Economic growth, population dynamics, and the value of land reserves: An integrated assessment*

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This version: September 2013

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Abstract

We study the macroeconomic effects of an expanding agricultural system and associated losses in biodiversity. We develop a two-sector endogenous growth model that captures a number of features specific to agriculture: (i) the demand for food is driven by population growth and percapita income; (ii) agriculture requires land as an input; and (iii) uniformity in crops favors the occurrence of pests and pathogens, so that increasing agricultural land has an external effect on future yields. We estimate the model using simulation methods based on data covering the period 1900-2010. We then employ the model to characterize how land conversion decisions affect population dynamics and economic growth, suggesting a role for land use management and R&D at the global level.

Keywords: Economic growth; Population dynamics; Demand for food; Land conversion; Biodiversity; Endogenous innovations

JEL Classification numbers: E10, O31, O44, Q15, Q16, Q57.

*We thank Alex Bowen, Derek Eaton, Sam Fankhauser, Antony Millner, and Marty Weitzman for helpful discussions and comments. Funding from the MAVA foundation is gratefully acknowledged. All errors are ours.

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1 Introduction

After hundreds of thousands of years of relative constancy, the human population has exploded with the advent of agriculture. Even more impressively, with the advent of a modern agricultural R&D sector, population has increased three hundred per cent over the last seventy-five years. While the large-scale expansion food production system is what provides sustenance for the extant population, it also provides a continuous flow of problems that must be addressed. Increasing yields have been associated with a significant increase of the variability in yields (Hazell, 1989), while at the same time 75% of crop biodiversity has been lost (FAO, 2010).[1] The continued conversion of lands to food production, and the uniformity of the genetic material used for production, implies that system will become wholly unstable as conversion goes to totality (Weitzman, 2000).

This paper provides a framework to analyze the role of global land use for population and economic growth. We formulate a dynamic, two-sector model of the global economy that distinguishes agriculture from other economic activities, and in which population dynamics are endogenously determined by fertility choices (Barro and Becker, 1989; Jones, 2001). Population determines the demand for food, while food supply itself derives from two sets of factors. The first is the availability of primary inputs, among which land plays a central role. Conversions of land to modern agricultural systems have contributed substantially to agricultural production over the past century or more.[2]

The second and more significant contributor to growth in agricultural output is technological progress. In our model we inconorporate research and development (R&D) through the Schumpeterian innovation model by Aghion and Howitt (1992). R&D is conducted through allocations of labor to increase sector-specific total factor productivity (TFP). In agriculture however TFP growth is the net result of man-made innovations generated in the R&D sector and the rate at which problems arise in the food production system. In particular, the continued expansion

[1] Increased variability is the result of an increasing correlation of yields (and problems) spatially, and also potentially the result of evolutionary forces being directed toward common crop varieties, inducing genetic spatial correlation (McCann, 2000). The associated risks have induced efforts to maintain genetic biodiversity by constituting seed banks in different locations (see Koo et al., 2003, for example).

[2] Increasing crop yield through technical change has not, over the past 50 years, been a substitute to land conversion (Ewers et al., 2009).

of modern agriculture increases opportunities for the proliferation of pests and pathogens, and hence the rate of arrival of biological hazards within that production system (Cardinale, 2012; Reich et al., 2012). We model the breakthrough of pathogens as a stochastic process whose realizations act as negative shocks to agricultural TFP. In this framework, net growth in agricultural TFP represents a race between pests and patents along a "technology ladder" (Goeschl and Swanson, 2003).[3] It follows that as the scale of the agricultural system expands, the risk that pests become more successful increases, reducing expected agricultural TFP growth over time.

This model is the first attempt to examine the fundamental relationship between economic growth, population and food demand, and the process of land conversion in an integrated framework. Economic growth relies on innovation, and the supply of labor to R&D sectors relies on population growth. In turn, growth in population and percapita income increase the demand for food, increasing the scale of the food production system and the demand for allocating land to agriculture. As the land gets converted for agricultural production, increased uniformity of the genetic material increases the flow of hazards into the production system, or generating a land-use externality. Allocating land to the reserves serves as a means of mitigating the rate of flow of hazards (i.e. reduce the land-use externality), but puts a constraint on the pathway of the economy.

To quantitatively assess the risks associated with a negative land-use externality, we structurally estimate key parameters of the model using simulation methods.[4] The time series we target to estimate the parameters are world population, GDP, land conversion, and TFP growth over the period 1900-2010. Despite the highly non-linear nature of the model, we are able to closely replicate observed trajectories, and the magnitude of the estimates are plausible given microeconomic evidence.

We then employ the model to carry out a number of policy experiments, projecting forward how the scale of agriculture and associated biological hazards determine optimal land allocation and R&D effort. Assuming that society continues along the observed pathway, where land

[3] Thus man-made R&D addresses biological hazards as they appear, or nature annihilates technological progress by rendering innovations obsolete. The integrated agricultural R&D sector represents a zero-sum game between people and pests, so that the state of technology is a measure of the relative success of society in appropriating total product from land.

[4] We refer to estimation rather than calibration because the number of parameters identified is much smaller than the number of targeted empirical moments.

conversion decision do not account for the negative externality, over the next 100 years population and converted land increase by 40% and 27% respectively. Assuming that such rapid pace of land conversion increases the probability of negative feedback from 1/200 in 2010 to a around 1/20 by 2100, the social optimum balances the impacts of system expansion against its costs. We find that the social planner would stop converting land immediately, and that he would leave part of the land currently exploited to be reconverted as a buffer against such risk. We further contrast the socially optimal land allocation with a case where fertility and land conversion choices are made by households that do not take into account the land use externality. In this setting, R&D is the only response to the negative externality associated with agricultural expansion. While the planner is able to mitigate the negative impact of global land conversion, it does so at the cost of lower percapita consumption, which significantly reduces the level of overall welfare relative to the social optimum.

To put these pathways into perspective, we also examine implications of the variability inherent within the system. We show that even the socially optimal pathways exhibit in-built variability around mean outcomes, on account of the stochasticity inherent in biological hazard. As the population increases, random fluctuations in food production imply increasing likelihoods of food-related mortality events (i.e. famines), with increasing scale over time. In effect, the scale of the wager increases the impact of biological hazards, potentially resulting in millions deaths each.

In sum, our paper is meant to demonstrate that land use is an important social problem from the global perspective. This is true for two reasons. First, evidence suggests that a continued expansion of the agricultural system is not a costless exercise, as continued land conversions imply an increased of arrival of biological hazards. Second, while man-made innovation flowing out of R&D can address these issues, there is a substantial cost to it, and an increasing population implies that the wager being placed is large and increasing. These populations can only be sustained with the continued application of the most optimistic technological assumptions concerning, or through the retention of reserves to mitigate the flow of hazards into the production system.

The remainder of the paper proceeds as follow. Section 2 describes the economy, Section 3 the R&D sector, and Section 4 the representation of social welfare. Section 5 discusses the

optimal control problem, estimation strategy, and presents simulated pathways as well as the implied variability of the system. Some concluding comments are provided in Section 6.

2 The economy

In this section provides an overview of our basic macroeconomic model of capital accumulation, population dynamics, land conversion, and labor allocation decisions. We then turn in the next section to the critical role of the R&D sector. We formulate the model from a social planner perspective, so that it can be solved as an optimization problem, and the assumed social welfare objective is also discussed subsequently.

2.1 Production and capital accumulation

The economy consists of two sectors, agriculture and other economic activities, which we refer to as 'manufacturing'. The aim of this distinction is to characterize two distinctive features of the agricultural sector. First, agriculture produces food, which determines the population that can be sustained. As a simplification, we will assume that agricultural output Y^{ag} in each period equals food consumption. Second, agriculture requires land as an input, so that the demand for food indirectly determines the demand for land conversion.

Formally, we represent production technology in both sectors with a Cobb-Douglas function.[5] In the agricultural sector, output in each period is a function of capital K^{ag}, labor N^{ag}, and converted land X:[6]

$$Y^{ag} \;=\; A^{ag}\,(K^{ag})^{\theta^K}\,(N^{ag})^{\theta^N}\,X^{1-\theta^K-\theta^N}\,, \tag{1}$$

where $\theta^{K,N} \in (0,1)$ are share parameters, with $\theta^K + \theta^N < 1$, and A^{ag} is an index of TFP in agriculture that period.

The manufacturing sector produces an aggregate final good with a Cobb-Douglas technology,

[5] This is a standard assumption, see Mundlak (2000) or Hansen and Prescott (2002) for example.

[6] We work in discrete time and denote the value taken by a variable V at time t as V_t. We suppress the time index unless some ambiguity arises.

and land requirements in this sector are negligible. Hence output is simply given by:

$$Y^{mn} \;=\; A^{mn}\,(K^{mn})^{\vartheta}\,(N^{mn})^{1-\vartheta}\;, \tag{2}$$

with $\vartheta \in (0,1)$. Manufacturing output can either be invested or consumed:[7] Market clearance for the manufacturing good is:

$$Y^{mn} = C + I^{\mathrm{K}}\,, \tag{3}$$

where C and I^{K} are total consumption and investment respectively. The accumulation of capital is then given by:

$$K_{t+1} = K_t(1-\delta^K) + I_t^{\mathrm{K}}\,, \quad K_0 \text{ given}\,, \tag{4}$$

where δ^K is the per-period depreciation rate.

2.2 Population dynamics

In each period, the change in population derives from the contemporaneous rate of fertility n_t and morality d_t:

$$N_{t+1} = N_t + n_t - d_t\,, \quad N_0 \text{ given}\,. \tag{5}$$

The representation of fertility (i.e. the increment to the stock of society's useful population) mirrors the traditional macroeconomic representation of households' fertility choices in a social planner setting. As per Barro and Becker (1989), the key determinant of fertility choices is the opportunity cost of rearing children in terms of labor supply. In our setting, this implies that additions to the population is a function of labor allocated to the child rearing activity:

$$n = \chi \cdot (N^N)^{\zeta}\,,$$

[7] The model is formulated as a social planner problem, so that savings cum investments decisions stem only from the manufacturing sector as in a one-sector economy. See Ngai and Pissarides (2007) for a similar treatment of savings in a social planner representation of multi-sector growth.

where the parameter $\chi > 0$ measure the time cost of children and $\zeta \in (0,1)$ is an elasticity representing scarce factors required in child rearing.[8]

Mortality is assumed to be driven by food availability, and results from a failure to supply sufficient food to maintain subsistence consumption for the existing population. In the simplest setting, food consumption is proportional to population, hence: $N\overline{c}(\cdot) = Y^{ag}$, where $\overline{c}(\cdot)$ is a measure of minimum food consumption. With this formulation, population growth determines how much food needs to be produced. Mortality is then given by a background rate $\overline{d}$, plus excess mortality occurring whenever the food production is insufficient to sustain the existent population. Formality, mortality in each period is given by:

$$d_t = N_t(\overline{d} + \delta_N(\overline{c}(\cdot) - Y^{ag}/N)^{\iota}) \tag{6}$$

where $\iota > 1$ measures the rate at which mortality increases when food production does not meet the demand for food, and $\delta_N > 0$ is the mortality rate when agricultural output tends to zero.

We will use the idea of a minimum food consumption, as measured by $\overline{c}(\cdot)$, to capture two concepts. The first is a physiological element underlying food consumption and represents minium per capita caloric intake. The second is a preference-based component, allowing us to represent the non-linear relationship between income and food expenditures. Hence over time food consumption increases with income but at a declining rate (see Section 4 below).

2.3 The allocation of labor, capital, and land

The fundamental questions we ask in this context concern the optimal allocation of labor, capital and land to generate production within this economy. Total population in each period is allocated to: production in the agriculture and manufacturing; R&D for the agricultural sector and for the manufacturing sector; land conversion activities; and child rearing. The stock of capital is likewise allocated to manufacturing and agricultural sectors. We will let both labor and capital be perfectly mobile across activities.[9]

[8] The assumption that $\zeta < 1$ captures the fact that the cost children over a period of time increases more than linearly with the number of children (see Barro and Sala-i Martin, 2004, p. 412).

[9] This implies that transitional dynamics following unexpected events will appear less costly than in a situation where capital and/or labor are sector specific. Thus while this assumption has no qualitative implications for our results, it should be kept in mind for the numerical results.

The allocation of labor to land conversion determines the amount of land available for agriculture in each period. In particular, land input to agriculture has to be converted from a total stock of available land $\overline{X}$ by applying labor (N^X). Thus over time, the stock of land used in agriculture develops as:

$$X_{t+1} = X_t(1-\delta^X) + \psi \cdot (N_t^X)^{\varepsilon}\,, \quad X_0 \text{ given}\,, \quad X_t \leq \overline{X}\,, \tag{7}$$

where $\psi > 0$ measures labor productivity in land clearing activities and $\varepsilon \in (0,1)$ is an elasticity. The assumption of decreasing returns to labor is standard. Note that depreciation δ^X allows the conversion process to be reversible, although only at a natural reconversion rate.

3 R&D sector

A core component of our framework is the role of the R&D sector in determining how production technology evolves over time, as measured by TFP growth. For both productive activities, man-made innovations are generated by applying labor to the R&D sector associated with each activity. In addition, the agricultural sector is subject to the arrival of biological innovations, as the scale of the system generates an inflow of hazards destructive of pre-existing technological progress. The net outcome of man-made and biological innovations is a single-dimension contest for the relative appropriation of product from land, in which moves up the ladder represent TFP growth and moves down the ladder represent the opposite.

3.1 Technology ladder

The evolution of TFP is based on the Schumpeterian model of Aghion and Howitt (1992) with no spillovers across sectors (see Acemoglu, 2002). In each period, a discrete jump in sectoral TFP occurs with probability $p \in (0,1)$:

$$A_{t+1} \;=\; (1+\rho)A_t\,,$$

where $\rho > 0$ measures the size of an innovation, or height of the steps on the technology ladder.[10]

3.2 R&D in the manufacturing sector

In the manufacturing sector the probability of an innovation in a given period is a function of labor applied to R&D activity that period, $N^{A^{mn}}$, and it is negatively related to the preexisting level of TFP:

$$p^{mn} = (A^{mn})^{\pi^{mn}} \cdot \lambda^{mn} \cdot (N^{A^{mn}})^{\mu^{mn}} , \tag{8}$$

where $\lambda^{mn} > 0$ is a productivity parameter and $\mu^{mn} \in (0, 1)$ is an elasticity, implying decreasing returns to labor in that sector. The parameter $\pi^{mn} < 0$ represents the idea that incremental innovations are more difficult to attain if the level of technology is already high, which we'll refer to as the intertemporal elasticity of innovation.[11]

We can define the expected growth rate in manufacturing TFP resulting from investing labor into R&D as:

$$E(g_t^{mn}) = (A^{mn})^{\pi^{mn}} \cdot \lambda^{mn} \cdot (N_t^{A^{mn}})^{\mu^{mn}} \rho^{mn} . \tag{9}$$

Given this representation R&D is subject to decreasing returns both within period and over time (see also Jones, 2001).

3.3 R&D in the agricultural sector

Agricultural R&D is where we integrate the economic and the biological aspects of the problem. As in the manufacturing sector, the technological progress, as measured by changes in TFP A^{ag}, is conceived as a standard technology ladder. The outcome of the innovation process in agriculture is determined by (i) the quantity of labor allocated to agricultural R&D to generate

[10] In general the "size" of an innovation in the Aghion and Howitt (1992) framework is taken to be the step size necessary to procure rights over the proposed innovation. For purposes of patent law, an innovation must represent a substantial improvement over existing technologies (not marginal change) which is usually represented as a minimum one-time shift.

[11] Assumptions that $\mu^{mn} < 1$ and $\pi^{mn} < 0$ avoid issues related to the population scale effect (see Jones, 1995). In our setting, since population is endogenous, both are required to prevent the growth process to become explosive.

man-made innovations, and by (ii) the scale of the modern food production system, as measured by the amount of land used for agriculture, through the rate of arrival of biological problems (pests and pathogens). We now discuss these in turn.

First, as in the manufacturing sector, in each period the probability that an innovation of size $\rho^{ag} > 0$ takes place is a function labor employed in agricultural R&D that period. Hence the expected growth rate of A^{ag} is:

$$E(g^{ag}) = (A^{ag})^{\pi^{ag}} \cdot \lambda^{ag} \cdot (N^{A^{ag}})^{\mu_{ag}} \rho^{ag} \,, \tag{10}$$

where $\lambda^{ag} > 0$, $\pi^{ag} < 0$, and $\mu^{ag} \in (0, 1)$ as above.

Second, the rate of arrival of problems for the R&D sector to address is endogenous and depends on the size of the agricultural system. As the amount of land allocated to the agriculture expands, the increased uniformity of production directs the process of evolution (e.g. Weitzman, 2000). This results in an increase in the likelihood of pests and pathogens that would negatively affect agricultural output. Following Goeschl and Swanson (2003), we represent the occurrence of such a pest-related event as the potential reduction of land productivity (i.e. a negative shock to agricultural TFP). The size of such a shock is standardized, so that agricultural TFP declines by $\epsilon < 0$ in the event of such a hazard. We will further standardize negative TFP shocks to be equivalent to the loss of one pre-existing innovation, i.e. the step-size of an innovation is set equal to the step-size that occurs in the event of a hazard ($\rho^{ag} = -\epsilon$).

The likelihood of the arrival of a biological problem (a hazard) is determined with the probability:

$$p_t^D = \lambda^D \cdot (X_t)^{\mu^D} \,, \tag{11}$$

where $\lambda^D > 0, \mu^D > 1$. This implies a convex relationship between land conversion and the occurrence of pests and pathogens. In each period, the expected rate of depreciation of $A^a g$ is thus a function of the stock of converted land and is given by

$$E(d_t) = \lambda^D \cdot (X_t)^{\mu^D} \, \epsilon \,.$$

Note that as X increases, both the expected growth rate and the coefficient of variability $\varphi(X) =$ Var(X)/E(X) increases.[12] Our model thus captures the stylized fact that the modern agricultural system is associated with increased variability in yields (Hazell, 1989).

The expected growth rate of agricultural TFP is the net result of the low of innovations out of the man-made R&D and the inflow of problems into the agricultural sector:

$$E(A_{t+1}^{ag}) = A_t^{ag}[1 + E(g_t^{ag}) + E(d_t)]\,, \tag{12}$$

where $E(g_t^{ag}) \geq 0$ and $E(d_t) \leq 0$. Hence the scale of the food production system determines the rate of arrival of problems and the associated expected rate of depreciation (technological regression, or moves down the technological ladder). The allocation of labor to the R&D sector determines the rate of innovation there, and hence the expected technological progress (or moves up the TFP ladder). Moreover, the fact that a decline in TFP makes innovation easier (through $\pi^{ag} < 0$) implies that both 'ecological' processes will equilibrate over time.

4 Social choice and welfare

We consider a social planner choosing paths for C_t, K_t^i and N_t^i, where i represent sectors, by maximizing social welfare over an infinite horizon, subject to technological constraints (1), (2), (3), (4), (5), (7), (9), (12). The solution to this problem involves the allocation of capital and labour in order to determine the optimal scale of the system, and the allocation of these resources between the differing sectors (manufacturing, agriculture, and sectoral R&D).

The assessment of the aggregate social welfare outcome along various possible production pathways concerns how to aggregate individual utilities and is complicated by the fact that population is endogenous. In the following, we first discuss the objective function of the social planner. We then turn to a second complication of land use allocation at the global level, namely the decentralized nature of land conversion and fertility decisions. When individual decision-

[12] We have: Var$(X) = p^D(X)(1 - p^D(X))\epsilon^2$, so that $\varphi(X) = (1 - p^D(X))\epsilon$, and hence

$$\frac{\partial \varphi(X)}{\partial X} = -\epsilon \frac{\partial pD(X)}{\partial X} > 0$$

.

makers do not internalize the increasing inflow of problems associated with land conversion, the decentralized allocation will differ from the social optimum. For this reason it is important to consider alternative assumptions concerning which variables are under the control of the social planner and which are not.

4.1 Social preferences and objective function

Given that the population level is endogenous, the representation of social preferences has strong implications in how society weigh off the size of the population with welfare experienced by each individual. In Appendix A we discuss the relevant axioms guiding the choice of the social evaluation criteria, as well as implications of alternative social welfare functions that can be applied in the present setting. We will focus on the *Classical Utilitarian (CU)* criteria, which is defined as

$$\text{SWF}^{CU} = \sum_{t=0}^{\infty} \beta^t N_t U(C_t/N_t)\,,$$

where $\beta < 1$ is a discount factor and $U(\cdot)$ is a instantaneous utility function. As discussed in Appendix A, a distinctive feature of the CU criteria is the presence of the population level (N) in the numerator of the welfare function. This is also its main drawback since it could favor pathways with large population levels and potentially very low percapita consumption. Therefore comparing and contrasting alternative social welfare functions is one part of the problem to be addressed, but it is left for future work.

The representation of the instantaneous utility function is based on the assumption that households derive utility from consumption, and consumption is restricted to the manufacturing good. More specifically, the utility function $U(\cdot)$ for individuals in this society, assumed to be isoelastic, is given by:

$$U(c) = \frac{\frac{C}{N}^{1-\gamma} - 1}{1-\gamma}\,,$$

where c denotes percapita consumption and γ is the elasticity of marginal utility.[13]

While households' preferences for food consumption does not explicitly enter the utility function, we incorporate households' demand for food as a subsistence requirement, i.e. the

[13] For $\gamma = 1$, this reduces to $U(c) = \ln\left(\frac{C}{N}\right)$.

amount of food that is required for maintaining an individual in a given society. Societies vary significantly in the average body mass per person, however, and a key feature of the demand for food and nutrients is that it is income elastic (Beatty and LaFrance, 2005). We thus treat $\overline{c}(\cdot)$ in equation (6) as an acceptable level of calories intake and model it as a concave function of income:

$$\overline{c} = \xi \cdot (Y^{mn})^{\kappa} .$$

In this representation the demand for food increase with income, and $\kappa < 1$ captures the fact that the demand for food increases less than proportionally with income (i.e. food is an "inferior good").

4.2 Centralized and decentralized choices

The problem of land use allocation at the global level is complicated considerably by reason of the decentralized nature of land conversion and fertility decisions. When individual decision-makers do not internalize the increasing inflow of problems associated with land conversion, the decentralized allocation will differ from the social optimum. For this reason it is important to consider alternative assumptions concerning which variables are under the control of the social planner and which are not, and weather the optimum program internalizes the land conversion externality or not. In particular, we consider three representations of the social planner problem: (i) the planner has control of all the variables and takes into account the land-use externality (shortened to "SPX"); (ii) the planner does not face a land-use externality, a case which we refer to as the "first best" scenario; and (iii) the planner does not control the scale of land conversion and population, and he has to respond to a land-conversion externality ("DCX"). We now discuss these in turn.

The *SPX* outcomes are derived from optimizing over the pathway when all variables are under the control of the social planner, and all information is known about the problem. In this case the negative feedback of converting land in terms of TFP is internalized, and population is optimally determined given the social cost of expanding the food production system. Intuitively, this is the social optimum.

The *first best* pathway is based on the assumption that land conversion does not generate an increased inflow of hazard to agricultural production. This pathway would prevail, for example,

if the risk of biological hazard remained insignificant, or if the planner was optimistic about the resilience of the natural system. Along this pathway there is no perceived cost to the expansion of the agricultural system. The sole constraint on expansion is the allocation of labor across various activities in each period, i.e. the competition for labor between conversion, child rearing, and production sectors. It is also assumed that the social planner has control over all variables (or, equivalently, that private and social objectives are congruent).

The *DCX* pathway focuses on the problem of decentralized choice regarding fertility and land conversion. Despite the recognition of the land conversion externality, the social planner under this regime cannot control population and land conversion. In other words, individual decisions about fertility and land conversion fail recognize the negative feedback from land conversion, and pursue a path equivalent to that of the *first best* pathway, so that private and social objectives diverge. In effect, the social planner has to deal with the fact that he has no control over the scale agriculture, and associated inflow of biological problems, by reallocating labor to R&D and production.

The *first best* pathway thus represents the unlimited view of the world, in terms of scale of population and agricultural system. The *DCX* pathway demonstrates how the pursuit of an unrestricted pathway (by reason of decentralized choice) in the presence of substantial negative feedback will result in the need for investment in R&D to deal with the resulting problems. The *SPX* pathways demonstrate how the centralized choice might instead reduce the scale of agriculture in response to the negative feedback from scale, resulting in a lower rate of land conversion, food production and population.

5 Results: Optimal control, estimation, and simulations

This section discusses implications of the modeling framework and provides some quantitative results. First, we provide further intuition about the solution to the optimal control problem we consider. Second, we report on the structural estimation of the key parameters of the model, and the numerical methods employed to solve the model. Third, we report simulated pathways. Fourth, we consider the implied variability around the optimal paths.

5.1 Intuition about optimal control

Economic growth and increasing percapita consumption are driven by TFP growth (Romer, 1994), and hence rely on the allocation of labor among production sectors (food, manufacturing, child-rearing, land conversion) and R&D sectors (agricultural, manufacturing). The growth engine in this model is thus population. However, decreasing returns to labor in R&D sectors imply that population growth will decline over time, and eventually attain a steady state.[14]

During the transition period, population growth requires the scale of the agricultural system to increases. Indeed while food consumption a does not enter the utility function directly, food is required to meet a subsistence level in oder to maintain a given level of excess mortality (see equation (6)). The demand for agricultural products is thus proportional to the population and to subsistence consumption, which increases with income, so that societies demand more food per person as manufacturing production expands.

The expansion of agricultural production requires resources to be allocated to production and R&D in that sector, and in particular requires land. As the scale of the land dedicated to agriculture increases, the rate of *arrival of hazards* increases. In our model, the rate of arrival is convex in the land surface used for agriculture. This is based on the idea that the agricultural production system induces its own hazards by reason of its uniformity (Cardinale, 2012; Reich et al., 2012).

Biological risks associated with genetic correlation can thus be mitigated through the existence of non-converted lands. The value of un-allocated lies in its role as buffer against the arrival of hazards, and hence acts as a substitute to man-made R&D. Moreover, while land is required to produce food, at the margin it can be substituted by other inputs, namely capital, labor, or more efficient technology. By reducing the surface of cultivated land area, the demand for other substitute input, but the required allocations of inputs to R&D is reduced.

5.2 Estimation strategy and computational method

The aim of the estimation is that the model endogenously reproduces observed trajectories over the period 1900 to 2010. Specifically, we target time series for world GDP (Maddison, 1995;

[14] See Bretschger (2013) for a discussion of demographic transition in the context of endogenous growth models.

Bolt and van Zanden, 2013), world population (United Nations, 1999, 2008), crop land area (Goldewijk, 2001; Alexandratos and Bruinsma, 2012) and TFP growth (Martin and Mitra, 2001; Fuglie, 2012).[15] The parameters of the model are summarized in Appendix B.

Our estimation strategy proceeds in four steps. First, a number of parameters are externally calibrated. We use standard values for $\beta = 0.985$, $\gamma = 1$, $\delta^K = 0.05$, and $\vartheta = 0.3$. The step size on the technology ladder is set to $\rho^{mn,ag} = -\epsilon = 0.05$.[16] Other parameters are taken from the literature. The value shares in agricultural production are taken from Madsen et al. (2010), with $\theta^K = 0.3$ and $\theta^N = 0.5$, and the intertemporal cost of innovation is estimated in Jones (2001), with $\pi^{mn,ag} = -0.5$.

Second, we calibrate a number of quantities so that the model in the first time period matches 1900 data. The initial stock of capital is set so that the marginal product of capital is initially 20%, an approximation of the return to capital at that time. Given the initial stock of capital, we then calibrate the initial level of TFP such that total output in $t = 0$ equals world GDP in 1900. We also calibrate the parameter measuring food consumption for unitary income (ξ) so that the demand for food in 1900 represents about one third of world GDP. While there is no precise data on agricultural GDP in 1900, this is consistent with empirical evidence from countries that are currently at an early stage of economic development.

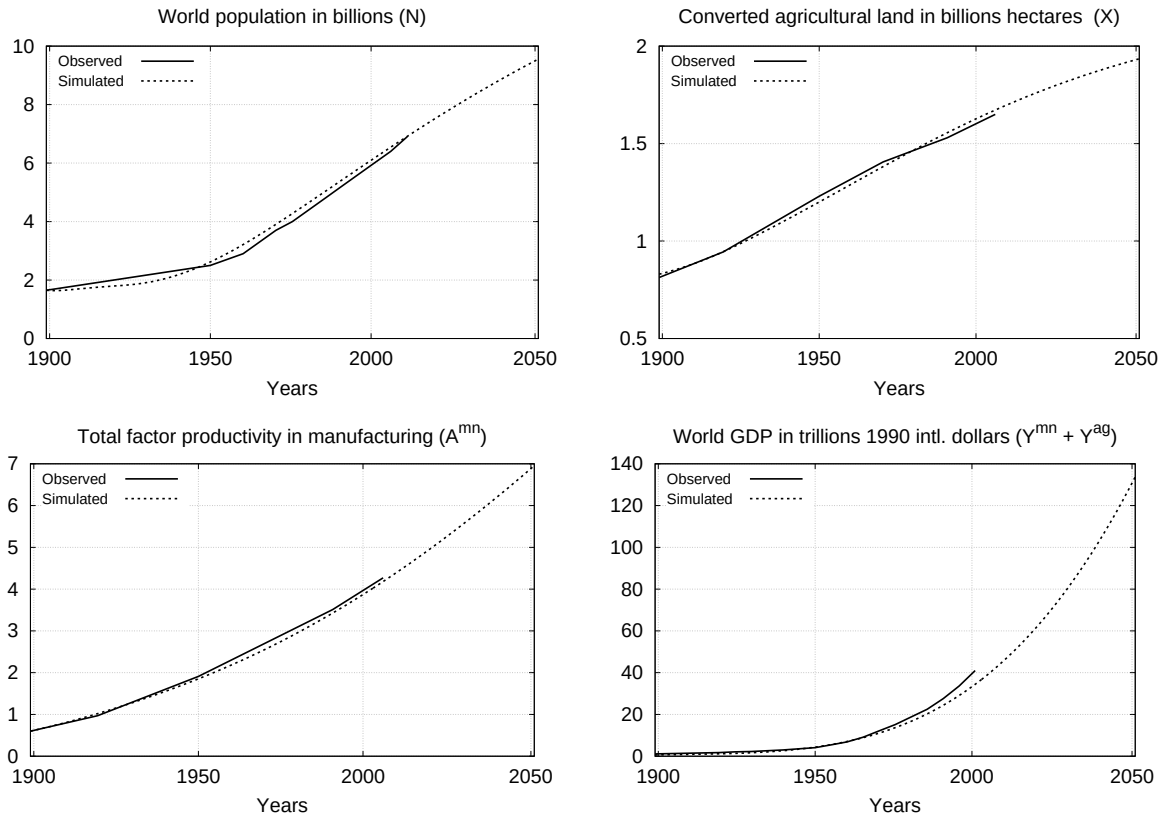
Third, for the remaining parameters $\{\chi, \zeta, \overline{d}, \delta_N, \iota, \kappa, \psi, \varepsilon, \delta^X, \lambda^{mn,ag}, \mu^{mn,ag}\}$, we define a grid spanning plausible values for each parameter, and solve the model for each possible combination of parameters. Since observed TFP growth is the net outcome of the R&D sector, we set the externality to zero ($\lambda^D = 0$). We then use a minimum distance criteria (least square deviations) to compare data points along the observed paths for GDP, population, crop lands and TFP against those simulated with our model.[17] Note that as for many simulation-based esti-

[15] TFP data cover only the period from 1960 to 2010, and is surrounded with uncertainty given they rely on the choice of a particular econometric methodology. Nevertheless, a robust finding of the literature is that the growth rate of TFP in both the agriculture and economy-wide is on average around 1.5-2% per year. For our purposes, we also note that observed TFP growth measure the net outcome of R&D sector, and will thus capture negative shocks to agricultural productivity due to land-use decisions.

[16] Formally our assumptions about functional forms do not allow us to identity the size of steps on the technology ladder ($\rho^{mn,ag}, \epsilon$) separately from productivity parameters ($\lambda^{mn,ag,D}$). To some extent, the choice of the step size is thus arbitrary, although it is consistent with evidence in Fuglie (2012) that do not report TFP growth rate above 3.5%.

[17] Model calibration is still work in progress. In particular to improve the fit of the model we are working on targeting both the levels and rate of changes for the four variables of interest.

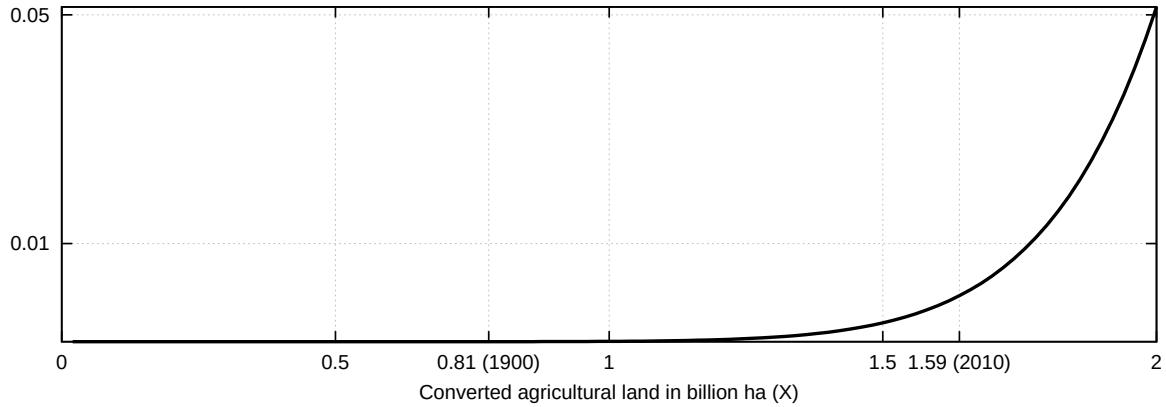
Figure 1: Calibrated model: 1900 – 2010



mation procedure involving highly non-linear models, the uniqueness of the solution cannot be formally proved (see Gourieroux and Monfort, 1996). Our experience with the model suggest however that the solution is stable, with the sum of squared errors increasing significantly for other choices for the parameters. This is mainly due to the fact that we target a large number of data points for several variables, which makes the selection criteria for parameters very demanding.

Observed trajectories and simulated trajectories based on the estimated parameters are shown in Figure 1. Overall simulations with the model closely track the data. The only exception is GDP from the year 2000 onwards, as simulated values are around 10% below observed ones. GDP projections with our model is thus likely biased downwards, and this should be kept in mind for the interpretation of our results. Nevertheless, paths for other variables suggest that the model can approximate observed dynamics quite well. Perhaps more importantly, projections after 2010 are in line with expectations, with world population slightly below 10 billions

Figure 2: Land conversion and hazard rate of agricultural TFP depreciation shocks



by 2050 and converted land still below 2 billon hectares by 2050 (see United Nations, 2008; Alexandratos and Bruinsma, 2012, for alternative projections on population and land conversion respectively). Note also that the growth rate of both population and land conversion decline with time.

The final input to the numerical model is the specification of the land-use externality (equation (11)). The hazard rate as a function of the total stock of converted land is illustrated in Figure 2. The function is posited to be highly convex, reminiscent of the threshold effect that characterizes many ecological processes. The ensuing hazard rate in 1900 is around zero, and reaches about 0.005 in 2010. Thus land use in 2010 implies a 1/200 year occurrence of a biological hazard that would reduce agricultural TFP by 5%. Furthermore, given projections with the estimated model, the total stock of converted land reaches about 2.2 billion hectares by 2150, which would translate into a hazard rate of around 0.15 per year.

The computational algorithm used to solve the model mimics the social planner welfare maximization program, given initial conditions and parameters derived from the data. Formally, we use standard non-linear optimization techniques, searching a local maximum of the objective function (the discounted sum of utility) while remaining in the set of feasible points defined by the constraints.[18] Since the model is designed as a convex problem, the local solution is also

[18] The program is implemented in the GAMS software and solved with the KNITRO (Byrd et al., 1999, 2006) which alternates between interior-point and active-set methods.

the global solution. Direct optimization techniques cannot accommodate an infinite horizon, however, as it would require maximizing a sum with an infinite number of terms subject to an infinite number of constraints. We thus truncate the horizon of the problem to T periods, and check that terminal effects do not affect the solution for the relevant (shorter) horizon of interest $T' < T$.[19]

5.3 Implementation of decision making scenarios

As described above we consider three scenarios determining global land-use management. The *SPX* scenario is simply the social optimum in which the planner controls every variables and takes into account the land-use externality.

In the *first best* scenario, the land-use externality is ignored by solving the model for $\lambda^D = 0$. This can be the case, for example, if the planner is optimistic about the resilience of the food production system. The only cost associated with land conversion is the labor required for land clearing activities. For the *first best* land conversion path, we evaluate the implied magnitude of the externality under the label "*First best – implied externality*". Note that the associated path for agricultural TFP would be the result of a 'myopic' social planer.

In the *DCX* scenario, decisions about land conversion and fertility are decentralized and do not take into account the land conversion externality. In the simulations, the paths for land conversion and population are exogenously fixed to those determined under the *first best* scenario. In this scenario the social planner takes into account the land-use externality, so that he can re-allocate resources to mitigate the negative effects of decentralized fertility and land conversion choices.

5.4 Results: optimal paths

In this section we employ the estimated model to simulate alternative policy scenarios from year 2010 onwards. Throughout the analysis, we treat the R&D processes as deterministic by

[19] If the time horizon of the problem is finite, the optimal shadow values of stock variables drop to zero in finite time. Hence typically, the solution of the finite horizon problem differs from its infinite horizon counterpart. This divergence is due to 'terminal effects', since only the very end of the program is affected by the finite horizon. A simple approach to evaluate the impact of terminal effects on the optimal paths is to vary T and calculate the difference in the optimal pathways over the relevant horizon T'. In practice, we have used $T = \{350, 400, 450\}$, finding no difference on the paths up to $T' = 200$.

assuming that the expected growth rate is the realized growth rate.[20] In the last subsection we consider the issue of variability around the expected path, and implication in terms of famine events and associated increase in mortality.

5.4.1 Aggregate impacts of the land conversion externality

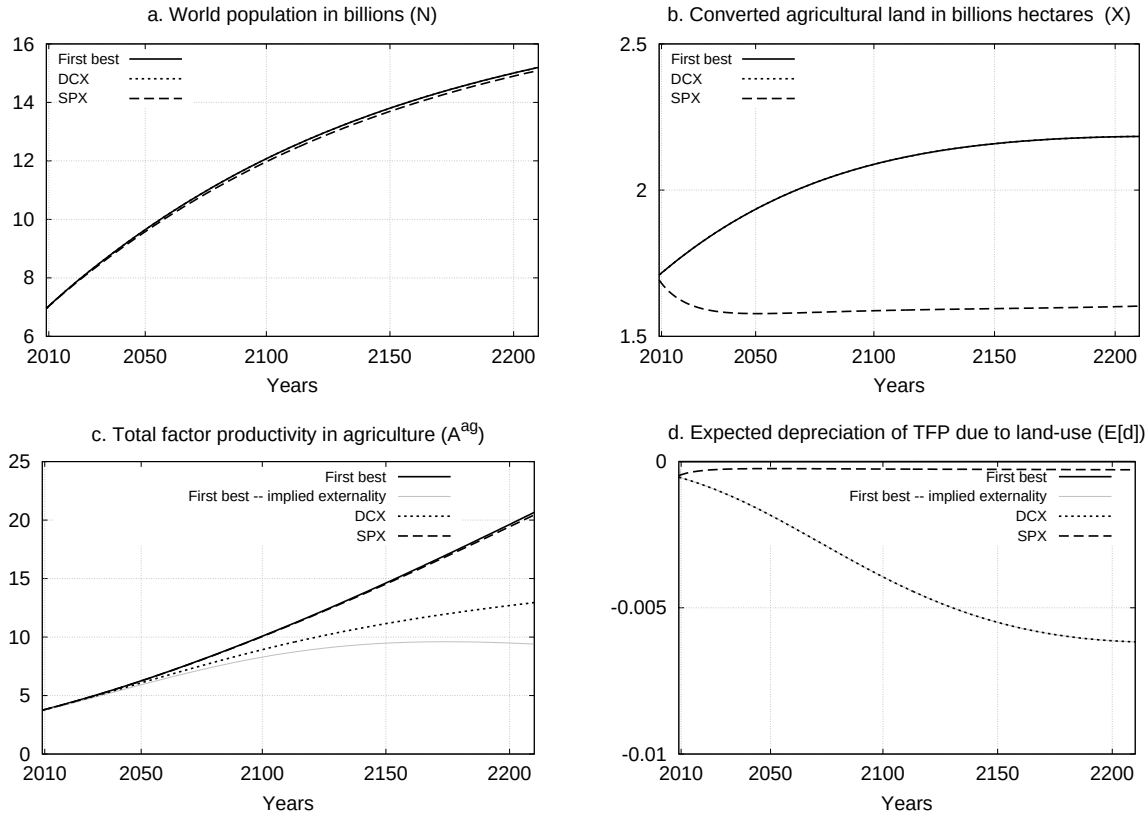
Figure 3 reports the fertility and land allocation decisions. For all scenarios, optimal paths for population (panel a) shows a slowdown of growth, stabilizing slightly above 16 billions in the very long run. Under *first best* and *DCX*, the amount of land used for agricultural production (panel b) increases significantly. However, the total amount of land that can be used for agriculture (around 3 billion hectares, see Alexandratos and Bruinsma, 2012) is never exhausted. Importantly, decisions about land use for these two scenario are only optimal under *first best*, and exogenously imposed in the *DCX* scenario.

Under *SPX*, where the planner faces the land-use externality and can adjust the amount of land being used, there is an immediate decline in the stock of land, reaching a steady state at around 10% below 2010 level. The planner thus leaves part of agricultural lands used in 2010 to convert back to natural lands in order to mitigate the risk of biological hazards. This is depicted in panels (c) and (d). In the *SPX* scenario the level of land-use implies that depreciation is below 0.1% per year, so that TFP growth approaches that prevailing under *first best* (although under the *first best* scenario, agricultural TFP grows solely through man-made innovations).

The level of externality implied by the *first best* path for land-conversion implies that TFP stabilizes after 150 years, as shown by the lines labeled *"First best – implied externality"*. Thus if the planner myopically neglects the land-use externality, man-made and biological innovations would equilibrate over time. While expected TFP depreciation (panel d) is the same under *first best – implied externality* and *DCX*, as land use path is the same, paths for TFP significantly diverges (panel c). This is due to the fact that under the *DCX* a lower TFP means a higher probability of innovation, as well as differences in the allocation of labor to agricultural R&D.

[20] We are in the process of formulating an expected utility representation of the problem.

Figure 3: Optimal paths for population, land conversion and agricultural TFP



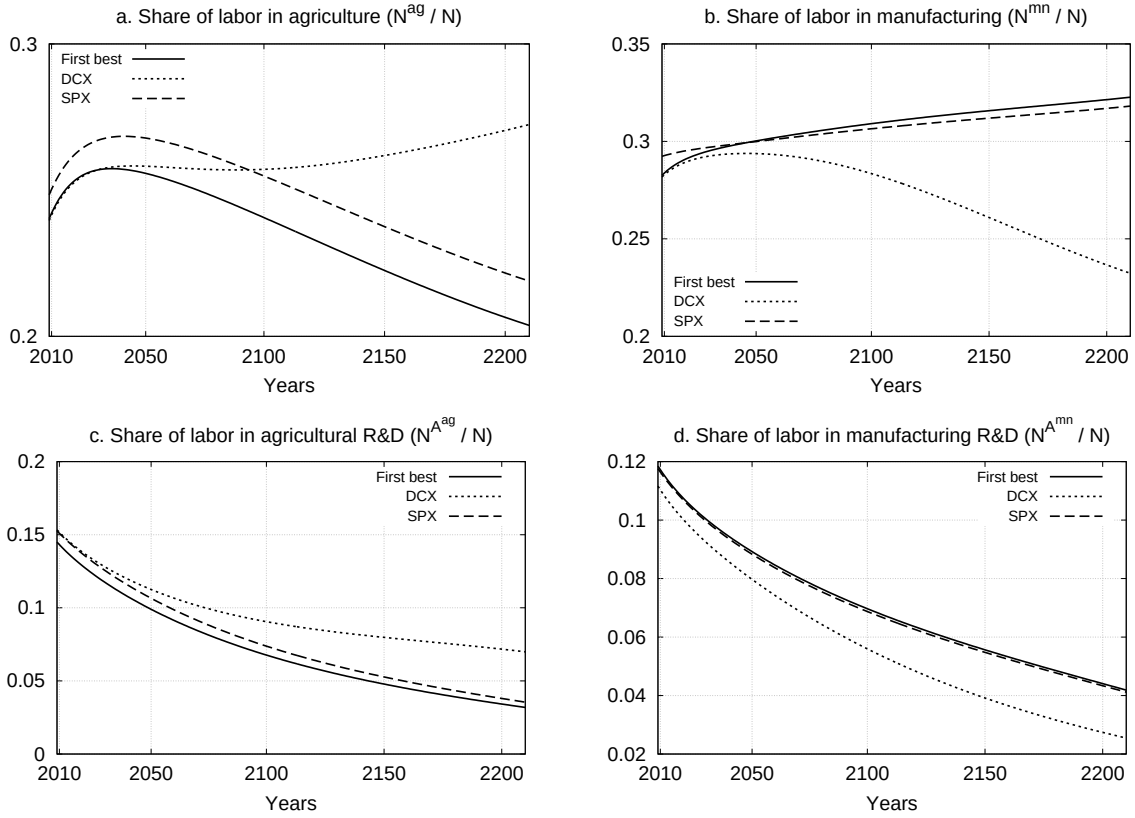
5.4.2 Labor allocation in agriculture and manufacturing

Figure 4 reports the optimal path of labor allocation decisions along the different scenarios. The most important difference in paths lies in comparing labor allocation in agriculture. The *DCX* path for allocating labor shows that a substantial share of labor must be allocated to agricultural R&D in order to balance the negative externality of land conversion. Similarly, the share of labor in agriculture increases over time to compensate the lower TFP growth. An implication of this is that the share of labor in manufacturing production and R&D are significantly lower under *DCX*.

Under the *SPX* path, the social planner maintains food supply by increasing the share of labor relative to *first best* in both agricultural production (panel a) and R&D (panel c). Change in the allocation to agricultural R&D is small, which shows that the planner substitutes land conversion to man-made R&D.

While the proportion of labor employed in the R&D can appear to be high (between 10

Figure 4: Optimal paths: Labor allocation

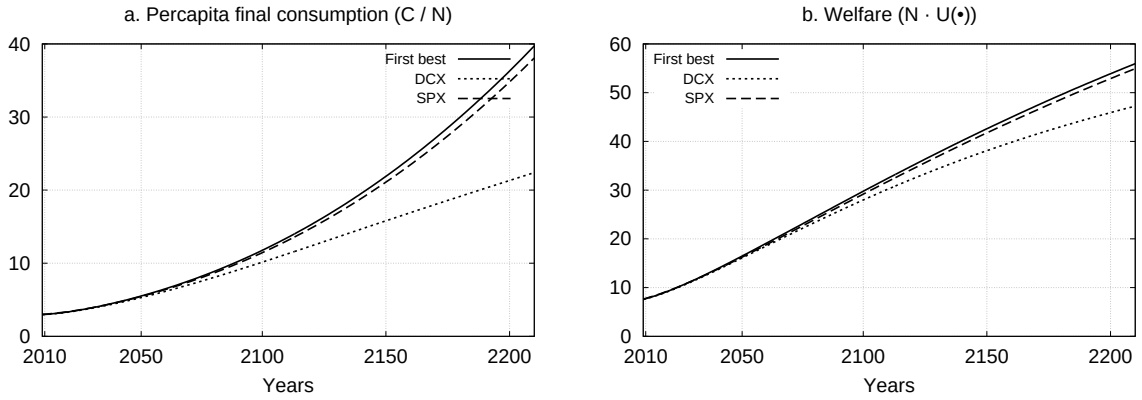


and 25%), it should be noted that any labor time dedicated at improving factor productivity is counted there. This includes many informal activities taking place in developing economies, such as seed selection or developing irrigation practices. Moreover, the downward trend in the share of labor allocated to R&D is due to decreasing returns to labor in man-made innovation, both within period and over time (equations 8 and 10).

5.4.3 Percapita consumption and social welfare

Figure 5 shows the implications of reallocating labor away from the manufacturing sector in terms of percapita consumption (panel a) and social welfare (panel b). While percapita consumption is relatively similar in both *first best* and *SPX* scenarios, it is significantly lower in the the *DCX* pathway. In turn, welfare diverges across scenarios. The *first best* scenario generates the highest welfare, which is due to both high population and high percapita consumption (see

Figure 5: Optimal paths: Welfare and percapita consumption



panel b). In the *SPX* scenario, both population and percapita consumption are sightly lower, but the path for welfare is very close. This suggest a large value for global land use management, as maintaining land reserves generates a substantial gain in welfare over time.

5.4.4 Interpretation

The results of the simulations demonstrate how the three basic scenarios drive very different futures in terms of land management and welfare.

The *first best* scenario is based on the assumption that there is no negative feedback from the scale of the system. This scenario provides a reasonably good fit to the massive rate of land conversion and population expansion of the past one hundred years. In the future it would describe a world in which land conversion increases by 30% relative to 2010.

The *DCX* scenario takes the fertility and land conversion decisions resulting from the first best scenario as a given, but assumes that the social planner can reallocate labor in response to the negative externality of land-use. The basic result is that the cost of the unmanaged global land must be divided between consumption and R&D. Given that land allocation decisions are made without consideration of the negative externality, the labor allocation must compensate through increased allocation to the R&D sector. This drives a gap between the first best and DCX scenarios, in terms of consumption and welfare. Thus depending on the size of the externality, DCX describes a path with rapid land conversion, and massive human population, but relatively

low welfare on account of the share of resources spent on R&D.

The *SPX* scenario describes the path where the social planner can control both fertility and land conversion decisions to manage the external cost of land conversion. In this case the decision maker will harness the expansion of the agricultural area in order to avoid a costly increase in the arrival of hazards. The overall impact is a much-reduced scale of the agricultural system. A sticking feature of the *SPX* path is that the social planner adjusts the amount of land but leaves the population trajectory almost unchanged. Thus while the additional cost of land conversion slightly reduce population, percapita consumption, and thus welfare, these trajectories remains close to their *first best* counterpart.

5.5 Variability analysis around optimal paths

In this section we demonstrate the importance of variability flowing from the agricultural production system, on account of its impacts upon population levels. In particular, the land use externality implies variability around the expected path, and given an increasing population even small variations in agricultural output could have size able consequences in terms of mortality. Note that since the social planner considers only the expected value of TFP shocks, he takes no account of variability in its decision making.[21]

One approach to quantify the magnitude of variability around the expected-value paths is to determine the number of expected casualties along the different paths. This can be interpreted as famine events that would be triggered by the arrival of biological hazard. In each period the realization of the negative TFP shock would reduce agricultural TFP by 5%, which would in turn reduce agricultural output that period. This would lead up to an increase of mortality. The expected number of casualties is thus given by the number of casualties that would be implied by a drop in TFP by 5% weighted by the probability that the negative shock occurs.

Table 1 reports the number of expected casualties that the planner expects along its optimal pathways from 2010 to 2050, cumulated over 10 years intervals. Under the DCX path, the number of famine events sharply increases over time, due to both an increase in the probability

[21] We are currently developing a stochastic version of the model that maximises expected welfare and will be used to evaluate the impact of risk preferences on optimal pathways. Note that it is possible to include additional costs in the welfare function flowing from events of unanticipated mortality, but it is difficult to place a weight on this objective relative to that of aggregate consumption.

Table 1: Expected cumulated casualties (in millions) due to famine events

	Time interval			
	2010 - 2020	2020 - 2030	2030-2040	2040 - 2050
DCX	8.1	14.4	28.2	52.2
SPX	4.6	4.4	5.7	7.8

of biological hazards and an increase in the population, magnifying the consequences of any particular negative TFP shock. In the SPX path (where reserve lands are kept to dampen the impact of hazards), the expected number of casualties remains low. Over time however, the number of casualties increases because of population growth.

6 Concluding comments

We have demonstrated in a two sector macroeconomic model the manner in which land/labor allocations interact with percapita income growth under a risk of biological hazards. First, we have explored the *first best* outcome, in which there is no negative feedback from continued expansion of agriculture. In this case, and under the most standard criteria for assessing welfare, it would be anticipated that land conversion would continue along with population growth. Our quantitative model projects a population approaching 16 billion persons over the next 200 years.

This outcome seems optimistic, and we have examined a scenario in which there is some negative feedback from the unabated expansion of the agricultural system. We have modeled this negative feedback as a stochastic hazard deriving from continued expansion of agricultural land. There is some empirical and theoretical justification for modeling the problem in this way. In our model, the ongoing expansion of agriculture generates an increased flow of hazards within the system, which must be addressed or experienced.

We have explored two possible scenarios for addressing this assumed negative feedback. One is the *DCX* scenario in which population and land conversion is determined in a decentralized manner, and the decision maker must react through labor allocation. In this scenario the R&D sector is used as a means of addressing the resulting biological feedback, although it results in a lower percapita consumption and aggregate welfare. The cost of addressing the hazards

resulting from expanded agriculture thus erode the benefits from higher population, and so the paths of aggregate welfare and population become dis-connected.

In the final scenario we have explored, *SPX*, the decision maker both perceives the negative feedback from scale and has all variables under control. In this scenario, the decision maker manages negative feedback through both the R&D sector and also through land conservation. In this scenario, the outcome is a lower human population, but with a high per capita level of consumption and welfare.

One reason to control land conversion at the global level is that the scale of the wager increases over time. As the human population continues to grow, minor shocks to agricultural output result in events of profound impact, with proportionally large increases in mortality. This is simply be a consequence of living in a world with so much wagered on the stability of a single system. Our conclusion is that the control of this system is very likely to turn out to be important.

Ultimately two sets of policy insights are to be pursued. First, the model allows to consider the extent of a tax on land conversion that would achieve the social optimum path in a decentralized setting. Given the race between pests and patents at play, an alternative would be to consider a subsidy to agricultural R&D could also compensate land-use externality. Second, results from this model on the "optimal" amount of land conversion can be used as an input in simulation models that focus on land-use patterns throughout the world. More specifically, high-resolution land-use models could be used to derive concrete implications about where land conversion policy should be directed.

We close by highlighting that our results so far focus on the classical utilitarian objective. Our preliminary experiments with other social objectives (average utilitarian and critical-level utilitarian; see Appendix A) suggest two things. First, a planner with an average utilitarian objective would favor percapita consumption over population, driving population to zero asymptotically. By making the decline in population slightly faster than that of manufacturing output, percapita consumption remains steady. Second, since changes in the size of population is already an indirect function of percapita consumption, the additional constraint imposed by a critical level of utility is not binding. This implies that our results hold for the critical-level utilitarian objective function, mitigating ethical concerns associated with the classical utilitarian objective that underlies our analysis.

References

Acemoglu, D. (2002) "Directed technical change," *Review of Economic Studies*, 69, pp. 781 – 809.

Aghion, P. and P. Howitt (1992) "A model of growth through creative destruction," *Econometrica*, 60 (2), pp. 323 – 351.

Alexandratos, N. and J. Bruinsma (2012) "World agriculture towards 2030/2050, The 2012 revision." ESA Working Paper No. 12-03.

Barro, R. J. and G. S. Becker (1989) "Fertility choice in a model of economic growth," *Econometrica*, 57, pp. 481 – 501.

Barro, R. and X. Sala-i Martin (2004) *Economic Growth - Second Edition*: Cambridge MA: MIT Press.

Beatty, T. K. M. and J. T. LaFrance (2005) "United states demand for food and nutrition in the twentieth century," *American Journal of Agricultural Economics*, 87 (5), pp. 1159 – 1166.

Blackorby, C., W. Bossert, and D. Donaldson (2005) *Population Issues in Social Choice Theory, Welfare Economics, and Ethics*: Econometric Society Monographs. Cambridge Univ Press, Cambridge, UK.

Bolt, J. and J. L. van Zanden (2013) "The first update of the maddison project; re-estimating growth before 1820." Maddison Project Working Paper 4.

Bretschger, L. (2013) "Population growth and natural resource scarcity: Long-run development under seemingly unfavourable conditions," *Scandinavian Journal of Economics*, 115 (3), pp. 722 – 755.

Byrd, R., M. E. Hribar, and J. Nocedal (1999) "An interior point method for large scale nonlinear programming," *SIAM Journal on Optimization*, 9 (4), pp. 877 – 900.

Byrd, R. H., J. Nocedal, and R. A. Waltz (2006) "KNITRO: An integrated package for nonlinear optimization," in G. di Pillo and M. Roma eds. *Large-Scale Nonlinear Optimization*: Springer-Verlag, pp. 35 – 59.

Cardinale, B. (2012) "Impacts of biodiversity loss," *SCIENCE*, 336, pp. 552 – 553.

Ewers, R. M., J. P. W. Scharlemann, A. Balmford, and R. E. Green (2009) "Do increases in agricultural yield spare land for nature?" *Global Change Biology*, 15, pp. 1716–1726.

FAO (2010) "The state of the worldŠs plant genetic resources for food and agriculture." Food and Agriculture Organization, Rome.

Fuglie, K. O. (2012) "Productivity growth and technology capital in the global agricultural economy," in K. O. Fuglie, S. L. Wang, and V. E. Ball eds. *Productivity Growth in Agriculture: An International Perspective*: Wallingford, U.K.: CAB International, pp. 335 – 368.

Goeschl, T. and T. Swanson (2003) "Pests, plagues, and patents," *Journal of the European Economic Association*, 1 (2), pp. 561 – 575.

Goldewijk, K. (2001) "Estimating global land use change over the past 300 years: The hyde database," *Global Biogeochemical Cycles*, 15 (2), pp. 417 – 433.

Gourieroux, C. and A. Monfort (1996) *Simulation-Based Econometric Methods*: Oxford University Press.

Hansen, G. D. and E. C. Prescott (2002) "Malthus to Solow," *American Economic Review*, 92 (4), pp. 1204 – 1217.

Hazell, P. (1989) "Changing patterns of variability in world cereal production," in J. R. Anderson and P. Hazell eds. *Variability in grain yields: Implications for agricultural research and policy in developing countries*: Baltimore: Johns Hopkins University Press, pp. 13–34.

Jones, C. I. (1995) "R & D-based mmodel of economic growth," *The Journal of Political Economy*, 103 (4), pp. 759 – 784.

——— (2001) "Was an industrial revolution inevitable? economic growth over the very long run," *Advances in Economics*, 1 (2), p. Article 1.

Koo, B., P. Pardey, and B. Wright (2003) "The economic costs of conserving genetic resources at the cgiar centres," *Agricultural Economics*, 29, pp. 287 – 297.

Maddison, A. (1995) "Monitoring the world economy 1820 Ű1992." Paris: Organization for Economic Cooperation and Development.

Madsen, J. B., J. B. Ang, and R. Banerjee (2010) "Four centuries of british economic growth: the roles of technology and population," *Journal of Economic Growth*, 15 (4), pp. 263 – 290.

Martin, W. and D. Mitra (2001) "Productivity growth and convergence in agriculture versus manufacturing," *Economic Development and Cultural Change*, 49 (2), pp. 403 – 422.

McCann, K. S. (2000) "The diversity-stability debate," *Nature*, 405, pp. 228 – 233.

Mundlak, Y. (2000) *Agriculture and economic growth: theory and measurement*: Harvard University Press, Cambridge M.A.

Ngai, L. R. and C. A. Pissarides (2007) "Structural change in a multisector model of growth," *American Economic Review*, 97 (1), pp. 429 — 443.

Reich, P. B., D. Tilman, F. Isbell, K. Mueller, S. E. Hobbie, D. F. B. Flynn, and N. Eisenhauer (2012) "Impacts of biodiversity loss escalate through time as redundancy fades," *Science*, 336, pp. 589 – 592.

Romer, P. M. (1994) "The origins of endogenous growth," *The Journal of Economic Perspectives*, 8 (1), pp. 3 – 22.

United Nations (1999) "The world at six billion." Department of Economic and Social Affairs, Population Division.

——— (2008) "World population prospects: The 2008 revision." Department of Economic and Social Affairs, Population Division.

Weitzman, M. (2000) "Economic profitability versus ecological entropy," *Quarterly Journal of Economics*, 115 (1), pp. 237 – 263.

Appendix A Endogenous population assessment criterion

For problems of endogenous population the social choice literature suggest five relevant axioms on which to build social evaluation criteria (see Blackorby et al., 2005):

Utility independence – the ranking of any two alternatives is independent of the utility levels of individuals who have the same utilities in both.

Existence independence – the ranking of any two alternatives is independent of the existence of individuals who ever live and have the same utility levels in both (therefore existence independence is a stronger independence axiom than utility independence, and it implies utility independence).

The negative expansion principle – the addition of a person to a utility-unaffected population is ranked as bad if the utility of the added person is negative.

Avoidance of the repugnant conclusion – it is not the case that any alternative in which each member of the population has a positive utility level, no matter how high, is ranked as worse than some alternative in which a larger population has a utility level that is above neutrality[22] but arbitrarily close to it (otherwise it may be recommended to create a large population, in which each person lives in poverty).

Priority for lives worth living – all alternatives in which each person is above neutrality are ranked better than all those in which each person is below it.

A key result of social choice theory is that no social welfare function satisfy all five population axioms together (plus other axioms, which are same-number analogues of the axioms used in characterizing social welfare functions in the case of exogenous population).[23] In particular, a major disadvantage associated with the use of the CU criteria is that it does not avoid the repugnant conclusion. An alternative is the *Average Utilitarian (AU)* criteria, which is given by

$$\text{SWF}^{AU} = \sum_{t=0}^{\infty} \beta^t U(\cdot)\,.$$

[22] Neutrality is defined as a utility level above which lives are worth living and below which lives are not worth living. It tends to be set to zero, without loss of generality.

[23] Since we consider a dynamic setting with uncertainty, the connexion with axioms postulated for static decision-making is somewhat loose. However Blackorby et al. (2005) do provide some extensions in their book that account for uncertainty and time/discounting.

AU does not satisfy either utility or existence independence, nor does it satisfy negative expansion.

The third is the *Critical-Level Utilitarian (CLU)*:

$$\text{SWF}^{CLU} = \sum_{t=0}^{\infty} \beta^t N_t \left[U(\cdot) - u_0 \right] ,$$

where u_0 defines the critical level of utility beyond which existence is not worth. CLU does not satisfy priority of lives worth living. The critical level utility should be purely a judgment on what is a minimum standard of living, and it cannot be defined without prior choice of the utility function's parameters.

The distinctive characteristic between these three social welfare functions is the presence or absence of the population level (N) in the numerator of the welfare function. In CU and CLU, population level is there, whereas in AU it is not present. For this reason, there is an in-built tendency for the CU version of welfare functions to favor pathways with large population levels, while the AU welfare function favors those with low population and high percapita consumption. In what follows we will only explore the CU welfare function as the least controversial of the various approaches to a social welfare in an endogenous population problem.

Appendix B Numerical values of the parameters

Parameter		Value
Imposed parameter		
β	Discount factor	0.985
γ	Elasticity of marginal utility	1
ϑ	Share of capital in manufacturing	0.3
θ^K	Share of capital in agriculture	0.3
θ^N	Share of labour in agriculture	0.5
δ^K	Capital depreciation	0.05
ξ	Food consumption for unitary income	0.31
$\overline{X}$	Total land suitable for agriculture	3
$\rho^{mn,ag}$	Maximum TFP growth	0.05
$\pi^{mn,ag}$	Intertemporal elasticity of innovation	-0.5
ϵ	Size of TFP depreciation shock	-0.05
λ^D	Scale of land-use externality	0.00005
μ^D	Convexity of land-use externality	10
Estimated parameters		
χ	Child rearing labor requirement	0.22
ζ	Elasticity of child rearing	0.29
$\overline{d}$	Exogenous mortality rate	0.023
δ_N	Consumption-based mortality	0.32
ι	Elasticity of mortality to food consumption	2
κ	Income elasticity of food demand	0.66
ψ	Land-conversion labour requirement	0.068
ε	Elasticity of land-conversion	0.9
δ^X	Rate of land reconversion	0.02
$\lambda^{mn,ag}$	R&D labor requirement	0.57
$\mu^{mn,ag}$	Elasticity of labour in R&D	0.3