

# Regulating ambient pollution when social costs are unknown

Charles Figuières\* and Marc Willinger†

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## Abstract

This paper offers a new mechanism in order to Nash-implement a Pareto optimal level of ambient pollution. It is well-known that non-point source emissions can only be regulated through instruments that are conditioned on aggregate emission, since individual emissions are not observable by the regulator. The novelty of our mechanism is that it does not require that the regulator knows the agents' preferences as is usually assumed. We show that the regulator can achieve the Pareto-optimal level of emissions under very weak informational assumptions. He only needs to know the number of polluting agents.

**Keywords:** non point source pollution, mechanism design, Nash-implementation.

**JEL Classification:** Q0, Q52, H23, C72, D82.

## 1 Introduction

The ambient tax (Segerson, 1988, Hansen, 1998, Shortle and Horan, 2001 to quote a few) is considered an efficient tool to implement a socially optimum level of emissions as a Nash equilibrium when the regulator cannot observe individual emissions. To achieve a target level of emissions with the ambient tax instrument the regulator only needs to know the level of ambient pollution, *i.e.* the aggregation of all individual emissions. There exists several variants of this instrument, but its basic principle is as follows: each polluter is liable for the ambient pollution in excess of the target, whether or not he is responsible for that outcome. That is, each polluter is required to pay for the total damage to society due to excess pollution. If pollution is below the target level, no tax is paid. The unique Nash equilibrium with the ambient tax

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\*INRA, UMR LAMETA, 2 place Viala, 34060 Montpellier, France. Email: Charles.Figuieres@supagro.inra.fr

†University Montpellier 3, Faculté d'Economie, UMR LAMETA, avenue Raymond Dugrand, C.S. 79606 / 34960 - Montpellier cedex 2, France. Email: willinger@lameta.univ-montpl.fr

25 instrument is such that aggregate emissions are equal to the target level defined by the regulator, so that at equilibrium none of the polluters is required to pay a tax for excess damage.

While the target can be set arbitrarily at any level by the regulator, the most interesting case is when it corresponds to a social optimum. To identify such a target the regulator requires additional information. Besides the ambient level of pollution the regulator also needs to know 30 the social damage function of the ambient pollution. Such knowledge is necessary in order to compute the optimum tax rate which corresponds to the level of emissions for which the marginal social benefit of the ambient pollution is equal to its marginal social damage.

In practice the social damage function is not known by the regulator, and information about it which may be available are questionable. Our purpose in this paper is to design an instrument 35 that does not rest on such pieces of information and nevertheless induces the agents to achieve the socially optimal levels of emissions as a Nash equilibrium. Our proposal applies to the class of non point source regulation problems where emitters are also the recipients of the externality. In order to implement it, the regulator needs to observe the level of ambient pollution and to know the number of agents involved in the problem. But neither the observation of individual 40 emissions nor the knowledge of the social cost are needed.

The mechanism operates as follows: the regulator asks each agent both to choose a level of emission and to provide his expectation about the aggregate level of emission, the ambient pollution. He also announces the following rule: if the actual level of ambient pollution is above the level predicted by the agent, the latter will be liable to a tax proportional to the gap. The 45 key of the mechanism lies in the tax rate which is equal to the proportion of other agents in the population, i.e.  $\frac{n-1}{n}$ . Under *laissez-faire* each agent chooses a level of emission such that the marginal utility gain of his emissions equals his marginal utility loss from pollution. Once the tax on expectation errors is introduced, each agent takes into account the externality he exerts on the  $n - 1$  other agents when choosing his own level of emission.

50 This note is organized as follows. The next section presents a non point source pollution framework. Section 3 explains our mechanism, the regulated game that goes with it, and it demonstrates that its Nash equilibrium exists, is unique under a normality condition and that it corresponds to a Pareto optimal allocation. The last section offers final remarks.

## 2 A non point source pollution framework

Consider  $n$  agents, indexed  $i = 1, \dots, n$  who emit individual pollutants  $e_i \in E_i = [0, \bar{e}^i] \subset \mathbb{R}_+$ , where  $E_i$  is a compact and convex subset of the positive real numbers. Individual emissions aggregate into an ambient pollution level

$$X = \sum_{h=1}^n e_h.$$

An *allocation* is a  $n + 1$  dimensional vector  $(e_1, \dots, e_n, X) \in \times_{h=1}^n E_h \times \mathbb{R}_+$ . A typical agent  $i$  is endowed with a preference ordering over the allocations. His ordering can be represented

by a continuous, differentiable utility function, which is strictly increasing with the agent's own emission  $e_i$  and strictly decreasing with the ambient pollution  $X$ :

$$U^i(e_i, X), \quad U_1^i > 0, U_2^i < 0,$$

where  $U_1^i = \partial U^i / \partial e_i$  and  $U_2^i = \partial U^i / \partial X$ . We also assume that the Hessian matrix  $Hess(U^i)$  is semi definite negative (hence utility functions are concave). Let:

$$MRS^i(e_i, X) = -U_2^i(e_i, X) / U_1^i(e_i, X)$$

55 stand for the absolute value of the *marginal rate of substitution* between agent  $i$ 's emission and ambient pollution.

Let us assume that corner decisions are dominated strategies, *i.e.*

**Assumption 1.**  $\forall i$  and whatever the others' strategies  $e_{-i} = \sum_{h \neq i} e_h$ ,

$$\lim_{e_i \rightarrow 0} MRS^i(e_i, e_i + e_{-i}) < 1,$$

$$\lim_{e_i \rightarrow \bar{e}^i} MRS^i(e_i, e_i + e_{-i}) > 1,$$

Under Assumption 1 Nash equilibria  $(\tilde{e}_1, \dots, \tilde{e}_n)$ , or *laisser-faire equilibrium*, are interior and solves the system of equations:

$$MRS^i(e_i, X) = 1, \quad i = 1, \dots, n. \quad (1)$$

On the other hand, interior Pareto optima  $(e_1^*, \dots, e_n^*)$  must satisfy the Bowen-Lindhal-Samuelson (BLS) condition:

$$\sum_h MRS^h(e_h, X) = 1. \quad (2)$$

In general (1) and (2) admit different solutions, *i.e.* *laisser-faire equilibria* are not Pareto optimal allocations. This comes as no surprise since in the absence of cooperation, when choosing his level of emission each agent  $i$  is weighing his marginal advantage  $U_1^i(e_i, X)$  compared to his individual marginal cost  $U_2^i(e_i, X)$ , while social efficiency requires to put individual advantages against the marginal *social cost*  $\sum_h U_2^h(e_h, X)$ . This social dilemma calls for some intervention from a benevolent authority.

65 It is worth noting that this model can be seen as a reduced form, which can accommodate two interpretations. In a first approach, each agent is a farm household, which consumes from the proceeds of its production, but production is accompanied by individual pollutions. In summary, more individual consumption is associated with more individual pollution. Households have preferences defined over consumption and aggregate pollution. Pareto optima in that case are allocations that maximize the social welfare of the community of farm households producing and suffering from pollution. In a second approach, agents are firms in the usual sense. In that case, functions  $U^i$  are better called profit functions. The ambient pollution is then an externality that adversely affects firms' cost and allocations that maximize the *producers' surplus* satisfy the BLS condition (2).

**Example 1.** Let the utility functions be:

$$U^i(e_i, X) = ae_i - \frac{b}{2}(e_i)^2 - cX, \quad a, b, c > 0, \quad a > nc,$$

and  $e_i \in [0, a/b]$ . At the Nash equilibrium, individual decisions are:

$$\tilde{e}_i = \frac{a - c}{b},$$

and the symmetric Pareto optimum is:

$$e_i^* = \frac{a - nc}{b} < \tilde{e}_i.$$

### 3 The mechanism

#### 3.1 The regulator's information and the regulated game

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From the point of view of the regulator, the information structure we consider has two aspects:

1. the regulator cannot observe individual emissions (or only at prohibitive costs). This *moral hazard* aspect defines the framework as a *non point source regulation problem*.
2. and he also ignores the agents' utility functions, that are subjective attributes to be elicited. This *adverse selection* aspect (information asymmetry) is at the root of an *implementation problem*.

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Those two informational conditions have been studied separately by two (related) strands of literature. The literature on ambient pollution considered only the moral hazard aspect. Collective penalties could therefore be defined by targeting a particular pollution level. If it turns out that this level is a Pareto optimal one, collective penalties induce a Pareto optimal outcome. Otherwise this optimality property has no reason to hold. It is conditional on the possibility to elicit agents' utility functions in order to compute the social cost, by some means. The second literature, the theory of mechanisms design, specifically addresses the adverse selection issue, information asymmetry about preferences, between agents and the regulatory authority. But it assumes that individual actions are perfectly observable, so that adequately designed transfers can be defined conditionally on the realization of those actions.

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Clearly, for a large class of non point source problems, internal consistency requires to merge those two aspects (this necessity is also emphasized by Duggan & Roberts, 2002, as an extension of their work). This is the way we initiate in this paper.

95 The regulator solicits information from the agents and uses the collected pieces of information to set up a system of incentive taxes, that cannot be based on individual emission because they are not observable, but can be based on ambient pollution.

Each agent is asked to state his expectation of the level of *aggregate pollution*, which will be denoted  $\widehat{X}_i \in [0, \sum_h \bar{e}_h]$ . Simultaneously he decides upon his level of emission, given that he will have to pay a penalty that depends only on the gap between the actual and his expected level of aggregate emissions:

$$T^i(\widehat{X}_i, X) = \begin{cases} k(X - \widehat{X}_i) & \text{if } X \geq \widehat{X}_i. \\ 0 & \text{otherwise,} \end{cases} \quad (3)$$

where  $k \in [0, 1]$  is a parameter that will play a crucial role in the following. Expression (3) is a tax for agent  $i$  only if the realized aggregate pollution is larger than his forecast. This formula 100 contains the original suggestion we make in this article, which departs from existing formulas in two respects: *i*) the target for the ambient pollution is not exogenously given to the agents. On the contrary, each agent announces his own target  $\widehat{X}_i$ , *ii*) the price  $k$  for units of pollution above the target is not set equal to the marginal social damage on others, since this information is not available to the regulator. However, we will see below that this parameter can be attributed 105 a value, based on available information, that will induce Pareto optimality.

This mechanism defines a class of games configured by parameter  $k$ . In a  $k$ -game each agent  $i$  has two decision variables to be determined simultaneously,  $e_i \in E_i \subset \mathbb{R}_+$  and  $\widehat{X}_i \in [0, \sum_h \bar{e}_h]$ . And faced with this scheme, his payoff (or utility) thus becomes:

$$\Pi^i(e_i, e_{-i}, \widehat{X}_i) \equiv \begin{cases} U^i(e_i - k(e_i + e_{-i} - \widehat{X}_i), e_i + e_{-i}) & \text{if } e_i + e_{-i} \geq \widehat{X}_i, \\ U^i(e_i, e_i + e_{-i}) & \text{otherwise.} \end{cases}$$

### 3.2 The $k$ -interior Nash equilibrium

Under the assumption that the game played by the agents is one of complete information and with common knowledge, the Nash equilibrium is a usual predictor for non cooperative decisions. In the regulated game, there are two kinds of interior Nash equilibria: *i*) those with announcements that exceed emissions (so there is no penalty), which themselves are those of the 110 laissez-faire scenario, *ii*) and those where announcements match aggregate emissions, which are modified compared to the laissez-faire scenario and configured by parameter  $k$ . We will focus our attention to the second kind of equilibria, which we call *k-interior Nash equilibria*.

**Definition 1.** A  $k$ -interior Nash equilibrium ( $k$ -INE) is a vector of individual emissions

$$(e_i(k), \dots, e_n(k)) \in \times_{h=1}^n E_h,$$

a profile of announcements:

$$(\widehat{X}_1(k), \dots, \widehat{X}_n(k)) \in \left[0, \sum_h \bar{e}_h\right]^n,$$

and a profile of transfers

$$\left( \mathbf{T}^1 \left( \widehat{X}_1(k), X(k) \right), \dots, \mathbf{T}^n \left( \widehat{X}_n(k), X(k) \right) \right) \in \mathbb{R}_+^n,$$

such that:

1. announcement decisions  $\widehat{X}_i(k), i = 1, \dots, n$  match aggregate emissions:

$$\widehat{X}_i(k) = \sum_h e_h(k), \quad i = 1, \dots, n.$$

2. Emission decisions  $e_i(k), i = 1, \dots, n$  solve the system of necessary conditions:

$$MRS^i \left( e_i, \sum_h e_h \right) = 1 - k, \quad i = 1, \dots, n. \quad (4)$$

To grasp item 1 in the above definition, consider the point of view of agent  $i$ . Given his best educated guess about the emissions by the other agents,  $\tilde{e}_{-i}$ , and given the own choice of emission  $\tilde{e}_i$  he contemplates, if he announces  $\widehat{X}_i > \tilde{e}_i + \tilde{e}_{-i}$  he pays no tax (and the risk of paying a tax is totally eliminated when the announcement is at the highest possible level  $\widehat{X}_i = \sum_h \bar{e}_h$ ). But the profile of emissions consistent with such an announcement is the *laissez-faire* (no tax). In other words, agents can escape the mechanism by coordinating on the *laissez-faire* equilibrium and announcing any level strictly higher than the corresponding ambient pollution. Now if the agent announces  $\widehat{X}_i < \tilde{e}_i + \tilde{e}_{-i}$  he pays a tax, which he can reduce by increasing his announcement (thereby increasing his utility) up to the point where  $\widehat{X}_i = \tilde{e}_i + \tilde{e}_{-i}$ ; with this choice he pays no tax. Regarding the equilibrium transfers, an implication of item 1 is:

$$\mathbf{T}^i(k) \equiv T^i \left( \widehat{X}_i(k), \sum_h e_h(k) \right) = 0, \quad i = 1, \dots, n.$$

115 Each equilibrium transfer is zero since, at a  $k$  - INE, announcements and ambient pollution are identical. This is an interesting advantage from a practical point of view, since mechanisms themselves have implementation costs that are likely to be larger for larger transfers. Here those costs are probably minimized.

120 The second item in the definition is the usual first order condition for interior optimal individual emissions. Interior decisions are more interesting than corner solutions in the context of this paper, because only in the first case individual behaviors fully integrate price information, and then the challenge is to find a price that indicates the social value of ambient pollution. Consistently with this perspective, it is easy to single out economic environments that discard corner decisions as dominated strategies:

**Assumption 2.** Assume,  $\forall i, \forall e_{-i}, \forall \widehat{X}_i$  :

$$\lim_{e_i \rightarrow 0} MRS^i \left( e_i - T^i \left( \widehat{X}_i, e_i + e_{-i} \right), e_i + e_{-i} \right) < (1 - k) \quad (5)$$

and

$$\lim_{e_i \rightarrow \bar{e}^i} MRS^i \left( e_i - T^i \left( \widehat{X}_i, e_i + e_{-i} \right), e_i + e_{-i} \right) > (1 - k). \quad (6)$$

125 Inequality (5) indicates that, starting from a zero emission level, the marginal utility gain from increased individual emission is greater than the marginal loss of utility caused by the increase of ambient pollution. Similarly, inequality (6) states that starting from the maximal level of individual emission, agent  $i$  has an marginal incentive to cut down pollution. Those inequalities are weaker forms of the so-called Inada conditions<sup>1</sup>. Besides, one has to bear in mind that  
130 they are not necessary conditions but only sufficient conditions to avoid corner decisions.

**Theorem 1 (Existence).** *The  $k$  - game admits at least one Nash equilibrium. Under Assumption 2, any Nash equilibrium is interior.*

*Proof.* See Appendix A.1. ■

For reasons that will become clear below, let us now focus on the case where  $k = 1 - 1/n$ . With this value for parameter  $k$ , each agent is taxed up to the share  $(n - 1) / n$  of the aggregate externality generated. And the first order conditions (4) rewrite:

$$MRS^i (e_i, X) = \frac{1}{n}, \quad i = 1, \dots, n.$$

It is as if the agent's marginal utility of polluting that goes through the first argument in his payoff function was cut down to only  $(1/n)^{\text{th}}$  of the original level, making individual emissions less advantageous. We are now in position to make a crucial observation. By summing-up the above equations over the agents:

$$\sum_{h=1}^n MRS^h (e_h, X) = 1,$$

one finds the BLS conditions (2). At the same time:

$$\mathbf{T}^i \left( \widehat{X}_i (1 - 1/n), X (1 - 1/n) \right) = 0, \quad \forall i.$$

135 Therefore a  $k$  - INE has the following desirable property:

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<sup>1</sup>Inada conditions would require:

$$\begin{aligned} \lim_{e_i \rightarrow 0} MRS^i \left( e_i - T^i \left( \widehat{X}_i, e_i + e_{-i} \right), e_i + e_{-i} \right) &= 0, \\ \lim_{e_i \rightarrow \bar{e}^i} MRS^i \left( e_i - T^i \left( \widehat{X}_i, e_i + e_{-i} \right), e_i + e_{-i} \right) &= +\infty. \end{aligned}$$

**Theorem 2 (Pareto optimality).** *A  $k$  - INE with  $k = 1 - 1/n$ , is a Pareto optimal allocation.*

In practice the exact number  $n$  of agents involved in the pollution problem might not be known exactly. In this case, a  $k$  - INE only approaches a Pareto optimal allocation. Intuitively, as parameter  $k$  gets larger the individual incentives to pollute at a  $k$  - INE are getting weaker, and when the number of agents used to construct the mechanism increases to get closer to its true value  $n$ , the  $k$  - INE eventually gets closer to a Pareto optimal allocation.

**Example 2 (ex. 1 continued).** *With the mechanism, and when the forecast is correct  $\widehat{X}_i = \sum_h \tilde{e}_h$ , the Nash equilibrium is now configured by parameter  $k$ :*

$$\tilde{e}_i(k) = \frac{a - \frac{c}{1-k}}{b} .$$

And when  $k = 1 - 1/n$ , the Nash equilibrium is Pareto optimal:

$$\tilde{e}_i\left(\frac{n-1}{n}\right) = \frac{a - nc}{b} = e_i^* .$$

In the above example, the  $k$  - INE exists and is unique. Uniqueness is an interesting property since the Nash implementation approach is often plagued with a serious problem of multiplicity of equilibria. To put it differently, Nash-implementation mechanisms often solve the social dilemma but at the cost of a coordination problem. Can we expect the uniqueness property to hold beyond our illustration? Under what condition can we guarantee this property?

To analyze this question, it will prove useful to reformulate each agent's problem. Define

$$\begin{aligned} c_i &= e_i - k(X - \widehat{X}_i) , \\ &= X - X_{-i} + k(\widehat{X}_i - X) . \end{aligned}$$

Or upon rewriting:

$$c_i - (1 - k)X = k\widehat{X}_i - X_{-i} . \quad (7)$$

With this rewriting, each agent's problem can be seen as one where the goal is to choose  $X$  in order to maximize:

$$U^i(c_i, X) ,$$

subject to (7) and  $X \geq X_{-i}$ . It looks like a consumer's problem, who is endowed with an exogenous "income"  $y_i = k\widehat{X}_i - X_{-i}$ , and has to allocate it optimally between his consumption of a private good  $c_i$  and of cleanup  $-X$ , whose price relative to  $c_i$  is  $p = 1 - k$ . Denote  $c_i = C^i(p, y_i)$  and  $X = D^i(p, y_i)$  the two demand functions derived from this problem. And assume that  $c_i$  and  $-X$  are normal goods, *i.e.* the demands  $C^i(p, y_i)$  and  $X = D^i(p, y_i)$  are respectively strictly increasing and strictly decreasing functions of income:

**Assumption 3 (Normality).**  $C_2^i(p, y_i) > 0$ ,  $D_2^i(p, y_i) < 0$ .

As far as  $X$  is concerned, the normality assumption is akin to a form of strategic complementarity: the larger the aggregate contributions of others,  $X_{-i}$ , the lower  $y_i$  and the larger  $X$ .  
 155 A more straightforward interpretation is as follows: the larger the tax (for instance because of a larger  $k$ ), the larger "income"  $y_i$  and the lower aggregate pollution.

**Theorem 3 (Uniqueness).** *If Assumption 3 holds, the  $k$  - INE is unique.*

*Proof.* See Appendix A.2. ■

160 This result applies also in the particular case where  $k = 1 - 1/n$ , which ensures that a Pareto optimal allocation can be implemented as a unique interior Nash equilibrium. Assumption 3 can be relaxed somewhat in order to hold only for the particular value  $k = 1 - 1/n$  that is consistent with Pareto optimality.

**Example 3 (ex.1 continued).** *Applying the reformulation (7) to the quadratic example, each agent's problem now reads as:*

$$\max_X U^i(c_i, X) = a[pX + y_i] - \frac{b}{2}[pX + y_i]^2 - cX.$$

subject to the constraint  $X \geq k\hat{X}_i - y_i$ . The two demand functions are:

$$\begin{aligned} c_i &= C^i(p, y_i) = \frac{a - c/p}{b} + (1 - p/b)y_i, \\ X &= D^i(p, y_i) = \frac{a - c/p}{bp} - \frac{y_i}{b}. \end{aligned}$$

As one can readily check, consumption and cleanup ( $-X$ ) are normal goods.

## 165 4 Conclusion

Three last remarks are in order.

Firstly, by now there exists a number of Nash-implementation mechanisms (Groves and Ledyard, 1977, Walker, 1981, Tian, 1990, Kim, 1993, Peleg, 1996, Falkinger, 1996, Duggan & Roberts, 1996, Chen, 2002, for instance). The mechanism proposed here has a similarity with  
 170 two of them in particular, those of Falkinger (1996)<sup>2</sup> and Duggan & Roberts (1996). Falkinger's mechanism (1996) uses a price for the externality which is calibrated by using the number of agents involved in the problem as the only piece of information. Actually, his formula for  $k$  is exactly the same as ours ( $k = 1 - 1/n$ ). But Falkinger's mechanism differs in that it penalizes or subsidizes deviations between own decisions and average decisions by others, whereas  
 175 our mechanism introduces a new decision variable for each agent (his forecast about ambient

<sup>2</sup>See also Falkinger, Fehr, Gächter & Winter-Ebmer (2000) for an experimental test of this mechanism.

pollution) and it only penalizes deviations between actual aggregate decisions and individual forecasts. A second difference lies in the number of Nash equilibria that can be unique with our mechanism (Theorem 3), whereas Falkinger's mechanism (1996) has a continuum of Nash equilibria<sup>3</sup> when  $k = 1 - 1/n$ . In Duggan & Roberts (1996) one finds the introduction of an announcement variable for each agent and the penalization of the gap between this announcement and the realizations. But the announcement is for pollution of the "neighbor" and thus relies on the ability to observe individual emissions. As Duggan & Roberts recognize themselves, their mechanism handles the problem of adverse selection but not that of moral hazard. Our mechanism bypasses this limitation by asking agents to make announcements on aggregate pollution.

Secondly, at a  $k$ -INE the budget of the regulatory authority is balanced. But this property need not hold outside equilibrium decisions. More precisely, as soon as one agent underestimates the aggregate emissions, a penalty will be paid and this will produce a budget surplus. Many mechanisms try, by construction, to avoid the imbalance of the budget. The reason comes from an implicit general equilibrium perspective, from which one deduces that any budget surplus necessarily returns to some economic agents (see for instance the discussion in Green and Laffont, 1979, Chap. 9). This redistribution, if correctly anticipated by agents, may compromise the good incentive properties of the mechanism under consideration. In defence of unbalanced budgets, one finds at least three possible reactions in the literature. The first one is that the regulator could announce that any redistribution of surplus will be lump-sum, hence without effects on incentives. The second one is to assume a form of myopia from the agents regarding the redistribution of the public budget. The last reaction does not rest on an assumption of bounded rationality. It assumes a partial equilibrium point of view, which is more in line with the problem studied in this paper. The economy has many agents and only a small subset of them is involved in the non point source pollution problem. A budget surplus, if any, can be redistributed to the rest of society, hence without interfering with the incentives given to the regulated agents. The budget surplus may even produce a double dividend, by financing interventions to promote other social goals.

Lastly, this paper is the first of its kind at the intersection of the literature on non point source pollution and on the design of mechanisms and may provide inspiration to sketch a research program. Just repeat the steps that each of the two literatures has already travelled by separately, while taking now into account both the constraints of non-observability and information asymmetry. For example, one could search for general possibility results of implementation in Nash equilibrium when only aggregate decisions can serve as support for incitative transfers.

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<sup>3</sup>However the Nash equilibrium under Falkinger's mechanism can be unique for arbitrarily lower values of  $k$ . For this reason, Falkinger suggests to use his mechanism with  $k \simeq 1 - 1/n$ , in order to implement near-optimal contributions to public goods.

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## Appendix

### A Existence and uniqueness of a $k$ - INE

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#### A.1 Existence

A standard set of sufficient conditions for the existence of pure strategy Nash equilibria is: *i*) strategy sets are non empty compact and convex sets, *ii*) for each player, his payoffs are continuous and quasi-concave with respect to his own decision variables.

The first condition is satisfied in the  $k$  - game. Regarding the second condition, payoff functions are continuous (though they are not differentiable everywhere because of the transfers), but quasi-concavity is an open question. For each agent  $i$ , taking  $e_{-i}$  and  $\widehat{X}_i$  as given parameters we must ascertain the quasi-concavity of the function:

$$\Pi^i(e_i) \equiv \begin{cases} f^i(e_i) & \text{if } e_i < \widehat{X}_i - e_{-i}, \\ g^i(e_i) & \text{if } e_i \geq \widehat{X}_i - e_{-i}. \end{cases} \quad (8)$$

where

$$\begin{aligned} f^i(e_i) &\equiv U^i(e_i, e_i + e_{-i}), \\ g^i(e_i) &\equiv U^i\left(e_i - k\left(e_i + e_{-i} - \widehat{X}_i\right), e_i + e_{-i}\right). \end{aligned}$$

220 Actually it can be shown that  $\Pi^i(\cdot)$  is concave (hence it is quasi-concave). The architecture of the proof is to show that the functions  $f^i(\cdot)$  and  $g^i(\cdot)$  are concave and that combination of both in (8) has a "min" property (see below). The latter property is important to establish the concavity of function  $\Pi^i(\cdot)$ . The concavity of both functions  $f^i(\cdot)$  and  $g^i(\cdot)$  is not enough to guarantee that  $\Pi^i(\cdot)$  is also concave. It is easy to exhibit examples where  $f^i(\cdot)$  and  $g^i(\cdot)$  are  
225 concave, whereas  $\Pi^i(\cdot)$  is not.

Note that  $f^i(\cdot)$  is concave by definition. And it can be shown that  $g^i(\cdot)$  is concave as well. Indeed its second order derivative is:

$$g^{i'''}(e_i) = (1 - k)^2 U_{11}^i + 2(1 - k) U_{12}^i + U_{22}^i.$$

And from the assumptions we made about utility functions:

$$v * Hess(U^i) * v^T = [v_1, v_2] * \begin{bmatrix} U_{11}^i & U_{12}^i \\ U_{21}^i & U_{22}^i \end{bmatrix} * \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \leq 0, \quad \forall v = [v_1, v_2] \in \mathbb{R}^2.$$

To obtain the sign  $g'''(e_i) \leq 0$ , showing that  $g^i$  is concave, it suffices to substitute the vector  $[v_1, v_2] = [1 - k, 1]$  in the above inequality.

The important "min" property mentioned above is the following: because  $U_1^i > 0$ , function  $\Pi^i(\cdot)$  can also be written as:

$$\Pi^i(e_i) \equiv \min \{ f^i(e_i), g^i(e_i) \} .$$

This is easy to check. For all  $e_i < \widehat{X}_i - e_{-i}$ , under  $g^i(\cdot)$  agent  $i$  would perceive a subsidy, hence  $f^i(\cdot) \leq g^i(\cdot)$ . And as soon as  $e_i \geq \widehat{X}_i - e_{-i}$ , under  $g^i(\cdot)$  agent  $i$  would be liable to a tax, thus  $g^i(\cdot) \leq f^i(\cdot)$ .

Recall that by definition a concave function is such that the set of points lying on or below its graph, *i.e.* its *hypograph*, is convex. Since  $f^i(e_i)$  and  $g^i(e_i)$  are concave, their hypographs:

$$\begin{aligned} hypo(f^i) &= \{ (e_i, \pi_i) \in E_i \times \mathbb{R} : f^i(e_i) \geq \pi_i \} , \\ hypo(g^i) &= \{ (e_i, \pi_i) \in E_i \times \mathbb{R} : g^i(e_i) \geq \pi_i \} , \end{aligned}$$

are convex. And the hypograph of  $\Pi^i(\cdot)$  is:

$$\begin{aligned} hypo(\Pi^i) &= \{ (e_i, \pi_i) \in E_i \times \mathbb{R} : \Pi^i(e_i) \geq \pi_i \} , \\ &= hypo(f^i) \cap hypo(g^i) , \end{aligned}$$

where the second line is obtained because  $\Pi^i(\cdot)$  is constructed as the min of  $f^i(\cdot)$  and  $g^i(\cdot)$ . So  $hypo(\Pi^i)$  is necessarily convex, as the intersection of two convex sets. Therefore  $\Pi^i(\cdot)$  is concave and the set of Nash equilibria for the  $k$ -game is not empty.

Of course, this does not mean that (some) Nash equilibria are interior. However, Assumptions 1 and 2 rules out corner decisions, which ensures in this case the existence of at least one interior Nash equilibrium.

QED.

## A.2 Uniqueness

The strategy of proof given here follows the elegant one offered by Andreoni and Bergstrom (1996) for the case of Nash contributions to a public good, though some modifications are required in our context, because action spaces also incorporate the announcements  $\widehat{X}_i$ ,  $i = 1, \dots, n$ .

To prove uniqueness in Theorem 3 we first need the following lemma:

**Lemma 1.** Suppose there exists two distinct  $k$  - INE, denoted  $(e'_1, \dots, e'_n, \widehat{X}'_1, \dots, \widehat{X}'_n)$  and  $(e''_1, \dots, e''_n, \widehat{X}''_1, \dots, \widehat{X}''_n)$ , and suppose without loss of generality that  $X'' \geq X'$ . Then for any agent  $i$ ,

i)  $k\widehat{X}''_i - X''_{-i} \leq k\widehat{X}'_i - X'_{-i}$ ,

ii)  $X''_{-i} = \sum_{h \neq i} e''_h \geq X'_{-i} = \sum_{h \neq i} e'_h$ .

*Proof.* Since  $(e''_1, \dots, e''_n, \widehat{X}''_1, \dots, \widehat{X}''_n)$  is a  $k$  - INE, it must be the case that  $X'' = \sum_{h=1}^n e''_h = D^i(p, k\widehat{X}''_i - X''_{-i})$ . By the same argument, it must also be the case that  $X' = \sum_{h=1}^n e'_h = D^i(p, k\widehat{X}'_i - X'_{-i})$ . And since by assumption  $X'' \geq X'$ , one must deduce that  $D^i(p, k\widehat{X}''_i - X''_{-i}) \geq D^i(p, k\widehat{X}'_i - X'_{-i})$ . And, if ambient clean-up is a normal good, one obtains item i) of the Lemma, i.e.  $k\widehat{X}''_i - X''_{-i} \leq k\widehat{X}'_i - X'_{-i}$ . To demonstrate item ii), rewrite item i) as  $k(\widehat{X}''_i - \widehat{X}'_i) \leq X''_{-i} - X'_{-i}$ . Finally, using the properties that  $\widehat{X}''_i = X''$ ,  $\widehat{X}'_i = X'$  at any  $k$  - INE, and the assumption  $X'' \geq X'$  one arrives at item ii) of the Lemma,  $X''_{-i} \geq X'_{-i}$ . ■

Equipped with this Lemma, assume by way of contradiction that there exists (at least) two distinct  $k$  - INE

$$(e'_1, \dots, e'_n, \widehat{X}'_1, \dots, \widehat{X}'_n),$$

and

$$(e''_1, \dots, e''_n, \widehat{X}''_1, \dots, \widehat{X}''_n).$$

Suppose, without loss of generality, that  $X'' = \sum_h e''_h \geq X' = \sum_h e'_h$ . Then according to item ii) in Lemma 1, it must be the case that, for any agent  $i$ ,  $X''_{-i} = \sum_{h \neq i} e''_h \geq X'_{-i} = \sum_{h \neq i} e'_h$ . And, since the two equilibria are distinct, for some agent  $i$  the previous inequality is strict,  $X''_{-i} > X'_{-i}$ , which implies:

$$X'' > X'. \tag{9}$$

Since  $c_i$  is a normal good and, from item i) in Lemma 1,  $k\widehat{X}'_i - X'_{-i} \geq k\widehat{X}''_i - X''_{-i}$ , then  $c'_i = C^i(p, k\widehat{X}'_i - X'_{-i}) \geq c''_i = C^i(p, k\widehat{X}''_i - X''_{-i})$  for all  $i$  and with a strict inequality for at least one agent. Therefore  $\sum_h c'_h > \sum_h c''_h$ . But in equilibrium it must also be:

$$\sum_{h=1}^n c'_h = \sum_{h=1}^n e'_h = X'$$

Applying the same observation to the second  $k$  - INE, one finds:

$$\sum_h c''_h = \sum_{h=1}^n e''_h = X''.$$

Necessarily, if  $\sum_h c'_h > \sum_h c''_h$  then  $X'' < X'$ , in contradiction with (9). So there cannot be two distinct  $k$  - INE. QED.

## References and Notes

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