and $h^Y=0$. See again (1') and the numerical section. Hence, the optimal slaughtering rate equals $h^Y=1-(1-s^X)/\psi bs^Y<1$. Therefore, again uncertainty affects the possibility for adult slaughtering only to be optimal, but not the size of the slaughtering rate.

When inserting $h^{Y*}=0$, $h^{x*}=1-(1-\psi bs^y)/s^x<1$, and $\lambda^*=E[U'(\pi)]w^X p^X/\rho$ from (8) into (10) and rearranging, the optimal number of animals is determined by the equality

$p^Y bs^Y(1-\psi)\left[E[w^Y((1+b)X,\Delta T,\varepsilon)]+\text{cov}(U'(\pi),w^Y((1+b)X,\Delta T,\varepsilon))/E\left[U'(\pi)\right]\right]$
$+p^X w^X(\psi bs^Y+s^X-1-\delta)=C'(X)-p^Y bs^Y(1-\psi)(1+b)X\partial w^Y/\partial X$. The left hand side is the expected marginal benefit of saving animals for male lamb and adult slaughtering next season, whereas the right hand side is the marginal cost of keeping animals for the next season. The second term on the right hand side reflects the weight loss on male lambs of keeping an additional animal for the next season. Because male lambs are slaughtered, a larger covariance (in absolute value) reduces the expected marginal benefit of keeping animals for future male lamb slaughtering, and hence, reduces the optimal number of animals in steady state.

Finally, if the expected marginal utility of lamb slaughtering equals the slaughtering value per adult, i.e. $p^Y\left[E[U'(\pi)]E[w^Y((1+b)X,\Delta T,\varepsilon)]+\text{cov}(U'(\pi),w^Y((1+b)X,\Delta T,\varepsilon))\right]=p^X w^X$, then it is optimal to slaughter both adult animals and female lambs. The shadow price is then given by $\lambda=E[U'(\pi)]p^X w^X/\rho$. When inserting this into (10) the state condition reads $p^Y bs^Y(\psi h^Y+1-\psi)\left[E[w^Y((1+b)X,\Delta T,\varepsilon)]+\text{cov}(U'(\pi),w^Y((1+b)X,\Delta T,\varepsilon))/E\left[U'(\pi)\right]\right]$ $+p^X w^X(\psi bs^Y(1-h^Y)+s^X-1-\delta)=C'(X)-p^Y bs^Y(\psi h^Y+1-\psi)(1+b)X(\partial w^Y/\partial X)$. Hence, the control condition and the state condition together with (1'), determine simultaneously the optimal slaughtering rates and the number of animals in steady state. For more details, see the numerical section below.

Dynamic analysis

When there is uncertainty, and hence the harvest is no longer linear in the objective function, we should expect a more gradually approach to the steady state through a saddle path approach (Clark 1990).

To sum up, with climate uncertainty present there is a risk attached to saving animals for future lamb slaughtering. The optimal steady state slaughter composition depends on the expected per animal slaughter values and the covariance between the marginal utility of profit and the lamb weights. The stronger the marginal utility of profits covariates with the lamb weight, the smaller is the likelihood for lamb slaughtering only to be optimal. Instead, slaughtering both lambs and adults, or even only adults, are possible better strategies if the covariance is strong.

9. Concluding remarks

This paper has analyzed the economics of sheep farming in a two-stage model of lambs and adult females (ewes). The analysis is at the farm level in a Nordic context with a crucial distinction between the outdoor grazing season and the winter indoor feeding season. The weight gain of lambs during the outdoor grazing season is subject to potential climate change effects and uncertainty. The farmer is assumed to be 'rational' and well informed, and aims to find the level of animal slaughter that maximizes profit, or the expected utility of profit when uncertainty is introduced, the accompanying number of summer grazing animals and the number of animals to be kept indoors during the winter.

In this two-stage model of lambs and ewes, the steady state harvesting decision is basically shaped by economic factors alone when there is no uncertainty. For the given price and market conditions whereby the value of lambs is higher than that of ewes, lamb-only slaughtering at the highest possible level is the optimal strategy. On the other hand, the optimal lamb slaughter fraction is determined by sheep biological factors alone. The reason for this sharp distinction between the effects of economic and biological forces is the lack of any density-dependent factors regulating sheep population growth. The exception is when the stocking rate and hence the grazing conditions is such that the per animal value is similar for ewes and lambs.

With uncertainty the steady state slaughter composition depends on the expected per animal slaughter values and the covariance between the marginal utility of profit and the lamb weights. The stronger the marginal utility of profits covariates with the lamb weight, the smaller is the likelihood for lamb slaughtering only to be optimal. The optimal slaughter fraction is, however, determined as in the deterministic case.

The numerical illustrations indicate that shifting economic conditions for the farmer have small effects on the stocking rate and vegetation utilization. Such shifts, at least within the actual range of parameter values, spill over to changing farm profitability. On the other hand, we find vegetation productivity to have crucial allocation effects. For example, when comparing two equally sized farms located in areas with differing productivity, the farmer that benefits from high productivity will find it rewarding to keep a significantly higher stocking rate than the other one. The high productivity farmer will receive substantial higher economic benefits as well. The optimal sheep farming decision may hence be more sensitive to changes in pasture quality and productivity than changes in economic conditions.

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Tables and figures

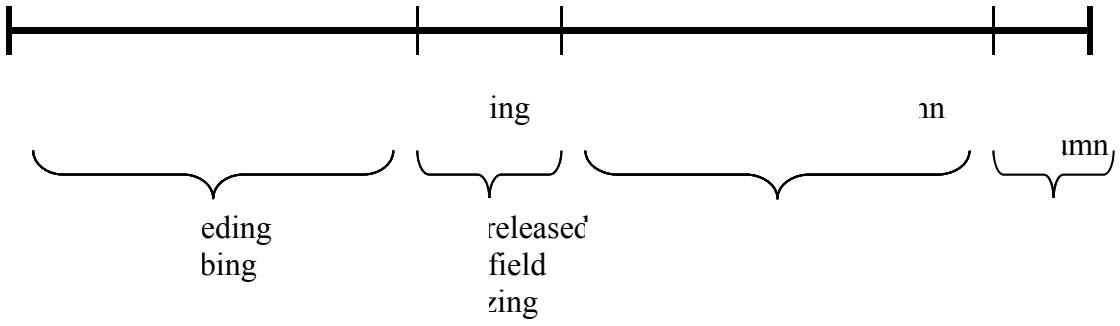

Figure 1: Seasonal subdivision in the Nordic sheep farming system.

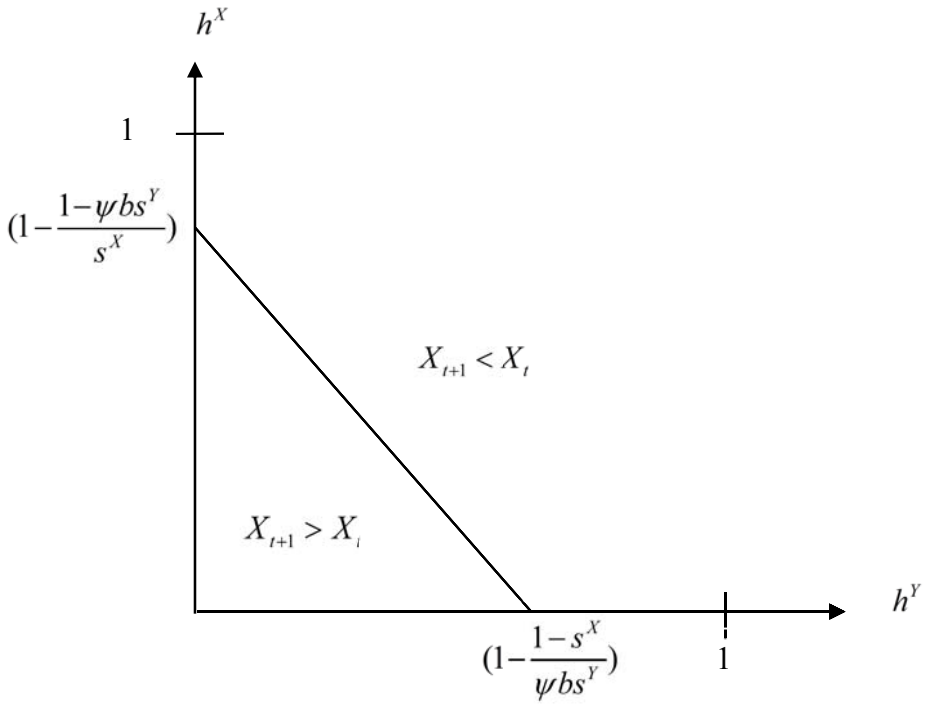

Figure 2: Equilibrium (constant animal population) harvesting relationship (Eq. 1'). h^Y, female lamb slaughtering fraction; h^X, ewe (adult female) slaughtering fraction.

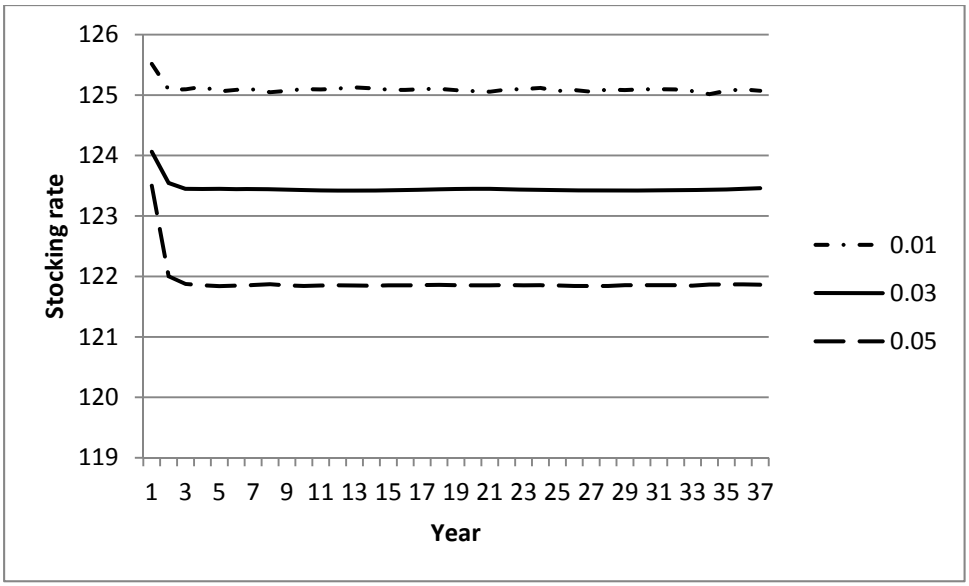

Figure 3: Baseline stocking rate dynamics, $\Delta T = 0$.

Table 1: Baseline ecological and economic parameter values

Parameter	Parameter description	Value
s^Y	Natural survival fraction lambs	0.91
s^X	Natural survival fraction ewes	0.95
b	Fertility rate	1.53 (lamb/ewe) (Mysterud et al. 2002)
ψ	Proportion of female lambs	0.50
k^0	Interaction term lamb weight function	22 (kg/animal)
k^1	Slope term lamb weight function, stocking rate	0,01 (kg/animal)
w^X	Adult (ewes) slaughter weight	30 (kg/animal)
p^X	Adult (ewe) slaughter price	35 (NOK/kg)
p^Y	Lamb slaughter price	60 (NOK/kg)
c	Cost coefficient	10 (NOK/animal)

Table note: Exchange rate: 1 Euro = 7.90 NOK (Jan. 2012).

Table 2: Steady-state sensitivity analysis, $\Delta T = 0$.

	Winter stock X^*	# lambs bX^*S^Y	h^{X*}	h^{Y*}	w^Y	$PV\pi$
Baseline[1)]	123	171	0.00	0.93	18.88	2 798 684
$\delta = 0.01$	125	174	0.00	0.93	18.84	4 012 937
$\delta = 0.05$	122	170	0.00	0.93	18.92	2 078 203
$P^Y = 70$	137	191	0.00	0.93	18.52	3 553 133
$P^Y = 50$	120	167	0.68	0.00	18.96	2 233 688
$P^X = 40$	137	191	0.68	0.00	18.54	2 962 197
$c = 15$	91	127	0.00	0.93	19.70	2 088 464
$c = 20$	72	100	0.00	0.93	20.18	1 655 500

1) See Table 1 for baseline parameter values.

Table 3: Steady-state sensitivity analysis, changing temperature.

Case	# ° C increase	Winter stock X^*	# lambs bX^*S^Y	h^{X*}	h^{Y*}	w^Y	$PV\pi$
Baseline[1)]	0	123	171	0.00	0.93	18.88	2 798 684
	1	126	175	0.00	0.93	19.19	2 890 650
North	3	130	181	0.00	0.93	19.83	3 078 806
	6	136	189	0.00	0.93	20.78	3 371 601
	1	120	167	0.00	0.93	18.28	2 630 882
South	3	122	170	0.68	0.00	16.84	2 371 012
	6	115	160	0.68	0.00	14.95	2 114 524

1) See Table 1 for baseline parameter values.