

# Optimal Species Preservation Policy in a Symbiotic Relationship between Species

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## **Abstract**

In recent years, economists and ecologists have become increasingly interested in optimal conservation policies to protect natural areas and the biodiversity embodied in them. A famous metaphor that describes this conservation policy is the Noah's Ark problem: Noah had to decide which species he should take aboard the ark to survive, and which were to become extinct (Weitzman, 1998). One of Weitzman's conclusions is that the optimal policy is an extreme policy. Thus, in the Noah's Ark model, almost all species go aboard either in full or not at all. In the case of a symbiotic relationship, the classic optimal policy of Noah's Ark problem might not stand. We examine optimal preservation policies in different costs functions and obtain a ranking criterion based on cost-effectiveness analysis.

*Keywords:* Biodiversity; Symbiotic relationship; Optimal policy; Keystone species

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## 1. Introduction

Biodiversity, is defined as the variety of microorganisms that live on the globe, and describes the relationships between the species and with ecosystems and more. The relationships between the species are often very important to the construction of the habitat's biodiversity. Damage to a certain species in the habitat could negatively affect the harmony of the habitat and of the species that depend on it. This opinion is common to both economics and ecology. The following quote demonstrates this: "Understanding how the strengths of species interactions are distributed among species is critical for developing predictive models of natural food webs as well as for developing management and conservation strategies..." (Berlow et al., 1999).

Financial resources for biodiversity conservation programs are not sufficient to protect all habitats and species. This requires choosing conservation priorities in order to support the most species at the least cost. Moran et al. (1996), (1997) pointed out that there is no single correct method for establishing biodiversity conservation priorities at any level of organization. Nevertheless, the question of how to determine priorities for maintaining or increasing biodiversity under a limited budget constraint has occupied the minds of environmental economists since the seminal work by Weitzman (1998). He developed "... *a more-or-less consistent conceptual framework and a more-or-less usable measure on the value of diversity that can tell us how to trade off one form of diversity against the other*" (Weitzman (1998)). The Noah's ark problem: Noah had to decide which species he should take aboard the ark to survive, and which were to die out. Weitzman used diversity theory (1993) to provide quantitative indicators of which species to preserve. Furthermore, in his seminal work on the Noah's Ark problem (1998) Weitzman assumed that the survival probabilities are independent and the costs function is linear. One of Weitzman's conclusions is that the optimal policy is an extreme policy. Therefore, in the Noah's Ark model almost all species go aboard either in full or not at all.

When central planners want to maximize the social welfare that is obtained from biodiversity, they must take into account the interactions between the species, since one species often cannot exist without the other. The extinction of a species due to the

absence of the species it depends on, damages species diversity directly and human society's benefit indirectly.

Therefore, the goal of this paper is to determine the optimal preservation policy, when there are many species that depend on others for their survival, and to find the ranking criteria by which to act. Species preservation requires a budget, which is limited (by human society), and consequently the central planner (for instance, the Israel Parks Authority) must determine a policy under budget constraints. This policy dictates the biodiversity for years to come, as well as the anticipated social benefits from species preservation. It should be noted that decisions concerning the preservation of certain species while ignoring their effects on other species, might harm the stability of the ecosystem and its recovery capacity. This would also cause a decline in the benefits to human society. For example, cutting down pine trees would harm the pine mushrooms (that live in commensalism with pine trees), which would in turn cause the habitat to change its species composition and thus damage other values such as recreation, landscape, soil quality and more.

There are other cases in which when the quantity of one species (that delineates the habitat's nature and landscape) is reduced, the ecosystem is damaged by the invasion of other species.

In the symbiotic Noah's Ark problem a central planner allocates a given budget to maximize the expectation of the biodiversity value function. One of the species is the keystone species, and the others are keystone-dependent species; these may be beneficiaries and/or predators, and they have a symbiotic relationship with the keystone species and vanishing biodiversity whenever the keystone species becomes extinct. In this case, the optimal policies in different costs functions regimes may be interior policies – some of the species may survive. Specifically, generalization of the biodiversity value function yields:  $W(\pi_0; \pi_1, \dots, \pi_K) = \pi_0 M_0 + \pi_0 \pi_1 E_1 + \dots + \pi_0 \pi_K E_K$ , where  $M_0, E_1, \dots, E_K$  are positive constants. For this special case, with a single keystone-dependent species, inserting a linear budget constraint, we obtain a concave function in  $\pi_0$  which might lead to an interior optimal policy. Moreover, in such an optimal policy at least half of the budget is invested in the keystone species' preservation. Also, the optimal preservation policy  $\pi_0^*$  of the keystone species is also

the optimal policy of the general case where there are several ( $K \geq 1$ ) similar keystone-dependent species, and is not dependent on  $K$ .

## **2. Biodiversity Preservation**

Decisions concerning protected species reflect their value, perception and the uncertainty derived from them to the society that makes the decisions. Choosing areas for preservation or maintenance based on economic criteria is rare in the literature. The uncertainty and the changes to the value of biodiversity require dynamic evaluation, which can facilitate the relearning of long-term preservation decisions.

In the modern world, we observe a multitude of development pressures that originate in demographic changes and changes that occur due to economic growth, and the consequent damage to the habitat is inevitable. Moreover, intensification of agriculture activities in landscape areas affects biodiversity and creates biodiversity-related conflicts (Henle et al., 2008)

Some claim that each generation must leave “enough and as good for the other” (namely, sustainable development). However, others argue that the present generation does not have the right to deplete basic resources (natural inheritance) from economic and other aspects. Since natural resources cannot be passed on intact, that is to say, unused, the proposal is that future generations are financially compensated for every devaluation of natural resources (that was caused by the present generation).

It is true that in recent years, there has been a deceleration in the rise of polluting emissions, but pollutants are still emitted. The destruction of natural habitats and other factors, which are believed to be the causes of species' extinction, can be very expensive. Therefore, it is only natural to ask what the value of biodiversity preservation is (Craft & Simpson, 2001). Even though, conservation efforts are increasing, it is shown that biodiversity continues to decrease (Rands et al., 2010).

One should bear in mind that we assume that there is an optimal level of environmental protection, which changes under various conditions. One can thus conclude that there is also an optimal level of biodiversity, linked to different uses of the soil. For example, agricultural land requires a different level of environmental protection than tropical

forests, and as a result, the optimal level of biodiversity required for agriculture is different from the optimal level required for the preservation and continuity of tropical forests.

The need for environmental protection does not necessarily mean that we must preserve all species in all places (an impossible situation), but it is essential to preserve the species which are used as products or services that people depend on (Perrings, 1999).

Assessing the value of biodiversity provides the quantitative basis for making decisions about biodiversity preservation. Biodiversity is faced with many – global and local – threats. It is important that all sectors (government, business and individuals) take into account that biodiversity contributes to our well-being, and invest in preservation actions. (Rands et al., 2010)

The financial resources of various biodiversity programs are not sufficient to protect all of the habitats and species. The human race is incapable of protecting everything. We must therefore decide on priorities in order to support and preserve most species at minimal costs. Moran et al (1996; 1997) argue that there is no one correct method to determine priorities of biodiversity preservation at all organizational levels. In other words, allocation of resources to maximize the classic goal of the economic problem is not possible. Moreover, the question how to set criteria to preserve and even increase biodiversity dependent on a given budget, has occupied many environmental economists, primarily following Weitzman's (1992; 1993) work. Weitzman developed "...a more or less consistent conceptual framework and a more or less usable measure on the value of diversity that can tell us how to trade off one form of diversity against the other" (Weitzman, 1992, p. 363 – see also Weitzman, 1995; 1998).

In his article, Weitzman (1998) tried to develop some sort of criteria that could serve to rank the priorities of biodiversity preservation. To answer this question, he compared biodiversity preservation subject to budget constraint to the problem of Noah's Ark (building the ark, food purchases, etc.) under budget constraint. The criterion that Weitzman proposed is the sum of the direct benefit that measures how much we value the existence of species  $i$  and the distance or difference of a species in the next unit, when the preservation project increases the survival probability of species  $i$  plus the cost of preservation.

Weitzman regarded species as if they were books in a library. His focal question was what the best way to preserve libraries is; the parallel question being, what is the best way to preserve species, that is, the problem of optimal preservation of biodiversity under budget constraints. The basic unit in the libraries model is the library, and in the abstract, each library unit represents a certain species.

An important result was that assuming independent survival probabilities between species and linear costs function, the optimal policy should be extreme (either preserve or die out). The only species that does not support this has interior survival probability determined by budget equality.

### **3. Theories to determine criteria of biodiversity preservation**

Various tools of conservation programs have been developed for the last twenty years. (The theories relate differently to the determination of preservation priorities for genetic diversity, species diversity and ecosystems diversity.)

Moran et al (1996; 1997) suggest an index to rate global investments. Any investment in biodiversity is influenced by damage (threat) and sustainability (success). Successful intervention by means of investment would depend on the probability or likelihood of success and the level of threat common in a certain country. This index is applied to determine regional ratings for various diverse areas in general, and rich in endemic species in particular (such as Central and South America, the Caribbean and others). The index binds a number of socio-economic variables, and uses the cost-effectiveness approach. This method provides available information about costs, but the effectiveness is defined in non-financial terms.

The Cost Effectiveness Priority Investment Index (CEPII) is derived by subtracting the result of the rating from information about threat and the probability of successful human intervention. The index is composed of a characteristic divided into threat or success ( $\rho$ ), multiplied by change to this characteristic in case of limited government intervention or no intervention at all. This value is normalized to chosen values of species abundance and endemism per square kilometer ( $\Delta\hat{B}$ ) and divided by the average cost per unit that derives from the intervention to preserve biodiversity between countries ( $C$ ). Since this information is unavailable, the closest estimate – the international level of investment – is used.

This index specifies the rating of countries that should be invested in, but it provides relative rather than absolute values. In addition, there is a lack of data about local priorities, demands and needs, so that the relative values we get are also lacking. In their 1997 paper, they defined  $\hat{B}$  as the sum of tall vegetation, mammals, birds, reptiles and amphibians. The authors concentrated on tropical forests (as opposed to the 1996 paper) and expanded the spatial level and location in addition to the habitat they examined.

In his papers, Weitzman relies on a biodiversity function he created, which primarily describes the distance and difference between the species. One of his seminal papers “On Diversity” (1992) illustrates how logical assessment of the “diversity value” of a collection of objects can be created from basic information about the difference-distance between any two objects in the group. He assumes the existence of one big universe that includes a large number of potential species, and suggests a distance between every pair of species that is not negative and symmetrical. Namely, the basic problem is to create appropriate assessment of the diversity of a group ( $Q$ ), indicated by  $V(Q)$ , when values of distance-difference are given. Weitzman portrays the group by means of a tree diagram, in which the length of each branch describes a family of existing species that have a historical-evolutionary connection. If two species are close and one of them is extinct, it is not a catastrophe. However, when both species are extinct, the evolutionary branch is compromised, and that might be a catastrophe. He presents various approaches to the discussion on biodiversity – the ecological approach and the economic approach, combined with a number of functions. The economic approach prefers species that are distant from each other, that is to say, if two species in a group are to be preserved, then it is preferable to preserve distant species. Another result obtained by Weitzman is that if one species is equal to other species in policy considerations, then its existence or extinction are irrelevant to the decision making process regarding its preservation or extinction relative to subgroups of other species. Weitzman also proves that according to the biodiversity function, the species with the biggest distance from other species has the highest value.

Weitzman’s 1993 paper shows how his “Diversity Theory” (as described in his 1992 paper) can be applied to analysis of preservation policy in the real world, in order to determine priorities of species preservation. In this paper, he applies the theory to types of cranes (wild birds). Weitzman also presents how humans tackle diversity. They value

species, such as cranes, for their beauty or the inspiration they provide. However, he insists that diversity is the measurement of the distance or difference, and attempts to assess a collection of differences. His primary research aim is preservation, which can include direct and indirect benefits that are not included in the value supplied by the biodiversity value function. Weitzman enumerates fifteen types of cranes and adds each type's extinction probability in the next 50 years. It is obvious that if a certain type of crane is extinct and does not have a close family relative, then the diversity loss is great. Furthermore, Weitzman calculates the present value of the biodiversity expectation function – the probabilities of extinction combined with the distance-difference matrix and the capitalization rate. He obtained various values of the substitution rate between species (marginal diversity) and the diversity elasticity (or preservation potential) of each family of cranes.

Weitzman (1995) defines the “diversity function” as a function that evaluates the diversity of a collection of given elements. The expectation of the biodiversity function is the sum of the diversity functions of a variety of collections of species weighted by the survival probabilities of all the collections. Its goal is to maximize the present value of diversity expectation (in addition to the direct values of the elements) subject to the preservation budget constraints. In this paper, for the first time<sup>1</sup>, he compares a species to a library. There are two reasons for this: One, a library does not have emotional value to people, it is neutral, and two, if we concentrate on libraries and the books they hold, it is much easier to focus the question of what is preserved, when speaking of the diversity function. Weitzman proposes the “bead model”, which describes a tree diagram in which the branches represent the evolutionary process over time, and the species are located at the tips of the branches. Each species is composed of a large number ( $M$ ) of beads bound together. In the metaphor, the species is a library and the beads are books. At the beginning of the process, there is one species. Later on, over time, more species are added that are slightly different from one another (in their genetic set, or in the library model – their book collection). All the species in the bead model are composed of equally long strings of beads. The difference or distance between each pair of species can be defined as the number of beads between the species. This is, in fact, the diversity function – a function that measures the distance or difference between species.

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<sup>1</sup> He expands on this in his 1998 paper.

Weitzman (1998) tries to present a practical way to think about the economics of biodiversity, mediated by an abstract mathematical model. He compares it to Noah's Ark's problem, which should be a basic problem expressed as an analytical problem, and which characterizes biodiversity preservation problems under budget constraint. One of the constraints in the allegory is the space constraint determined by budget and Noah's labor. The central question he asks is how to determine basic priority of preservation or biodiversity enhancement. Specifically, what is the cost-benefit formula or criterion that can serve to rate priorities between various projects of biodiversity preservation?

Another way to present the problem is by means of a shared game of a phylogenetic tree (Haake et al, 2008), in which the players are  $N$  species and the tree (T) represents the evolutionary ties between the species. The average marginal diversity added to each group is a Shapely value, which is a term borrowed from cooperative games theory, and is given here in a coalitional form as a coalition function  $V(S)$ . The Shapely value depends on the end weights and topology of the tree. This application of the Shapely value is consistent with the biodiversity preservation theory. The Shapely value can also serve as a ranking criterion, since it supplies an estimate of the contribution of each species added to the group's biodiversity.

#### **4. Externalities, functions and ecosystems processes**

Biodiversity has a direct and indirect impact on human society, but has an external impact on nature itself, ecosystems, habitats and other species (Rands et al., 2010). An ecosystem can add innumerable interactions between species, in which each species represents a certain niche and contributes to the resilience and stability of the ecosystem. In this section, we describe the links between the species and the ecosystem, and their interactions with society.

In ecosystems, the "players" are organisms, which add up to the species. A species can have direct and/or indirect benefit for the human race, as raw materials to produce goods and commodities or to support other species, but they have incomplete markets. Crocker & Tschirhart (1992) describe the stationary equilibrium of an ecosystem, in which the demand of species for other species is equal to the supply of other species, and energy is preserved. The ecosystem, consequently, is integrated in the economy and

the impact of human intervention can be monitored. Human intervention creates externalities that affect the ecosystem and shifts the system's equilibrium to a new place, which in turn influences the benefit and/or human production processes.

Ecosystems exhibit complex and dynamic connections, but so do economic systems. Economic systems were built successfully, so if there is similarity between the complexity of economic systems and that of ecological systems, a model can be built in the same way. The similarity is manifested in the demand, supply, competition, and marginal substitution rates of ecology.

Crocker & Tschirhart's approach enables to incorporate an ecological model in a simple economic model, for example to liken decision makers in ecosystems to organisms that unite at the species level. They presented a model of the influences of human intervention on the ecosystem, as if only one consumer and one producer existed in the world. The consumer prefers a single end product (bread) and spare time. The end product is produced by labor and an interim product (wheat). The interim product is produced by labor, and the consumer has no direct benefit from it. The ecosystem, which also plays a role, is composed of a predator that eats a mouse that eats wheat kernels. The predators provide the consumer with negative benefit. Therefore, the consumer's utility function is composed of bread, spare time units in which the predators can be killed and spare time. The production function is composed of labor to produce wheat kernels and dependence on predators. The dependence is the source of the externalities on the ecosystem.

It can be said that externalities on the ecosystem presents a dual problem. The first problem is that we require sufficient biological information to understand the interrelations in an ecosystem. The second problem is that we require some kind of framework to use this information in the economic analysis. The authors attempt to solve the second problem.

##### **5. Biodiversity – connections between species**

There is a connection between species diversity and the ecosystem, because the variety of species largely determines the functional characteristics of the ecosystem (Woodward, 1993).

There are interrelations between the various species – interstitial and structural – and other interrelations. Some species are structural and some are interstitial (Huston, 1994). Structural species create the physical structure of the ecosystem (or habitat), for instance trees or corals. Interstitial species live in the environmental system created by the structural species, and are directly influenced by structural species, for instance microorganisms or fungi. The influence is one-way – interstitial species almost do not affect structural species. It is known that in an ecosystem, populations of various microorganisms live side by side, mixed with each other in some ways, and we can therefore expect, in the nature of things, some relations between the individuals of various species populations<sup>2</sup> (Perevolotsky & Pollak, 2001).

The pairs of populations involved and their mutual influences are often used to classify other interactions. Following are six possible results:

- ❖ 0,0 Neutral: Both partners to the interaction are indifferent to each other.
- ❖ 0,+ Commensalism<sup>3</sup>: One partner benefits from the presence of the other, and the other neither benefits nor loses.
- ❖ +,+ Mutuality: Both partners in the interaction benefit from each other.
- ❖ +,- Predation<sup>4</sup> (including herbivorism<sup>5</sup> and parasitism<sup>6</sup>): The fitness of one organism increases at the expense of the other organism.
- ❖ -,0 Amensalism: Disturbed competition – one partner is indifferent in its fitness to the presence of the other, which is damaged.
- ❖ -,- Competition: The fitness of both partners decreases because of their interaction.

Another type of relationship exists between a dominant host species called “keystone species” (Kotliar et al, 1999) that is defined as “a species the removal of which would have a disproportionately big influence on its environment” (Shkedy, 2009) and its dependent. The dependent species is called “keystone species dependent” (Berlow et al, 1999). Not every system has keystone species, namely, the presence of a species that

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<sup>2</sup> In each type of interaction and mutual influence, the individual influences and is influenced, but the result is evident on the population level – it grows, decreases, moves, etc.

<sup>3</sup> Commensalism is a type of non-competitive interaction between organisms, in which one species benefits from the proximity and the other species is not affected either way.

<sup>4</sup> An interaction in which one organism feeds on the other organism; the predator’s fitness increases while the prey’s fitness decreases.

<sup>5</sup> An animal species eats a plant species, partially or fully.

<sup>6</sup> Two species in an obligatory relationship; the parasite is metabolically dependent on the host.

improves the habitat and increases the chances of other species to exist, without damaging the survival chances of the (beneficial) facilitating species.

Another relationship is of landscape modulator species, which determine the makeup of the species in the habitat, for instance the common oak or mastic tree (Shkedy, 2009).

## 6. Characterization of the Expectation of the Biodiversity Value Function

We denote by  $\pi_0$  the survival probability of the keystone species and by  $\pi_k$  for  $k=1, \dots, K$ , the conditional survival probability of the  $k$ -th keystone dependent species, conditioned on the event that the keystone species survives.

Analogously, let  $A_0$  be the set of books in the central library, and denote by  $M_0$  the number of books in  $A_0$ . For  $k=1, \dots, K$ ,  $A_k$  denotes the set of books in the  $k$ -th professional library.

We assume:

(A.1) For  $1 \leq k' \neq k'' \leq K$   $A_{k'} \cap A_{k''} \subset A_0$ . (Thus, e.g. the books' collection of the Physics library, which are not included in  $A_0$ , is disjoint from the books' collection of the Chemistry library, which are not included in  $A_0$ ). We denote the number of books in the  $k$ -th professional library, which are not included in the Central library, by  $|A_k \setminus A_0| = E_k > 0$

(A.2) The total number of the books in a set  $S$  of libraries, where  $S \subset \{0, 1, \dots, K\}$  (which is the diversity of books) is:

$$(1) \quad V(S) = \begin{cases} M_0 + \sum_{k \in S \setminus \{0\}} E_k & 0 \in S \\ 0 & otherwise \end{cases}$$

(A.3) The probability of survival of library 0 and the  $k$ -th library together is  $\pi_0 \cdot \pi_k$  (based on the multiplicative formula that:  $Prob(A \cap B) = P(A) \cdot P(B|A)$ ).

**Theorem 1:** The expectation of the biodiversity value function satisfies in this model:

$$(2) \quad W(\pi_0; \pi_1, \dots, \pi_K) = \pi_0 M_0 + \pi_0 \pi_1 E_1 + \dots + \pi_0 \pi_K E_K.$$

In particular  $\frac{\partial W}{\partial \pi_0} > 0$  and  $\frac{\partial W}{\partial \pi_k} > 0$  for all  $k=1, \dots, K$  and  $\pi = (\pi_0; \pi_1, \dots, \pi_K) \in (0,1]^{K+1}$

Proof:

Denote the *random biodiversity value function* by  $V(\bullet) = X_0(\bullet) + \sum_{k=1}^K X_k(\bullet)$  where

$$X_0(\bullet) \text{ is the random variable } X_0(S) = \begin{cases} M_0 & 0 \in S \\ 0 & \text{otherwise} \end{cases}$$

$$\text{and } X_k(S) = \begin{cases} E_k & k \in S \\ 0 & 0 \in S \\ 0 & \text{otherwise} \end{cases} \quad \text{for } k=1, \dots, K.$$

Since  $EX_0(\bullet) = M_0\pi_0$  and  $EX_k(\bullet) = E_k\pi_0\pi_k$ , we obtain:

$$W(\pi_0; \pi_1, \dots, \pi_K) = EX_0(\bullet) + EX_1(\bullet) + \dots + EX_K(\bullet) = \pi_0 M_0 + \pi_0 \pi_1 E_1 + \dots + \pi_0 \pi_K E_K$$

We get from this formula for the *biodiversity value function*, that by differentiation, the partial derivatives of  $W$  are strictly positive in  $(0,1]^{K+1}$ .

Corollary 1: Assume  $E_1 = \dots = E_K$ , then  $W(\pi_0; \pi_1, \dots, \pi_K) = \pi_0 M_0 + \pi_0 E_1 (\pi_1 + \dots + \pi_K)$

## 7. The Central Planner Problem

The central planner maximizes the expectation of the biodiversity value function under budget constraint  $B > 0$ . In our model the range of  $\pi$  is  $I^{K+1}$ , where  $\pi = (\pi_0; \pi_1, \dots, \pi_K)$  and  $I = [0,1]$  is the unit interval. The central planner solves the problem of:

$$\text{MAX } W(\pi_0; \pi_1, \dots, \pi_K) = \pi_0 M_0 + \pi_0 \pi_1 E_1 + \dots + \pi_0 \pi_K E_K$$

$$\text{s.t. } C(\pi_0; \pi_1, \dots, \pi_K) \leq B$$

$$\pi \in I^{K+1}$$

Proposition 1: Assume that  $C(\pi_0; \pi_1, \dots, \pi_K)$  is defined on  $I^{K+1}$  and is a continuous and monotonic increasing function, and  $C(0;0, \dots, 0) = 0$  and  $C(1;1, \dots, 1) \geq B$ . Then the central planner problem has a non-empty set of maximizers and each optimal policy satisfies the budget equality.

Proof:

The set  $\{(\pi_0; \pi_1, \dots, \pi_K) \in I^{K+1} : C(\pi) \leq B\}$  is nonempty, closed and bounded set in  $I^{K+1}$ . By the continuity of  $W(\pi_0; \pi_1, \dots, \pi_K)$ , the result follows. Next, to prove the budget equality we assume that  $C(1; 1, \dots, 1) \geq B$  or in the linear case:  $C_0 + KC_1 \geq B$ , then there is budget equality for  $\pi \in I^{K+1}$ .

## 8. The Existence of an Optimal Policy

**Theorem 2:** In our symbiotic model with keystone species and K keystone-dependent species, under assumption (A.1) and if the budget  $B > 0$  and the costs function  $C(\pi_0; \pi_1, \dots, \pi_K)$  is defined on  $I^{K+1}$  and is a continuous and monotonic increasing function, and  $C(0; 0, \dots, 0) = 0$  and  $C(1; 1, \dots, 1) \geq B$ , then  $\hat{\pi}_0 > 0$ .

**Proof:** As we mentioned in proposition 1, by the continuity of  $W(\pi_0; \pi_1, \dots, \pi_K)$ , there is an existence of optimal policy, because continuous function over non-empty closed and bounded set get maximum and minimum. Since the biodiversity value function is increasing over  $R_+^{K+1}$  and  $C(1; 1, \dots, 1) \geq B$  there is a budget constraint. Particularly, there is budget equality for any optimal policy. Obviously, if  $B > 0$  than  $\hat{\pi}_0 > 0$  for any optimal policy  $\hat{\pi}$ . The assumption about increasing monotony is derived from the partial derivation of the biodiversity value function (i.e. the marginal utilities):

- i.  $\frac{\partial W}{\partial \pi_0} = M_0 + \pi_1 E_1 + \dots + \pi_K E_K > 0$
- ii.  $\frac{\partial W}{\partial \pi_k} = \pi_0 E_k > 0 \quad k = 1, \dots, K$

## 9. The Interiority and Uniqueness of the Optimal Policy

Assume now that  $C(\pi_0; \pi_1, \dots, \pi_K)$  is a separable costs function:

$$C(\pi_0; \pi_1, \dots, \pi_K) = C_0(\pi_0) + C_1(\pi_1) + \dots + C_K(\pi_K) \quad \text{and assume that:}$$

$$(A.4) \quad C_0(\pi_0): [\underline{\pi}_0, \overline{\pi}_0] \rightarrow \mathfrak{R}_+$$

$$(A.5) C_0(\underline{\pi}_0) = 0$$

$$(A.6) C'_0 > 0, C''_0 \geq 0$$

$$(A.7) C_k(\pi_k) : [\underline{\pi}_k, \overline{\pi}_k] \rightarrow \mathfrak{R}_+ \quad \forall k = 1, \dots, K$$

$$(A.8) C_k(\underline{\pi}_k) = 0$$

$$(A.9) C'_k > 0, C''_k \geq 0$$

$$(A.10) C(\overline{\pi}) = C_0(\overline{\pi}_0) + \sum_{k=1}^K C_k(\overline{\pi}_k) > B > 0$$

$$(A.11) C_0(\overline{\pi}_0) \leq B$$

**Theorem 3:** Under the above assumptions and a continuous convex costs function  $C(\pi)$  and positive budget  $B > 0$ , if  $\hat{\pi}$  is an optimal policy in the symbiotic model, then  $\hat{\pi}_0 = \tilde{\pi}_0$ . i.e., the survival probability of the keystone species is unique. In particular, if  $C_0(\pi_0)$  is convex in  $\pi_0$  and  $C(0; \pi_1, \dots, \pi_K)$  strictly convex in  $(\pi_1, \dots, \pi_K)$ , i.e.  $C(\pi) = C_0(\pi_0) + C_1(\pi_1) + \dots + C_K(\pi_K)$ , there exist a unique optimal policy  $\hat{\pi} = \tilde{\pi}$ .

**Proof:** see Zemah Shamir and Shitovitz (2012).

Assume now that:

$$(A.12) C'_0(\overline{\pi}_0) = \infty$$

**Theorem 4:** Under assumptions (A.4)-(A.12), for any optimal policy  $(\hat{\pi}_1, \dots, \hat{\pi}_K)$ , the unique survival probability of the keystone species  $\hat{\pi}_0$  is interior, i.e.  $0 < \hat{\pi}_0 < \overline{\pi}_0$  and  $C_0(\hat{\pi}_0) < B$ .

**Proof:** Since  $C(\hat{\pi}) = B < C(\overline{\pi})$ , assume by negation that  $\hat{\pi}_0 = \overline{\pi}_0$ , therefore there is a species  $1 \leq k \leq K$  in which  $\hat{\pi}_k < \overline{\pi}_k$ . In particular, for species 0 and k, since  $\hat{\pi}$  is an optimal corner solution at  $\hat{\pi}_0 = \overline{\pi}_0$  and  $\hat{\pi}_k < \overline{\pi}_k$ . We obtain that F.O.C. for corner solution is:

$$\frac{MW_{\pi_0}}{MW_{\pi_k}} = \frac{M_0 + \sum_{k=1}^K \pi_k E_k}{\pi_0 E_k} \geq \frac{MC_{\pi_0}}{MC_{\pi_k}} = \frac{C'_0(\overline{\pi_0})}{C'_k(\hat{\pi}_k)} = \frac{\infty}{C'_k(\hat{\pi}_k)} = \infty$$

A contradiction

## **10. The Species Preservation Ranking Criterion of Noah's Ark**

Weitzman (1998) set a ranking criterion based on the distance between the species and their expected utilities weighted with their survival probabilities. In our symbiotic model, we derive a ranking criterion based on the marginal contribution of the species to its marginal preservation costs.

### 10.1 The General Case

Applying Theorem 4, the central planner problem is now:

$$\begin{aligned} \text{MAX} \quad & M_0 + (\pi_1 E_1 + \dots + \pi_K E_K) \\ \text{s.t.} \quad & \underline{\pi}_1 \leq \pi_1 \leq \overline{\pi}_1 \\ & \vdots \\ & \underline{\pi}_K \leq \pi_K \leq \overline{\pi}_K \\ & C_1(\pi_1) + \dots + C_K(\pi_K) \leq B - C_0(\hat{\pi}_0) \equiv \tilde{B} \end{aligned}$$

Note that, based on theorem 4,  $\tilde{B} = B - C_0(\hat{\pi}_0) > 0$ .

The central planner problem equivalent (since  $M_0$  is constant) to this maximization problem:

$$\begin{aligned} \text{MAX} \quad & \pi_1 E_1 + \dots + \pi_K E_K \\ \text{s.t.} \quad & \tilde{B} - C_1(\pi_1) - \dots - C_K(\pi_K) \geq 0 \\ & \underline{\pi}_1 \leq \pi_1 \leq \overline{\pi}_1 \\ & \vdots \\ & \underline{\pi}_K \leq \pi_K \leq \overline{\pi}_K \end{aligned}$$

Using Kuhn-Tucker's theorem and its conditions, we obtain a parameter  $\alpha^* > 0$ , such that for any  $k=1, \dots, K$ :

If  $\frac{E_k}{C'_k(\hat{\pi}_k)} < \alpha^*$ , then  $\hat{\pi}_k = \underline{\pi}_k$ , i.e. the species dies out.

If  $\frac{E_k}{C'_k(\hat{\pi}_k)} > \alpha^*$ , then  $\hat{\pi}_k = \overline{\pi}_k$ , i.e. the species survives.

For  $\frac{E_k}{C'_k(\hat{\pi}_k)} = \alpha^*$ , no specific conclusion.

## 10.2 The Linear Case

**Theorem 5:** Assume that  $\underline{\pi}_1 = \underline{\pi}_2 = \dots = \underline{\pi}_K = 0$ ,  $\overline{\pi}_1 = \overline{\pi}_2 = \dots = \overline{\pi}_K = 1$  and the marginal costs function for the keystone species is  $C'_0(\overline{\pi}_0) = \infty$ , while the rest of the costs function is the form of  $C_k(\pi_k) = C_k \cdot \pi_k$ , then for any optimal policy  $\hat{\pi}$ , there is  $\alpha^* > 0$ , such that for any  $k=1, \dots, K$ :

If  $\frac{E_k}{C_k} < \alpha^*$ , then  $\hat{\pi}_k = 0$ , i.e. the species dies out.

If  $\frac{E_k}{C_k} > \alpha^*$ , then  $\hat{\pi}_k = 1$ , i.e. the species is fully survived.

## 11. The General Case of $K \geq 2$ Keystone-Dependent species

In this section, we introduce the central planner problem in diverse scenarios. We examine the symmetric case in different costs functions.

### 11.1 The case with K identical keystone-dependent species – the general symmetric case

Assume now that:

(A.13)  $C(\pi_0; \pi_1, \dots, \pi_K)$  is a convex and symmetric function in all of its  $K$  last coordinates.

**Conclusion 1:** There is an optimal policy  $\hat{\pi} \in D$ , where  $D$  is a non-empty closed subset of  $I^{K+1}$ , with the Equal Treatment Property, which means that for identical species (with symmetric costs and for all  $1 \leq k' \neq k'' \leq K$   $E_{k'} = E_{k''}$ ) the probabilities in this optimal policy satisfies  $\hat{\pi}_{k'} = \hat{\pi}_{k''}$ .

Remark 1: A simple but not trivial example of symmetric convex cost function at its last  $K$  variables is:  $C(\pi_0; \pi_1, \dots, \pi_K) = C(\pi_0; 0, \dots, 0) + \pi_1^2 + \dots + \pi_K^2$ .

**Theorem 6:** Assume now  $E_1 = E_2 = \dots = E_K \equiv E_1$ , under assumptions (A.1) and (A.13) and the assumption that  $C(\pi_0; \pi_1, \dots, \pi_K)$  is convex and symmetric in  $\pi_1, \dots, \pi_K$ , there is an optimal policy  $\hat{\pi}$  where  $\hat{\pi}_0$  for the keystone species and symmetric optimal policy for the keystone-dependent species  $\tilde{\pi}_1 = \frac{\pi_1 + \dots + \pi_K}{K}$ . Particularly, our  $(K+1)$  species' problem is equivalent to a single keystone species and a single keystone dependent species (see Zemah shamir, 2011).

**Proof:** For  $\hat{\pi} \in D$  derived by conclusion 1, the expectation of the biodiversity value function is:

$$\begin{aligned} W(\pi_0, \pi_1, \dots, \pi_K) &= \pi_0 M_0 + \pi_0 \pi_1 E_1 + \dots + \pi_0 \pi_K E_K = \\ &= \pi_0 M_0 + \pi_0 \tilde{\pi}_1 E_1 K \end{aligned}$$

While the costs function is:

$$C(\hat{\pi}_0, \tilde{\pi}) \equiv C(\hat{\pi}_0; 0, \dots, 0) + KC(0, \tilde{\pi}_1) \leq C(\hat{\pi}_0, 0, \dots, 0) + C(0, \pi_1, \dots, \pi_K),$$

This derived from the symmetry and convexity, therefore the average of  $(\hat{\pi}_1, \dots, \hat{\pi}_K)$  is a symmetric optimal policy within  $C(0, \pi_1, \dots, \pi_K) \geq KC(0, \tilde{\pi}_1)$ .

## 11.2 The case with $K$ identical keystone-dependent species – the linear symmetric case

**Theorem 7:** Assume now  $E_1 = E_2 = \dots = E_K \equiv E_1$ , under assumptions (A.1) and (A.13) and the assumption that  $C(\pi_0; \pi_1, \dots, \pi_K)$  is linear and symmetric in  $\pi_1, \dots, \pi_K$ , i.e.  $C_1 = C_2 = \dots = C_K \equiv C_1$ , there is an optimal policy  $\hat{\pi}$  where  $\hat{\pi}_0$  for the keystone species and symmetric optimal policy for the keystone-dependent species. In particular,

this (K+1) species' problem is equivalent to a single keystone species and a single keystone dependent species with linear costs function (see Zemah Shamir, 2011).

The central planner problem in this case is:

$$\begin{aligned} \text{MAX} \quad & \pi_0 \left( M_0 + \sum \pi_k E_1 \right) \\ \text{s.t.} \quad & \pi_0 C_0 + \pi_1 C_1 + \dots + \pi_K C_1 \end{aligned}$$

This central planner problem is equivalent to the two-species model. Where

$$\tilde{\pi}_1 = \frac{\pi_1 + \dots + \pi_K}{K} \in [0,1] \text{ and } \tilde{C}(\pi_0, \tilde{\pi}_1) = C\left(\pi_0, \frac{\tilde{\pi}_1}{K}, \dots, \frac{\tilde{\pi}_1}{K}\right).$$

By solving the maximization problem above, and assuming linear cost function, an interior optimal policy might occur. In this case

$$\begin{aligned} \text{MAX} \quad & \pi_0 M_0 + \pi_0 \underbrace{(KE_1)}_{\tilde{E}_1} \tilde{\pi}_1 \\ \text{s.t.} \quad & \pi_0 C_0 + \tilde{\pi}_1 \underbrace{KC_1}_{\tilde{C}_1} = B \end{aligned}$$

In this equivalent problem in which  $(\hat{\pi}_0, \hat{\pi}_1)$  is an interior optimal policy and  $\hat{\pi}_0 C_0 > \frac{B}{2}$ .

## 12. Discussion

In his paper “Noah’s Ark’s Problem”, Weitzman presented the classic problem of species preservation under budget constraint. Many studies were conducted in his footsteps (Van der Heide et al, 2005; Weikard, 2002; and others) which expanded the problem and introduced criticism of it. A major criticism was the lack of reference to the relationships between the species. Although it was addressed on the level of ranking criteria (Van der Heide et al, 2005) and probabilities (Baumgartner, 2002), the wider issue of solving the central planner’s problem was not dealt with, specifically, solving Noah’s Ark’s problem when there is a keystone species and the species boarding the ark depend on the keystone species' survival. In addition, various studies did not address the linear cost function presented by Weitzman.

In this chapter, we expanded Noah's Ark's problem to an uncertainty environment, which includes a keystone species and other species dependent on the survival of the keystone species.

We used a variety of cost functions to obtain comprehensive results for various applications. We examined a linear cost function, an increasing convex monotonous function, i.e. marginal cost tends to infinity for the upper bound.

We obtained results that suggest an optimal policy different from the one presented by Weitzman. In the case of  $K > 1$  keystone-dependent species, under various assumptions about the cost function, we find that the keystone species has a unique interior optimal survival probability. When we determine the optimal policy for the keystone species, we can establish ranking criteria for species preservation in Noah's Ark. A keystone-dependent species whose marginal cost ratio is lower than the shadow price will not board Noah's Ark, that is to say will not survive. If the shadow price is lower than the marginal cost ratio, the species will be preserved. When the ratio is equal, we cannot know the fate of that species with certainty.

There is always an optimal policy for Noah's Ark with keystone-dependent species. We examined various types of Noah's Arks. One type is if the ark is symmetrical, namely the cost of the dependent species is equal for each  $k$  species, as is their biodiversity value. In such a case, the optimal symmetric policy is single (and an infinite number of asymmetric optimal policies are possible, all with a single optimal survival probability  $\hat{\pi}_0$  for the keystone species). In this case, we found that the keystone species' (i.e. Noah's) survival probability is single, interior and does not depend on the number of keystone-dependent species. One can therefore say that the cost of the keystone species is not dependent on the number of  $K$  species.

In case of Noah's Ark with asymmetrical effective cost-benefit between the species, we can determine preservation-ranking criteria. The criteria is determined by the rank of cost-effective analysis according to the inequalities, and in case of a strong inequality, then all the optimal survival probabilities to the left of the strong inequality get 0 probability, and all the optimal survival probabilities to the right get 1 probability. In the center, we get one single interior symmetric survival probability, which is determined by budget equality.

The applications to Noah's Ark are manifold, starting from Weitzman's library model, in which the library collections (the books) could be destroyed by fire. In this study, we have posited one central library and  $K \geq 2$  professional libraries (for instance, one national library and any number of professional libraries such as physics, chemistry, math, etc.) and the scientific diversity of the library group is the number of the various books in all the professional libraries that belong to that group. In this case, the lending process is performed through the central library (inter-library lending).

Another example is from the field of computers – a central server that supplies electronic information transfer services to various computers throughout an organization or a country. If the central server collapses, no information can be transferred between individuals or other entities.

An important application is in the field of banking. If the central branch of a bank is out of action for some reason, then data transfer to other branches is incomplete. In this case, payments and other fiscal operations could be damaged.

These applications are suitable to other organizations that have a central "head office" and branches in various disperse locations. The Israel Nature and Parks Authority is an example. The Head Scientist and management are located in Jerusalem, and the country is divided into districts. In this case, the cost for each district is lower than the cost of the head office.

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