Performance of Three Multi-Award Reverse Auction Mechanisms

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Abstract

This paper compares the performance of three multi-award reverse auction mechanisms using lab experiment. The first mechanism is called the Uniform Price Reverse (UPR) auction, where each winning bidder is paid the lowest rejected bid. The second mechanism is called the First Price Reverse (FPR) auction, where winning bidders are paid their submitted bids. The third mechanism is called the Generalized Second Price Reverse (GSPR) auction, where each winning bidder is paid the bid that is immediately higher. Theoretically, I derive the equilibrium bidding strategy for each auction mechanism and show that a symmetric equilibrium strategy may not exist under the GSPR auction. Empirically, lab experiment results show that UPR and GSPR auctions lead to a higher efficiency level compared to FPR, while UPR auction yields the lowest auctioneer surplus. From a valuation perspective, UPR and GSPR auctions are preferred to FPR auction.

Keywords: Procurement Auction, Uniform Price Reverse Auction, First Price Reverse Auction, Generalized Second Price Reverse Auction, Experiemntal Economics

JEL: C91, C92, D44

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1 Introduction

Auctions are considered as effectiveness ways to allocate scare resources (McAfee and McMillan, 1987) under a defined set of rules. Popular auction items include United States Treasure bills, artworks, antiques, mineral rights and others. In an ordinary (or "forwarding") auction, buyers compete to purchase goods or services by offering higher prices to the sellers. In the reverse auction, the buyers and sellers' roles are reversed and sellers compete to sell goods or service by offering lower prices. Buyers usually award the contract to sellers with the lowest prices. The reverse auctions (or sometimes called the "procurement auction") are commonly used by commercial and government agencies to reduce the cost incurred to achieve a given target or to maximize benefits under a finite budget. Recently, reverse auction has been used by the Federal Communications Commission (FCC) to relinquish broadcast spectrum rights (Cramton et al., 2015).

Two general auction approaches have been used in reverse auctions. The first approach is called the Uniform Price Reverse (UPR) auction, where all winning bidders are compensated with a same price that is determined by the buyer's budget or a pre-defined objective. The second approach is the First Price Reverse (FPR) auction, or the Discriminatory auction, where each winning bidder receives a compensation equals to her submitted bid. The performance of Uniform Price and First Price auctions have been studied extensively in both ordinary and reverse auctions under various contexts (Bower and Bunn, 2001; Cason and Gangadharan, 2004, 2005; Holt Jr, 1980). Theoretically, UPR auction is preferred to FPR auction from a valuation perspective as individuals have the incentive to reveal their true types when all winners are paid the same price, while FPR auction is shown to have a better overall market performance when the buyer is facing a budget constraint (Cason and Gangadharan, 2005). In a reverse auction setting, bidders significantly bid higher than their true cost to receive information rents when FPR auction is used.

In reverse auctions, the buyer may purchase multiple items at the lowest prices, which departs from an ordinary auction where usually one item is auctioned and the highest bidder wins the auction. For a single item auction, the First Price (Cox et al., 1988; Harrison, 1989; Laffont et al., 1995; Riley and Samuelson, 1981) and Second Price (Graham and Marshall, 1987) auctions are the most widely used auction formats. For multiple items auctions, common auction approaches include the First Price auction (Cox et al., 1984), Generalized Second Price auction (Che et al., 2017; Edelman et al., 2007; Gomes and Sweeney, 2014; Jeziorski and Segal, 2015; Varian, 2009, 2007), Uniform Price auction and the VCG auctions (Cason and Gangadharan, 2004, 2005; Fukuda et al., 2013). In the literature, equilibrium solution concepts are mostly based on the locally envy free equilibrium (Edelman et al., 2007; Fukuda et al., 2013) and Bayesian Nash equilibrium (Gomes and Sweeney, 2014) for the Generalized Second Price auction. Lab experiment is a primary tool used to compare auction performances under the controlled environment. Che et al. (2017), Fukuda et al. (2013), and McLaughlin and Friedman (2016) experimentally test different ordinary auction mechanisms inspired by the sponsored search engine advertisement auctions including the Generalized Second Pricing auction. There are few studies investigate the performance of auction mechanisms when applied in reverse auctions. In this paper, I study a novel reverse auction mechanism called the Generalized Second Price Reverse (GSPR) auction in which each bidder is compensated with a price that equals the bid that is immediately higher. For example, an individual with the lowest bid will get compensated with the second lowest bid, the subject with the second lowest bid will get compensated with the third lowest bid, and so on. The GSPR auction is the Generalized Second Price auction applied in a reverse auction context and has not been studied theoretically or empirically.

Gomes and Sweeney (2014) characterizes the Bayesian Nash equilibrium in a Generalized Second Price auction under the context of auctioning advertising positions in the search engines. Under the sponsored advertising position auctions, each position has a different click-through rate. They find that if the click-through rates are the same, it is possible that an efficient Bayesian Nash equilibrium does not exist, and more broadly, a symmetric Bayesian Nash equilibrium may not exist as well. In this paper, the Generalized Second Price auction is studied in a reverse auction context with no click-rate variations for different positions. I derive a generic solution for the GSPR auction and demonstrate that a Bayesian Nash equilibrium may not exist under certain circumstances. The GSPR auction is compared with two standard multi-award reverse auction mechanisms including the UPR auction and FPR auction.

A fundamental challenge in auction literature is to balance the tradeoffs between incentive compatibility and the auctioneer's surplus. In general, the bidders are often compensated with an excessive amount of surplus in order for them to reveal their true types at the cost of the auctioneer's surplus, such as in the UPR auction. However, the auctioneer can also discriminate bidders by using the FPR auction where the bidders have no incentive to reveal their true types. As a result, the auctioneer is often unable to get a precise estimate of the bidders' true cost without specific assumptions or structural estimates. This paper demonstrates that the GSPR auction can be an ideal candidate for multi-award reverse auctions since under the GSPR auction, individuals' bids are close to their true cost and the auctioneer still obtains a high level of surplus similar to the FPR auction,

This paper also compares the social efficiency and the allocation of social surplus across the three reverse auction mechanisms. The allocation of social surplus can substantially influence the bidders' incentives and result in different economic and social impacts when reverse auction is used to achieve a social objective. For example, in an environmental conservation context, landowners (e.g., farmers or wetland managers) may receive too many subsidies for enrolling in better environmental management practices when a UPR auction is used to decide the distribution of the conservation fund, because the lowest cost landowner receives the same amount of compensation as the marginal landowner. Even though the UPR auction is considered incentive compatible and the landowners will, in principle, reveal their true opportunity cost, such an approach is far from efficient from a conservation perspective as some low cost landowners exhaust a considerable amount of conservation budget by receiving a huge surplus from a uniform compensation as they are paid much higher than the true cost. Alternatively, even though FPR auction is shown to perform better than UPR (Cason and Gangadharan, 2005), strategic bidding incentives prevent the auctioneer from getting a good estimate of the landowners' true opportunity cost, which serves as the basis for a robust benefit-cost analysis or designing conservation polices. In this paper, lab experiment results indicate that the GSPR may overcome the surplus allocation and incentive issues and, empirically, perform well both from the efficiency and the valuation perspectives. Though this study is motivated by the information asymmetry issue and optimizing incentive payment in various environmental conservation programs (Ferraro, 2008), the GSPR auction can be applied to other types of reverse or procurement auctions when there are multiple winners from one auction outcome.

This paper provides the first evidence on the performance of the GSPR auction, which is compared with the more commonly used UPR and FPR auctions. Experimental results show that FPR auction leads to the highest bids on average, then GSPR auction, and the UPR auction leads to the lowest average bid. The bidding prices are similar under UPR and GSPR auctions. The sellers acquire a significantly higher surplus in the UPR auction, and acquire the lowest surplus in the FPR auction. In terms of social efficiency, the UPR and GSPR auctions achieve a very high social efficiency level while the FPR auction performs significantly worse than the other two counterparts.

The remainder of the paper is organized as the follows. In Section 2, I set up the auction framework and present the theoretical benchmarks based on the Bayesian Nash equilibrium solution concept. In Section 3, I describe the experiment procedure and summary statistics. In Section 4, I present the experiment results focusing on individual bidding behaviors and social efficiency. Section 5 concludes the paper.

2 Theoretical Remarks

This section discusses theoretical property of the three reverse auction mechanisms. The theoretical analyses of Uniform and First Price reverse auctions are mainly based on Vickrey (1962), Harris and Raviv (1981) and Cox et al. (1984) while the analyses of the Second Price Reverse auction are constructed following the conceptual framework of Borgers et al. (2007), Edelman et al. (2007) and Varian (2007), and the deriviation of Bayesian Nash equilibrium is based on Gomes and Sweeney (2014).

2.1 Uniform Price Reverse Auction

Assume there are N risk-neutral bidders competing for a total of M homogenous contracts ($M < N$, $M \geq 2$, $N = 1, 2, ..., N$) and each bidder can be awarded at most one contract. In the UPR auction, each bidder submits a bid for a single unit and the M lowest bidders will be awarded with the contract at the price of the lowest rejected bid, or the $M + 1th$ lowest bid. The winning price, paid by the auctioneer to the bidder for completing the contract, is denoted as p_i. Let c_i be the bidder i's cost for completing the contract and assume each c_i is independently drawn from a distribution with a probability density function $f(\cdot)$ and cumulative probability distribution function $F(\cdot)$, with the support on $[\underline{v}, \bar{v}]$. Let $b_i = s(c_i)$ be the bidder i's bid function.

I maintain the standard assumptions that the bidding function $s(\cdot)$ is increasing, concave and differentiable. If a bid b_i is accepted, bidder i's monetary payoff is the winning price p_i (i.e., the lowest rejected bid) minus cost c_i. When a bid is not accepted, in this case, if the bid b_i not one of the lowest M bids, bidder i's payoff is zero. Let $s^{-1}(\cdot)$ denote the inverse of the bidding function $s(\cdot)$, thus, $s^{-1}(s(c_i)) = s^{-1}(b_i) = c_i$.[1]

Since M lowest bidders are chosen as the winners, the probability that a bid b_i wins the auction is denoted by $G(s^{-1}(b_i))$, which equals the probability at most $M-1$ of the values drawn by other bidders are less than $s^{-1}(b_i)$. As a result, following Harris and Raviv (1981), the calculated probability of being one of the lowest bids equals the probability $s^{-1}(b_i)$ below the Mth order statistics among the other $N-1$ bids. The probability of winning is

$$Pr(b_i) = Pr(b_i < y) = Pr(s^{-1}(b_i) < c_{(M)}) = 1 - Pr(s^{-1}(b_i) > c_{(M)}), \tag{1}$$

where y is the M lowest bid of the other $N-1$ bidders, $c_{(M)}$ is the M order statistics among the $N-1$ private cost of other bidders. The probablity of winning with a cost c is

$$G(c) \quad = \quad \sum_{k=1}^{M} \begin{pmatrix} N-1 \\ k-1 \end{pmatrix} (1-F(c))^{N-k}(F(c))^{k-1}. \tag{2}$$

Therefore, individual i' expected profit is

$$\pi(c_i) = G(s^{-1}(b_i))(p_i - c_i), \tag{3}$$

where p_i is the lowest rejected bid and $p_i = b_{(M+1)}$, $b_{(k)}$ is the kth lowest bids and $G(\cdot)$ is calculated based on equation (2). According to equation (3), if one bids higher than the private cost c_i, the probability of winning decreases while the profit conditional on winning is still the same. If one bids lower than the private cost, when $b_i < c_i < p_i$, the bidder still wins the bid with the same profit; if $b_i < p_i < c_i$, the bidder can underbid to to win the auction but will result in a negative profit; if $p_i < b_i < c_i$, the bidder will lose the bid. Therefore, under the UPR auction with multiple awards, an equilibrium bidding strategy is (after dropping subscript i)

$$s^{UPR}(c) = c. \tag{4}$$

[1] I also assume a random tie breaking rule in case there are multiple bids end at the same price.

2.2 First Price Reverse Auction

Under the FPR auction with multiple awards, individual i' profit is

$$\pi(c_i) = G(s^{-1}(b_i))(p_i - c_i) = G(s^{-1}(b_i))(b_i - c_i), \tag{5}$$

where the winning probability is also determined by equation (2). The bidder i chooses b_i to maximize expected profit $\pi(c_i)$. The first order condition implies

$$G(s^{-1}(b_i)) - (b_i - c_i)g(s^{-1}(b_i))\frac{1}{s'(s^{-1}(b_i))} = 0. \tag{6}$$

A symmetric Nash equilibrium requires that $b_i = b(c_i)$ for all i, therefore, equation (6) can be simplified as

$$G(c_i) - (s(c_i) - c_i)g(c_i)\frac{1}{s'(c_i)} = 0. \tag{7}$$

Rearrange equation (7), we have

$$s' = (s(c_i) - c_i)\frac{g(c_i)}{G(c_i)}. \tag{8}$$

Therefore, the equilibrium bidding strategy $s^{FPR}(c_i)$ is determined by equation (8) and can be expressed as (after dropping subscript i)

$$s^{FRP}(c) = c + \frac{\int_c^{\bar{c}} G(\tilde{c})d\tilde{c}}{G(c)}, \tag{9}$$

which is higher than s^{UPR} almost everywhere except at $\bar{c}$, where $s^{FRP}(\bar{c}) = s^{URP}(\bar{c}) = \bar{c}$.

2.3 Generalized Second Price Reverse Auction

Under the GSPR auction, since M lowest bidders are chosen as the winners, the winning price is a function of the ranks. The probability of each rank needs to be calculated separately. To proceed, the probability that a bid b_i being the lowest is denoted by probability $G_1(s^{-1}(b_i))$, which is also the probability that zero of the values drawn by other bidders are less than $s^{-1}(b_i)$. Similarly, the probability that a bid b_i is ranked no higher than the second lowest (including the lowest and the second lowest position) is denoted by probability $G_2(s^{-1}(b_i))$, which is also the probability that at most one of the values drawn by other bidders are less than $s^{-1}(b_i)$. In general, we use $G_k(s^{-1})$ to denote the probability that at most $k-1$ of values drawn by other bidders are less than $s^{-1}(b_i)$.

Thus, the probability of being the kth lowest bid equals

$$Pr_k(b_i) = Pr(y_{(k-1)} < b_i < y_{(k)}) \tag{10}$$

when $k > 1$. When $k = 1$,

$$\begin{aligned} Pr_1(b_i) &= Pr(b_i < y_{(1)}) \\ &= 1 - Pr(s^{-1}(b_i) > c_{(1)}) \end{aligned} \tag{11}$$

where $y_{(k)}$ is the kth lowest bid of the other $N-1$ bidders, $c_{(k)}$ is the kth order statistics among the $N-1$ cost of other bidders. Therefore, in the GSPR auction with multiple awards, bidder i's expected profit function is

$$\pi(c_i) = \sum_{k=1}^{M} G_k(c_i)(b_{(k+1)} - c_i), \tag{12}$$

where $G_k(c)$ is

$$G_k(c) = \begin{pmatrix} N-1 \\ k-1 \end{pmatrix} (1-F(c))^{N-k}(F(c))^{k-1}. \tag{13}$$

Based on Edelman et al. (2007), it is easy to show that the GSPR auction is not incentive compatible and truthful bidding is not an equilibrium strategy under complete information. For example, assume there are five bidders with private cost profile $\{20, 17, 15, 10, 9\}$, if everyone bids her true cost, then the bid profile is also $\{20, 17, 15, 10, 9\}$. In this case, the profit for the lowest cost bidder (bidder 1) is $\pi_1 = b_{(2)} - c_1 = 10 - 9 = 1$, the profit for the bidder with the second lowest cost (bidder 2) is $\pi_2 = b_{(3)} - c_2 = 15 - 10 = 5$. It is clear that bidder 1 can increase her bid to 11 and receive a much higher profit. The new bid profile is $\{20, 17, 15, \mathit{11}, 10\}$ and the profit for two bidders are $\pi_1 = b_{(3)} - c_1 = 15 - 9 = 6$ and $\pi_2 = b_{(2)} - c_2 = 11 - 10 = 1$. Therefore, there is incentive for low cost bidders to overbid in order to get a higher winning price under complete information.

The profit function for the GSPR auction is quite complicated with incomplete information.

Under the GSPR with multiple awards, the equilibrium bidding strategy $s^{GSPR}(c_i)$ maximizes profit equation (12). Note that past research has used the local envy-free Nash equilibrium (LEFNE) to analyze the theoretical property of Generalized Second Price auction assuming complete information (Edelman et al., 2007) and tests its predictions using lab experiment method and computer simulations (Fukuda et al., 2013; Thompson and Leyton-Brown, 2017). In this paper, I attempt to apply the more standard Bayesian Nash equilibrium solution to set a common theoretical benchmark across all reverse auctions under incomplete information. Thus, under the GSPR auction, the equilibrium strategy, if exists, should satisfy,

$$s^{GSPR}(c) \quad = \quad c + \sum_{k=1}^{M} \gamma_s(c) \int_{c}^{\bar{c}} (c + s(\tilde{c}))(1 - F(\tilde{c}))^{N-k-1} f(\tilde{c}) d\tilde{c}. \tag{14}$$

where

$$r_s(c) \quad = \quad \frac{(k-1)(N-k)\frac{G_s(c)}{F(c)(1-F(c))^{N-k-1}}}{\sum_{t=1}^{M}(N-t)G_t(c)}. \tag{15}$$

See Appendix for step by step solutions. Below I solve the equilibrium strategy based on the parameters in the experiment and set up a theoretical benchmark for data analysis.

2.4 Equilibrium Bidding Strategy

In the experiment, individual cost followes a uniform distribution on the interval [5, 20]. The group size is 5 and there are two winning awards in each group. For the UPR auction, the equilibrium bidding strategy is

$$s^{UPR}(c) = c. \tag{16}$$

For the FPR auction, the equilibrium bidding strategy is

$$s^{FRP}(c) = c + \frac{\int_{c}^{20} G(\tilde{c}) d\tilde{c}}{G(c)}, \tag{17}$$

where

$$G(c) \quad = \quad \sum_{k=1}^{2} \begin{pmatrix} N-1 \\ k-1 \end{pmatrix} (1 - F(c))^{N-k} (F(c))^{k-1} \tag{18}$$

As a result, the equilibrium bidding strategy can be calculated according equations (17) and (18) for any bidder with a private cost $c \in [5, 20]$. Thus

$$s^{FPR}(c) = c + \frac{(20-c)(c+5)}{5c}. \tag{19}$$

For the GSPR auction, substitute $N = 5$ and $M = 2$ into equation (14). The equilibrium strategy, if exists (i.e., $s^{GSPR}(c)$ is strictly increasing (Gomes and Sweeney, 2014)), satisfies

$$s^{GSPR}(c) = c + \frac{3\int_c^{\bar{c}}(c + s(\tilde{c}))(20-\tilde{c})^2 d\tilde{c}}{(20-c)^2(c+5)}. \tag{20}$$

In the Appendix, I show that a symmetric equilibrium does not exist under the GSPR auction. Thus, empirical evidences are needed to assess the performance of GSPR.

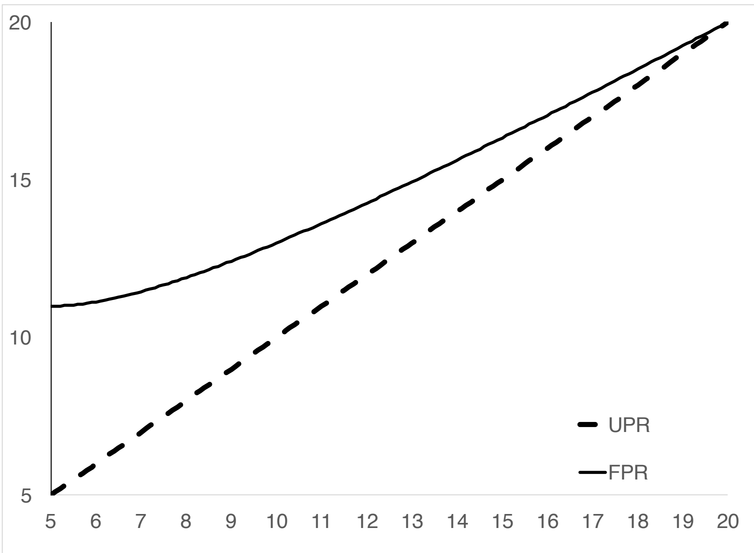


Figure 1: Equilibrium Bidding Function under the UPR and FPR Auctions

3 Experiment Procedure

Eighteen experimental sessions were conducted at the University of Connecticut (UCONN). A between subject design was implemented including six sessions of UPR auction, six sessions of FPR auction and six sessions of GSPR auction. All subjects were recruited through the UCONN Daily Digest, an online daily newsletter, where advertisements were placed to solicit volunteer participants in economic experiments. Experimental tasks were not specified in the advertisements to minimize the concern of self-selection effects.

Selected students would receive a confirmation email indicating the economic experiment involves multiple periods of decision making.[2] All subjects included in the experiment expressed a willingness to participate in economic experiments and they need to reply to the advertisement to be eligible for participation. Participants' names and email addresses were checked, before confirming their attendance, to ensure each subject participated only once in this sequence of experiments. Experiments were conducted through networked computer terminals using z-tree (Fischbacher, 2007). Inter-participant communications during the experiment were prohibited and subjects could not observe each others' choices.

Subjects who appeared on-time were told that they had already earned a $5 show-up fee. Experiment's instructions were read aloud while participants read along. Subjects were paid in cash once the experiment was finished. One experimental session usually lasted about fifty minutes yielding an average individual payoff around $16. There were ten subjects in each experimental session. They were further divided into two groups of equal size in the reverse auction game.

In the experiment, each treatment is replicated by six independent sessions. The between-subject experimental design avoids potential order effects and correlated observations arising from a within-subject design (Charness et al., 2012). A total of sixty subjects went through the same treatment, which enables us to detect about one-half of standard deviation change in the outcome variable with a power of 0.80 at a 0.05 significant level when only one outcome is observed for each subject. In the implementation, each subject is asked to make decisions in 50 periods, with a different private cost and group composition in each period. Therefore, the sample size and experimental design provides sufficient statistical power to detect treatment differences.

In each session, subjects were randomly assigned to one of two groups and were asked to make decisions. At the beginning of each decision period, subjects were told the number of awards, the group size, as well as their private cost to complete the contract. As noted before, private cost followed a uniform distribution on the interval [5, 20]. All the induced cost were rounded to the nearest tenth. In the experiment, the distribution of private cost, the number of awards and the number of subjects in each group were all public information. After each

[2] All experiment spots were reserved on a "first come, first serve" basis.

decision period, subjects would receive a different private cost and the group memberships were reshuffled. In each decision period, subjects were not allowed to bid more than \$30.[3] A total of 180 subjects participated in the experiment, producing 9,000 individual-period level observations, or 3,000 observations by 60 subjects in each treatment resulted from the between-subject design.

Table 1 below shows that summary statistics regarding key experimental parameters and outcomes. Results show that the FPR auction generates a significant higher bids compared to UPR auction (Wilcoxon rank-sum test, $z = 10.61, p < 0.001$) and GSPR auction (Wilcoxon rank-sum test, $z = 10.97, p < 0.001$), while there is no significant difference between the UPR auction and GSPR auction (Wilcoxon rank-sum test, $z = 0.110$, $p = 0.917$). In the Appendix, Figure A1 also plots the cumulative destiny function (CDF) for the bids distribution under the three auctions. Results show that UPR and GSPR follow a similar CDF (two sample Kolmogorov-Smirnov test, UPR and GSPR: $p = 0.062$; UPR and FPR: $p < 0.001$; FPR and GSPR: $p < 0.001$).

Table 1: Summary Statistics

Treatment	Number of Sessions	Cost Distribution	Awarded Units	Bids Mean (Median)	Min (Max)	Std. Dev.	No. of Obs.
UPR	6	U(5,20)	2	12.74 (12.6)	5(30)	4.46	3000
FPR	6	U(5,20)	2	13.97 (13.5)	5.1(30)	3.96	3000
GSPR	6	U(5,20)	2	12.73 (12.5)	5(30)	4.37	3000

4 Experiment Results

This section summarizes the experimental results based on the equilibrium predictions, individual bidding strategy as well as the surplus and efficiency outcomes.

4.1 Equilibrium Predictions

In order to test the equilibrium predictions, I plot the bids against the equilibrium bidding strategy for UPR and FPR auctions in Figure 2. The horizontal axis represents individual's private cost, ranging from 5 to 20; the vertical axis presents the bid level, ranging from 5 to 30,

[3]In the equilibrium, the maximum bid is 20 experimental dollars. Results show that, in the actual experiment, this restriction only applies to situations where high cost individuals with a very low probability of winning occasionally bid the maximum allowable amount.

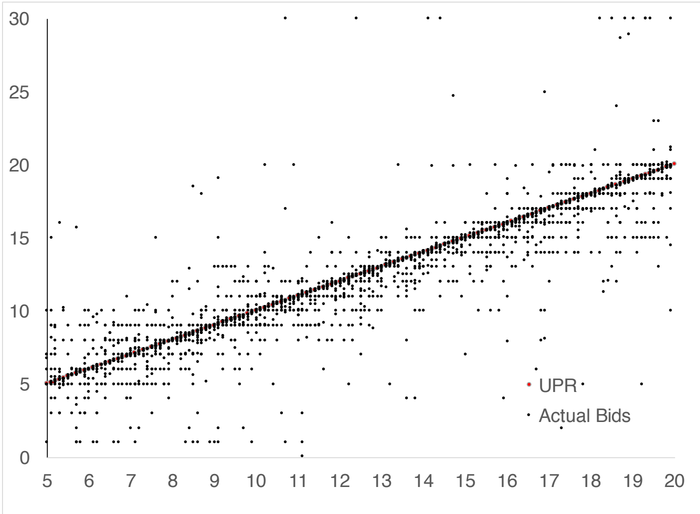
the maximum allowable bid in the experiment. Figure 2a shows that bid distribution and the equilibrium bidding strategy under the UPR auction. The majority of bids concentrate around the equilibrium biding strategy $s^{UPR}(c) = c$. A total number of 1613 bids (53.76%) are within a \$0.1 interval of the actual cost, an additional of 743 bids (24.77%) are within an \$1 of the actual cost. Only 644 bids (21.47%) deviate the private cost by more than \$1. During the last 25 periods, 786 bids (67.93%) are within a \$0.1 interval of the actual cost, an additional of 397 bids (26.47%) are within an \$1 of the actual cost. Overall, the equilibrium strategy provides a good prediction regarding the bid distribution under the UPR auction.

Figure 2b shows that bid distribution and the equilibrium bidding strategy under the FPR auction. The majority of bids are below the equilibrium strategy, which seems to place an upper bound on the bids. There are a very few bids (46 observations, or 1.53%) placed below one's private cost. Most of the underbids come from subjects with high private cost. Clearly, bidding lower than one's private cost is not an optimal strategy since one will either receive zero or negative profits, which is strictly worse off than just bidding one's true cost. In Figure 2c, the bid distribution under GSPR auction is similar to Figure 2a with the UPR auction. As noted before, the Wilcoxon rank-sum test results show that the bids are not significantly different between GSPR and FPR auctions ($z = 0.110, p = 0.917$). The experimental results are also consistent with Cason and Gangadharan (2005) where they find the most offers (bids) are within 2% of the cost, while the offers in the discriminative price auction (or FPR auction) are significantly greater (about 8% more) than the cost.

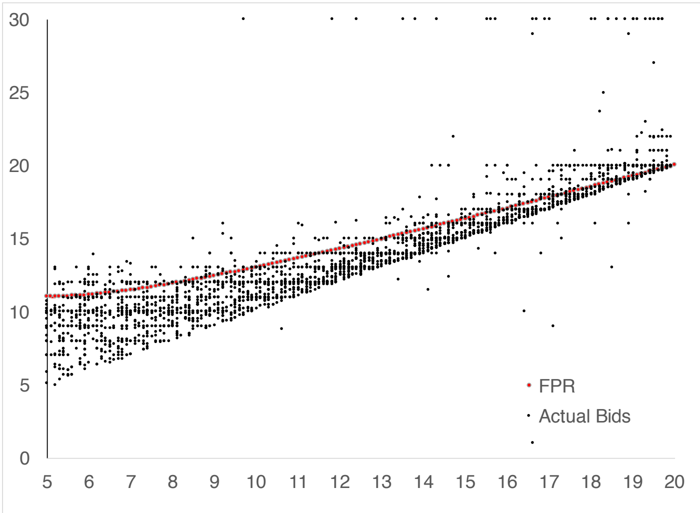
Figure 3a and 3b plot the mean and the median bid at each cost level (from 5 to 20 at a 0.1 interval) based on the experiment data for all 50 decision periods. Our results show that the mean and median bids are below the equilibrium bidding strategy for the UPR auction. The mean and mean bids are very close under the GSPR and UPR auctions, except at low cost range, where the mean and median are slightly higher under the GSPR auction. Interestingly, under the FPR auction, the bid pattern matches better with the equilibrium strategy when there is only one award. Figure 4 shows the mean bid for each cost level (from 5 to 20 at a 0.1 interval), suggesting that the equilibrium strategy with only one award (e.g., only the lowest bid will win) fits the experimental data noticeably better.[4] This result may imply that individuals underestimate the winning probability when more than one award is available and they may just

[4]In the Appendix, Figures A2, A3, A4 also show the median bid at each cost level for each experiment session.

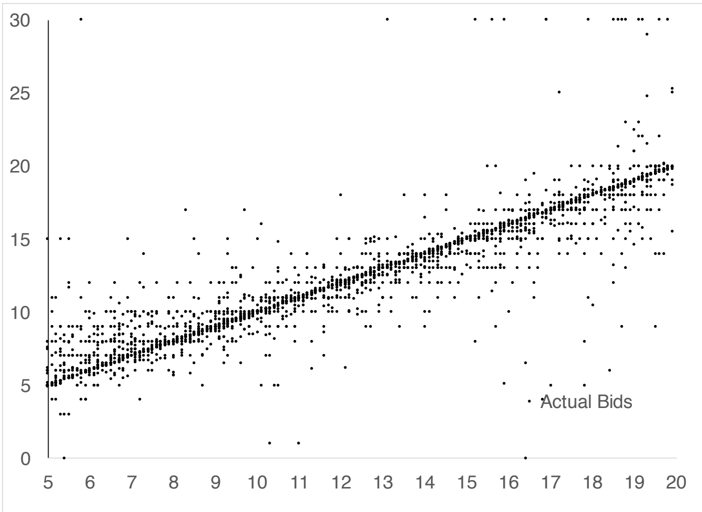
compete to win the auction as if there were only one award. The auctioneer, on the other hand, could potentially take advantage of this behavioral bias and extract a higher surplus from the bidders in a reverse auction.



(a) Uniform Price Reverse Auction

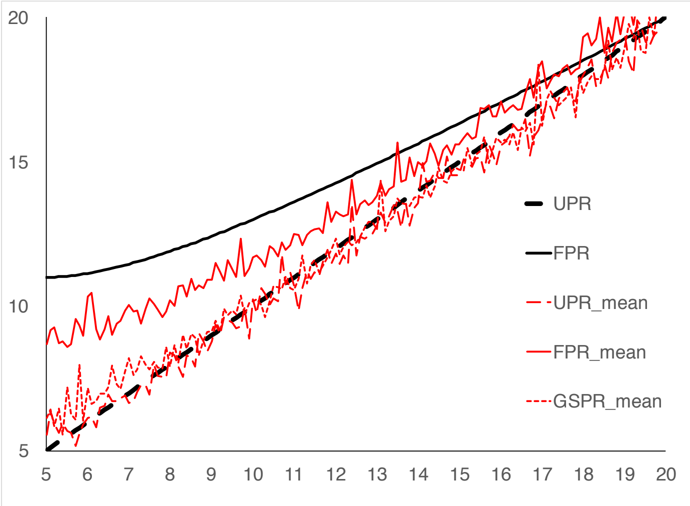


(b) First Price Reverse Auction

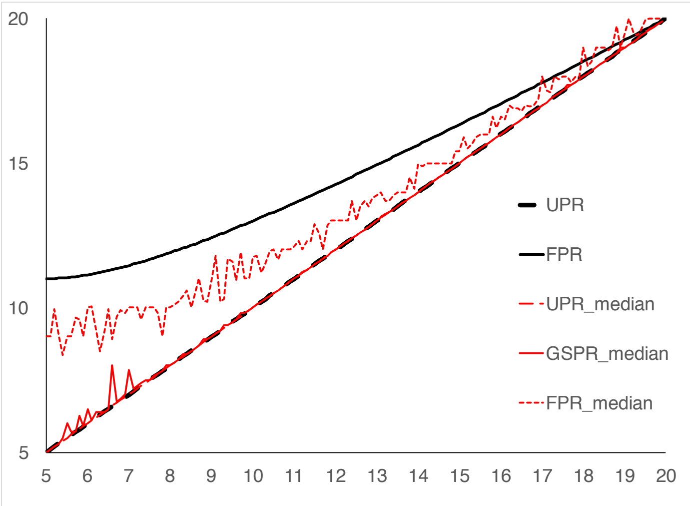


(c) Generalized Second Price Reverse Auction

Figure 2: Equilibrium Strategies and Bids Distribution.



(a) Equilibrium Strategies, Bid Mean



(b) Equilibrium Strategies, Bid Median

Figure 3: Equilibrium Strategies, Bids Mean and Median.

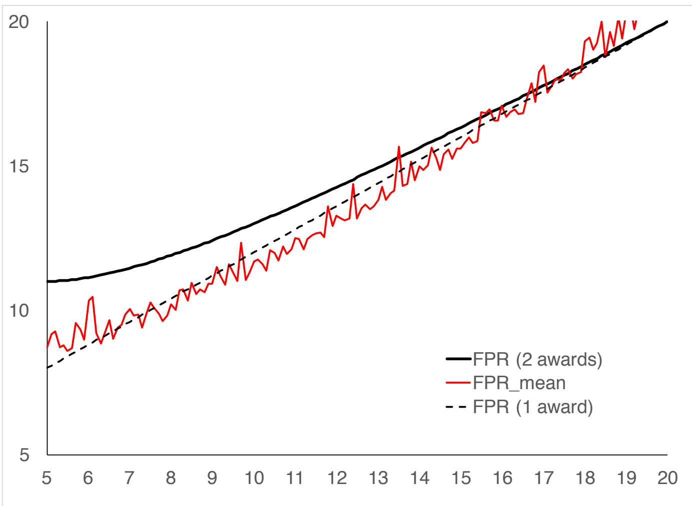


Figure 4: Equilibrium Strategies under First Price Reverse Auction

In general, experiment data reveals that the equilibrium strategy yields predictable patterns regarding individual bidding behaviors, even though under UPR auction, around 20% of bids still deviate from individual cost by more than $1. Also, the equilibrium bidding strategy seems to yield an upper bound on the overall bidding pattern for the FPR auction. Under the GSPR auction, when a Bayesian Nash equilibrium does not exist, the bid distribution seems to be close to the distribution under the UPR auction where the submitted bids are close to individual cost.

4.2 Individual Bidding Strategy

This section uses regression models to compare the differences across the three reverse auction mechanisms. The regression analysis is based on a two-factor random effects model including group and session specific factors (Marks and Croson, 1998). Specifically,

$$b_{it} = X_{it}\beta + \mu_i + v_t + \epsilon_{it}, \tag{21}$$

where b_{it} represents the bidding amount for individual i in period t, with the two random effects denoted by μ_i and v_t, respectively, and X_{it} is a set of regressors that may include treatment dummies, individual cost, as well as their interaction effects. For comparison purpose, ordinary least squares estimates are also included in the regression results.

Table 2 shows the regression results using data from all 50 experimental periods. Table 3 shows the regression results based on data only from the last 25 experimental periods. Comparing Tables 2 and 3, results are pretty consistent in terms of significance levels and coefficient magnitudes. Therefore, I use the regression results based on the last 25 periods for interpretations.

Based on Models (3) and (4) in Tables 3, on average, individuals significantly increase their bids in the FPR auction ($\beta_{FPR} = 1.846$, $p < 0.01$) compared to the baseline treatment UPR auction. Note that since the interaction term $FPR \times cost$ is negative significant ($p < 0.01$), individual with a low private cost is more likely to bid a higher amount in the FPR than in the UPR auction, which is consistent with the equilibrium strategy implied by equation (19). On average, when the cost increase by $1, the difference between UPR and FPR reduces by $0.195 as individuals balance the winning probability and expected profit under the FPR auction. Under the GSPR auction, according to Model (3), individuals do not significantly increase their bids compared to the UPR auction ($\beta_{GSPR} = 0.345$, $p = 0.112$). However, since the interaction term $GSPR \times cost$ is also negative significant ($p < 0.01$), the difference between UPR and GSPR will reduce by about $0.045 if the cost increases by $1.

Table 2: Regression Results, All Periods

	(1) OLS	(2) OLS	(3) R.E.	(4) R.E.
cost	0.908***	0.971***	0.907***	0.971***
	(0.00436)	(0.00744)	(0.00434)	(0.00742)
FPR	1.815***	3.658***	1.700***	3.519***
	(0.113)	(0.173)	(0.201)	(0.239)
GSPR	0.452***	0.865***	0.242	0.799***
	(0.113)	(0.172)	(0.201)	(0.239)
FPR×cost		-0.147***		-0.146***
		(0.0105)		(0.0105)
GSPR×cost		-0.0437***		-0.0447***
		(0.0106)		(0.0105)
Constant	0.978***	0.159	0.965***	0.174
	(0.163)	(0.178)	(0.152)	(0.169)
N	9000	9000	9000	9000
Adjusted R^2	0.8352	0.8389	N/A	N/A
log-likelihood	N/A	N/A	-17984.4	-17883.6

Notes: Standard errors in parentheses. Results are based on the bids from all 50 periods. Regression models (1) and (2) controls for session and period fixed effects. Models (3) and (4) are based on two-factor random effects model. Adjusted R^2 are calculated for models (1) and (2), log-likelihood statistics are calculated for model (3) and (4). * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

In the Appendix, I include a quadratic cost term to detect the possibility a non-linear relationship between the cost and the bids. Regression results in Table A1 implies a convex bidding function under the FPR auction, which is consistent with the equilibrium predictions.

Winning Bids Figure 5 shows the box plot of the bid distribution for each rank (position) under different reverse auction mechanisms using data from all 50 experimental periods. The black-lined boxes cover the interquartile range, the line in the box presents the median of the bids at each position, and the vertical line segments stretch to 5% and 95% percentile. Results show the FPR auction leads to a higher median bid compared to UPR and GSPR auctions at each position. Note that under the FPR auction, there is a large proportion of bids higher than $20, which may reflect subjects' aggressive bidding choice in face of a small winning probability.

Table 4 shows the impacts of auction treatments on the bid distribution at each rank, based

Table 3: Regression Results, Last 25 Periods

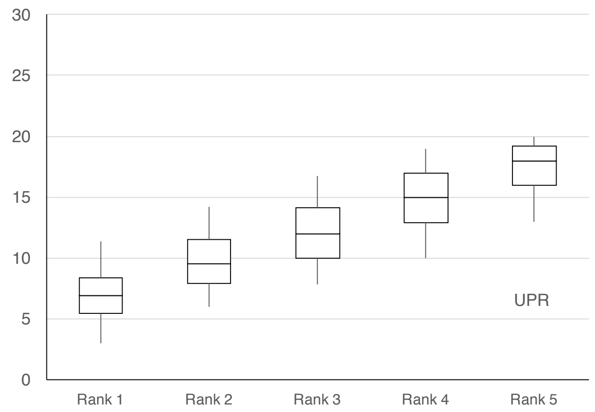
	(1) OLS	(2) OLS	(3) R.E.	(4) R.E.
cost	0.906***	0.990***	0.906***	0.990***
	(0.00596)	(0.0101)	(0.00594)	(0.0100)
FPR	1.830***	4.237***	1.846***	4.271***
	(0.154)	(0.232)	(0.214)	(0.277)
GSPR	0.765***	1.084***	0.345	1.082***
	(0.154)	(0.191)	(0.214)	(0.276)
FPR×cost		-0.194***		-0.195***
		(0.0143)		(0.0142)
GSPR×cost		-0.0573***		-0.0588***
		(0.0143)		(0.0142)
Constant	0.687***	-0.303	0.868***	-0.181
	(0.179)	(0.201)	(0.168)	(0.195)
N	4500	4500	4500	4500
Adjusted R^2	0.8352	0.8389	N/A	N/A
log-likelihood	-8800.1	-8704.1	-8840.3	-8744.0

Notes: Standard errors in parentheses. Results are based on the bids from the last 25 periods. Regression Models (1) and (2) controls for session and period fixed effects. Models (3) and (4) are based on two-factor random effects model. Adjusted R^2 are calculated for models (1) and (2), log-likelihood statistics are calculated for model (3) and (4). * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

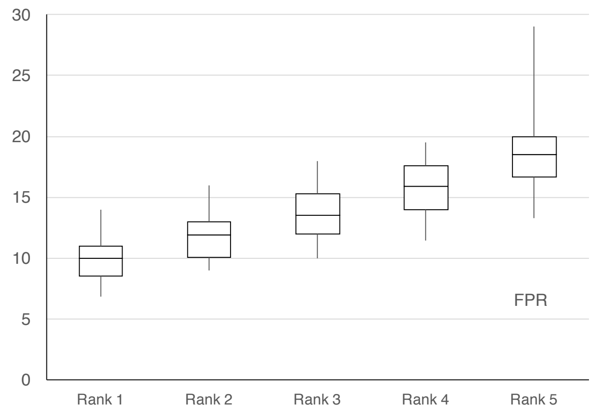
on both OLS and random effects regressions using all 50 periods data. Results shows that the FPR auction significantly increases the bids compared to the baseline treatment UPR for all positions. The GSPR auction only increases bid for the lowest position. There are no significant differences observed between UPR and GSPR at other positions. Table 5 shows the impacts of auction treatments on the bids distribution at each position using only the last 25 periods data. The FPR auction still significantly increases the bids compared to UPR for all positions and such difference increases for the lowest two positions during the last 25 periods. The difference between GSPR and UPR also becomes significant at the lowest position during the last 25 periods, which suggests the incentive to overbid under the GSPR auction among low cost bidders.

Deviations from the Cost Another way to analyze the data is to compare the individual bid deviations from their actual cost across treatments. Table 6 shows the regression results where the dependent variable is the difference between individual bid and cost from Model (1) to (4). The dependent variable is changed to the absolute difference between individual bid and cost from Model (5) to (8). The UPR auction is still treated as the baseline. According to Table 6, the difference between individual bid and cost under FPR auction is significantly higher compared to the UPR auction, while the difference and the absolute difference are not significant under GSPR in all random effects models. Apparently, FPR auction leads to a larger deviation from individual cost compared to UPR and GSPR, while the bid deviations are similar in UPR and GSPR.

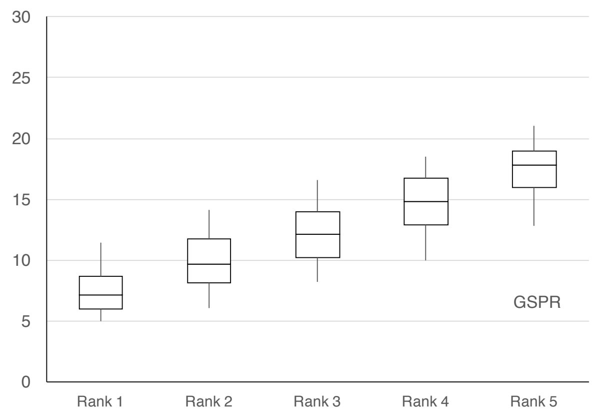
Figure 5: Bids Distribution by Relative Rank in a Group, All Periods



(a) Bids Distribution by Relative Rank in a Group, UPR



(b) Bids Distribution by Relative Rank in a Group, FPR



(c) Bids Distribution by Relative Rank in a Group, GSPR

Notes: Figures are based on the data collected from all 50 periods. The black-lined boxes show the interquartile range, the line in the box is the median, and the vertical line segments stretch to 5% and 95% percentile.

Table 4: Regression Results, Ranked Bids, All Periods

	(1) OLS/1	(2) R.E./1	(3) OLS/2	(4) R.E./2	(5) OLS/3	(6) R.E./3	(7) OLS/4	(8) R.E./4	(9) OLS/5	(10) R.E./5
FPR	2.157***	2.855***	1.971***	2.040***	1.603***	1.503***	0.986***	1.153***	0.946**	1.007**
	(0.314)	(0.266)	(0.332)	(0.172)	(0.365)	(0.166)	(0.365)	(0.163)	(0.457)	(0.396)
GSPR	0.402	0.607**	-0.222	0.239	-0.0550	0.105	-0.146	0.150	-0.190	0.0265
	(0.314)	(0.266)	(0.332)	(0.172)	(0.365)	(0.166)	(0.365)	(0.163)	(0.457)	(0.396)
Constant	6.626***	2.189***	9.424***	2.724***	12.34***	3.026***	14.75***	2.945***	17.22***	3.906***
	(0.428)	(0.226)	(0.453)	(0.153)	(0.497)	(0.155)	(0.498)	(0.171)	(0.624)	(0.499)
N	1800	1800	1800	1800	1800	1800	1800	1800	1800	1800
log-likeli.	N/A	-3481.5	N/A	-2824.1	N/A	-2730.1	N/A	-2672.2	N/A	-4250.4
Ad.R^2	0.2858	N/A	0.1557	N/A	0.0792	N/A	0.0379	N/A	0.0342	N/A

Notes: Standard errors in parentheses. The dependent variable is the bid in the lowest position in Model (1) and (2), second lowest position in Model (3) and (4), and so on. Odd numbered regression models control for the Session and Period fixed effects. Even numbered models are based on two-factor random effects model. Models are based on the data from the all 50 periods. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5: Regression Results, Ranked Bids, Last 25 Periods

	(1) OLS/1	(2) R.E./1	(3) OLS/2	(4) R.E./2	(5) OLS/3	(6) R.E./3	(7) OLS/4	(8) R.E./4	(9) OLS/5	(10) R.E./5
FPR	6.272***	3.140***	3.776***	2.270***	2.726***	1.561***	1.424***	1.222***	0.470	1.145***
	(0.459)	(0.327)	(0.474)	(0.221)	(0.528)	(0.183)	(0.523)	(0.179)	(0.652)	(0.395)
GSPR	1.358***	0.700**	0.284	0.421*	0.600	0.136	0.522	0.239	0.608	0.128
	(0.459)	(0.326)	(0.474)	(0.221)	(0.528)	(0.183)	(0.523)	(0.179)	(0.652)	(0.394)
Constant	6.202***	1.838***	9.392***	2.524***	11.91***	2.731***	14.08***	2.826***	17.41***	2.682***
	(0.496)	(0.291)	(0.512)	(0.202)	(0.571)	(0.192)	(0.565)	(0.222)	(0.704)	(0.606)
N	900	900	900	900	900	900	900	900	900	900
log-likeli.	N/A	-1757.3	N/A	-1378.1	N/A	-1360.7	N/A	-1366.7	N/A	-2073.6
Ad.R^2	0.3482	N/A	0.1768	N/A	0.0816	N/A	0.0370	N/A	0.0198	N/A

Notes: Standard errors in parentheses. The dependent variable is the bid in the lowest position in Model (1) and (2), second lowest position in Model (3) and (4), and so on. Odd numbered regression models control for the Session and Period fixed effects. Even numbered models are based on two-factor random effects model. Models are based on the data from the all 50 periods. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

4.3 Allocation of Realized Surplus and Efficiency

This section investigates the allocation of realized surplus between the bidders and the auctioneer (or the buyer). To provide a benchmark, I assume each completed contract is worth \$20 to the buyer (or $v_b = 20$). The surplus is calculated as the average profit an individual winner or the buyer receives from the completion of one contract. Note that even though the bidders' surplus is theoretically unaffected by the buyer's value toward the project, the percentage of surplus split between buyer and seller is determined by the buyer's value. Figure 6a shows the allocation of social surplus between the buyer and the bidders. The bidder's surplus is the

Table 6: Regression Results, Deviations from the Cost

	(1) OLS/diff	(2) OLS/diff	(3) R.E./diff	(4) R.E./diff	(5) OLS/|diff|	(6) OLS/|diff|	(7) R.E./|diff|	(8) R.E./|diff|
FPR	1.616***	1.699***	1.815***	1.700***	0.711***	0.747***	0.937***	0.747***
	(0.115)	(0.202)	(0.113)	(0.201)	(0.107)	(0.210)	(0.107)	(0.210)
GSPR	0.341***	0.251	0.321***	0.242	-0.210*	-0.0578	-0.0657	-0.0627
	(0.115)	(0.202)	(0.113)	(0.201)	(0.107)	(0.210)	(0.107)	(0.210)
cost			-0.0924***	-0.0928***			-0.0504***	-0.0501***
			(0.00436)	(0.00434)			(0.00412)	(0.00411)
Constant	-0.152	-0.188	0.978***	0.965***	0.802***	0.807***	1.418***	1.430***
	(0.157)	(0.143)	(0.163)	(0.152)	(0.147)	(0.149)	(0.154)	(0.157)
N	9000	9000	9000	9000	9000	9000	9000	9000
log-likeli.	N/A	-17930.0	N/A	-17984.4	N/A	-17433.1	N/A	-17487.0
Ad.R^2	0.1682	N/A	0.2080	N/A	0.0815	N/A	0.0965	N/A

Notes: Standard errors in parentheses. The dependent variable is the difference between bid and cost in Model (1) to (4). The dependent variable is the *absolute* difference between bid and cost in Model (5) to (8), Regression models (1), (2), (5) and (6) control for session and period fixed effects. Models (3), (4), (7) and (8) are based on two-factor random effects model. Models are based on the data from the all 50 periods. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

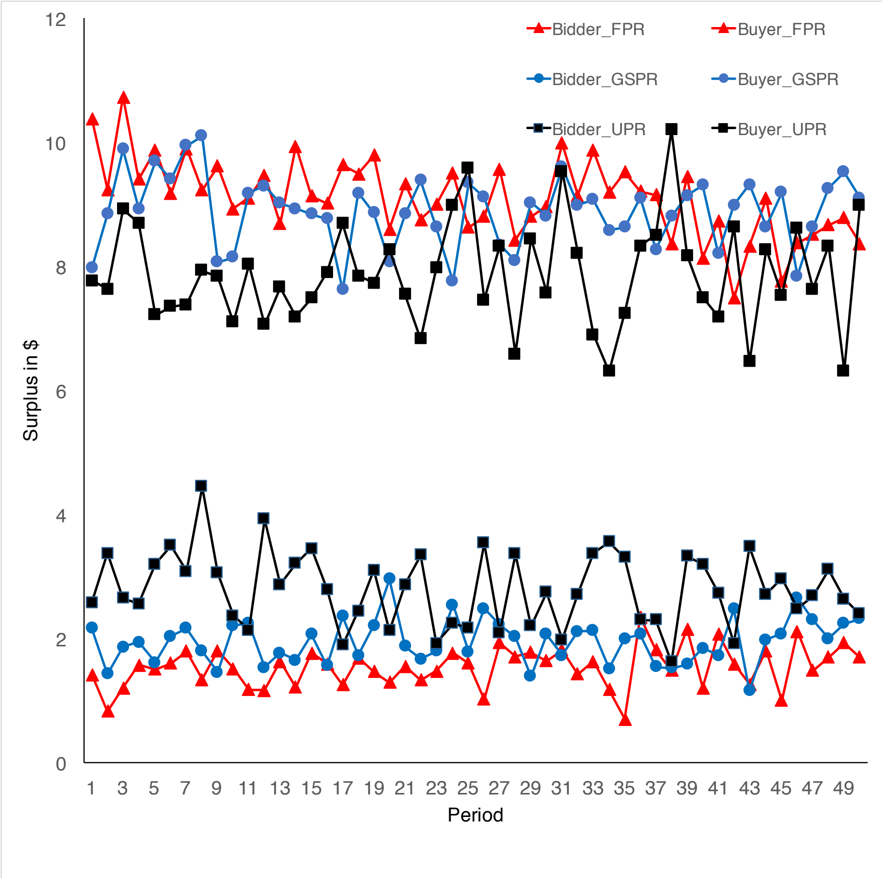
lowest under the FPR auction and the highest under the UPR auction. The buyer's surplus is the opposite. Under the FPR auction, the buyer obtains the highest surplus while under the UPR auction, the buyer obtains the lowest surplus.

Figure 6b shows the total realized social surplus under different auction mechanisms. From Figure 6b , the realized social surplus is approximately the same among three mechanisms. The efficiency level is calculated using the realized social surplus divided by the maximum possible social surplus. Figure 7 shows the efficiency levels across all periods under three auction rules. Results suggest that all auction rules generate a high efficiency level, with an average around 96.8% for FPR, 98.1% for GSPR and 98.1% for UPR. Inefficiency arises when the low cost bidders lose contracts to high cost bidders.

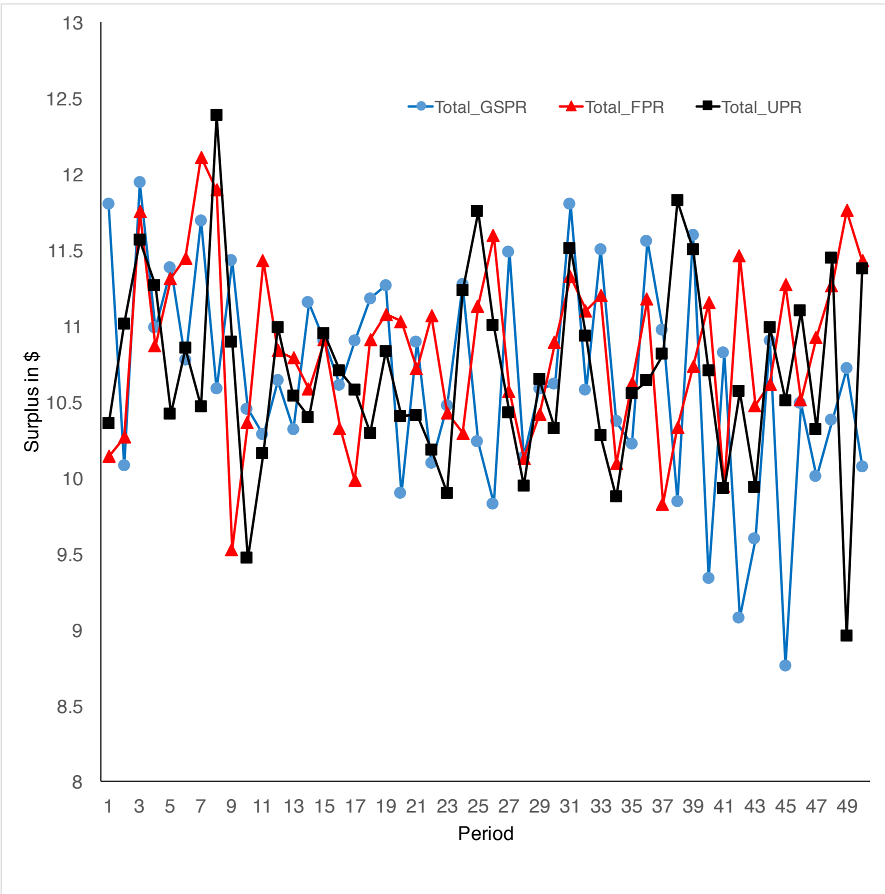
Regression analyses reveal that significant differences among three auction rules in terms of surplus and efficiency. Table 7 compares different reverse auction rules in terms of realized surplus (profit) for bidders and the buyer using OLS and two-factor random effects models. Results show that FPR auction yields the lowest bidders' profit and the highest auctioneer's profit. The GSPR auction is ranked between the other two auctions in both bidders' and auctioneer's profit. The UPR auction yields the highest bidders' profit and lowest auctioneer's profit among the three auction mechanisms. Based on Model (1) to (4), Table 7, the bidders' surplus is significantly reduced under the FPR and GSPR auctions, though the reduction in surplus is smaller during the last 25 periods. Based on Model (5) to (8), Table 7, the auctioneer's surplus is significantly higher under the FPR and GSPR auctions. During the last 25 periods, GSPR yields a higher auctioneer's surplus compared to FPR (Model (8), $\beta_{FPR} = 0.935 < \beta_{GSPR} = 0.973$), even though the surplus difference between FPR and GSPR is not significant at a 5% level.

Table 8 compares the auction efficiency calculated at the group level using realized social surplus divided the maximum social surplus. Regression results suggest the FPR auction lead to the lowest efficiency level, with a tendency to further decrease in later periods. The GSPR auction is not statistically different from UPR auction in terms of efficiency. Note the the auctioneer's profit and the efficiency can be potentially influenced by the auctioneer's value. In the Appendix, I also compare the influence of different buyer's values on the efficiency level and find the main conclusion is unchanged (Table A2 and A3).

The above results suggest important tradeoffs when choosing the reverse auction mechanism in a multi-award environment. The UPR and GSPR auctions are more efficient, while FPR and GSPR auctions yield a higher surplus for the auctioneer. From the perspective of bidders, UPR auction is preferred as it yields the highest bidders' surplus. From the valuation perspective, UPR auction is the preferred approach as bidders have the incentive to reveal their true private cost. The GSPR auction is also desirable from the valuation perspective as the bids are close to the true cost in the experiment. Thus, from the auctioneer's perspective, GSPR combines some nice properties of UPR and FPR auction and has the potential to become a popular method applied in practice.



(a) Allocation of Realized Social Surplus



(b) Total Social Surplus

Figure 6: Allocation of Realized Surplus and Total Realized Social Surplus.

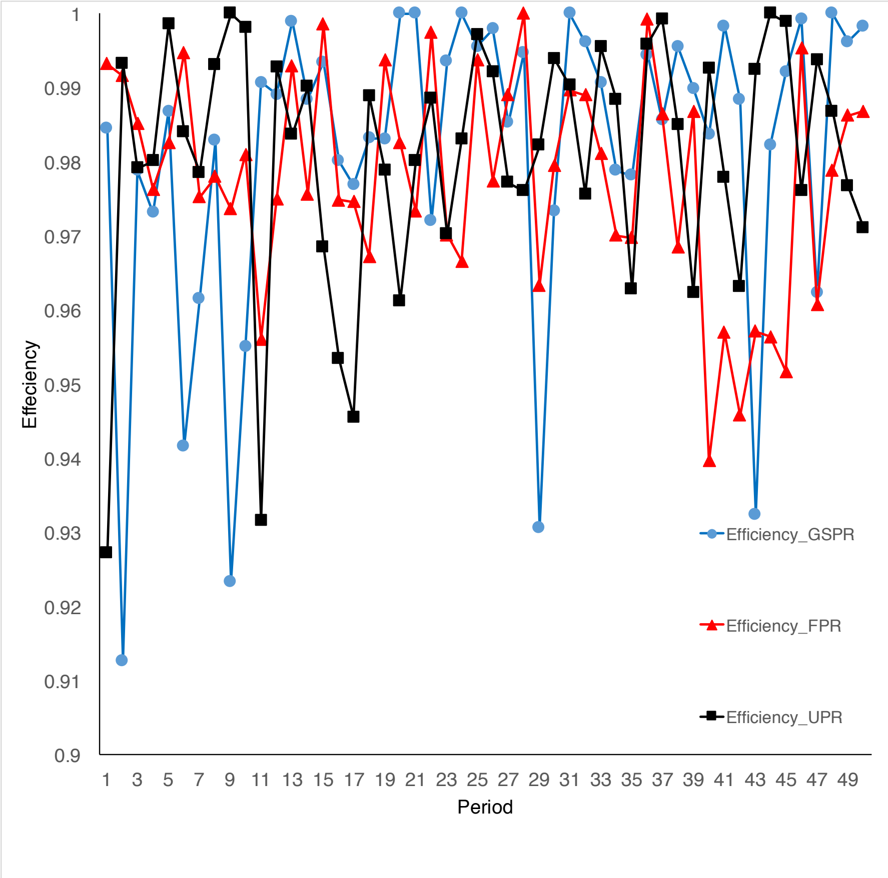


Figure 7: Realized Efficiency under Different Auction Mechanisms.

Table 7: Regression Results, Social Surplus

	(1) profit All 50	(2) profit All 50	(3) profit Last 25	(4) profit Last 25	(5) b.profit All 50	(6) b.profit All 50	(7) b.profit Last 25	(8) b.profit Last 25
FPR	-1.260***	-1.260***	-1.129***	-1.129***	1.227***	1.227***	0.935***	0.935***
	(0.0926)	(0.0920)	(0.133)	(0.132)	(0.137)	(0.137)	(0.198)	(0.197)
GSPR	-0.850***	-0.850***	-0.784***	-0.784***	1.009***	1.009***	0.973***	0.973***
	(0.0926)	(0.0920)	(0.133)	(0.132)	(0.137)	(0.137)	(0.198)	(0.197)
Constant	3.020***	2.803***	2.900***	2.751***	8.036***	7.877***	8.579***	7.891***
	(0.285)	(0.390)	(0.306)	(0.401)	(0.422)	(0.126)	(0.456)	(0.164)
N	1800	1800	900	900	1800	1800	900	900
log-likelihood	N/A	-3406.9	N/A	-1719.5	N/A	-4111.4	N/A	-2074.5
Adjusted R^2	0.3056	N/A	0.2948	N/A	0.0538	N/A	0.0410	N/A

Notes: Standard errors in parentheses. The dependent variable is average profit for a winning bidder from Model (1) to (4) and for the buyer from Model (5) to (6). Odd numbered regression models control for the Session and Period fixed effects. Even numbered models are based on two-factor random effects model. Model (1), (2), (5) and (6) are based on the data from all periods. Model (3), (4), (7) and (8) are based on the data from the last 25 periods. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 8: Regression Results, Efficiency Level

	(1) Efficiency All 50 Periods	(2) Efficiency All 50 Periods	(3) Efficiency Last 25 Periods	(4) Efficiency Last 25 Periods
FPR	-0.0120***	-0.0120***	-0.0264***	-0.0264***
	(0.00454)	(0.00454)	(0.00759)	(0.00759)
GSPR	0.000531	0.000531	0.00122	0.00122
	(0.00454)	(0.00454)	(0.00759)	(0.00759)
Constant	0.977***	0.981***	0.984***	0.983***
	(0.0140)	(0.00589)	(0.0175)	(0.00946)
N	1800	1800	900	900
log-likelihood	N/A	2016.2	N/A	853.9
Adjusted R^2	0.0287	N/A	0.0592	N/A

Notes: Standard errors in parentheses. The dependent variable efficiency is calculated at the group level. Regression models (1) and (3) control for the Session and Period fixed effects. Models (2) and (4) are based on two-factor random effects model. Model (1) and (2) are based on the data from all periods. Model (3) and (4) are based on the data from the last 25 periods. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

5 Conclusion

This research expands the current design in the reverse auction mechanisms by exploring the performance of three reverse auction rules when multiple homogenous contracts are awarded to different bidders. A series of lab experiment results suggest noticeable differences among the three reverse auction approaches in terms of bid distribution, surplus allocation and achieved efficiency. The First Price Reverse auction leads to the highest bids on average, while Uniform Price Reverse auction leads to the lowest bids that are closer to individual private cost. Theoretically, a symmetric Bayesian Nash equilibrium may not exist in the GSPR auction. Empirically, the GSPR auction can achieve a very high efficiency level similar to the UPR auction and significantly reduce the auctioneer's cost, as well as yield the auctioneer a surplus level similar to the FPR auction. Furthermore, the GSPR auction performs well from a valuation perspective as the observed bids well approximate individual true cost.

The ordinary ("forward") Generalized Second Price auctions have been used widely by search engines to sell online advertisements. This paper first applies the Generalized Second Price auctions in a reverse auction context and compares its performance with other commonly used reverse auction formats in a multi-award environment. As a first step, the ratio of awarded units and the total number of units is fixed in the experiment. Future experiments can vary the number of units awarded in the experiment and investigate the effects of winning probability on observed bidding behaviors. Furthermore, future studies could explore the possibility of designing the winning price as a function of others' bids as well as the winning position, similar to the differentiated click-rate in the sponsored search auctions.

Note that in a reverse or procurement auction, the auctioneer often has different targets. Similar to this study, the auctioneer may have a pre-defined objective such as purchasing a certain unit of goods from the sellers, then the auctioneer's objective might be minimizing the total cost or maximizing the total benefit (Schilizzi and Latacz-Lohmann, 2007). Alternative, the auctioneer may face a finite budget, then the objective is to maximize the total number of units that the auctioneer can purchase in a reverse auction market (Cason et al., 2003; Cason and Gangadharan, 2005). Future research can compare the empirical performance of different reverse auctions when the auctioneer has different objectives.

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Appendix

A Equilibrium Strategy in GSPR

Following Gomes and Sweeney (2014), denote the bidder i's (with a private cost c) expected payment in equilibrium by $E(p(c))$, by the Revelation Principle, the $E(p(c))$ has to satisfy

$$c \in \underset{\hat{c}}{argmax} E(p(\hat{c})) - \sum_{k=1}^{M} G_k(\hat{c})c. \tag{A1}$$

According to the Integral-form Envelop Theorem (Milgrom and Segal, 2002), equation (A1) implies that,

$$E(p(c)) - \sum_{k=1}^{M} G_k(c)c = \sum_{k=1}^{M} \int_c^{\bar{c}} G_k(\tilde{c})d\tilde{c} + E(p(\hat{c})), \tag{A2}$$

where $E(p(\hat{c})) = 0$ as the highest cost bidder expects to receive zero payment with a zero probability of winning the auction. According to the definition of GSPR,

$$\begin{aligned} E(p(c)) &= E(b_{(k+1)}) \\ &= \textstyle\sum_{k=1}^{M} G_k(c)E(s(c_{(k+1:N)})|c_{(k:N)} \leq c \leq c_{(k+1:N)}) \\ &= \textstyle\sum_{k=1}^{M} G_k(c)E(s(c_{(1:N-k)})|c \leq c_{(1:N-k)}) \\ &= \textstyle\sum_{k=1}^{M} G_k(c) \int_c^{\bar{c}} s(\tilde{c}) \frac{(N-k)(1-F(\tilde{c}))^{N-k-1}f(\tilde{c})}{(1-F(\tilde{c}))^{N-k}} d\tilde{c}, \end{aligned} \tag{A3}$$

Based on equations (A2) and (A3)

$$\sum_{k=1}^{M} G_k(c)c + \sum_{k=1}^{M} \int_c^{\bar{c}} G_k(\tilde{c})d\tilde{c} = \sum_{k=1}^{M} G_k(c) \int_c^{\bar{c}} s(\tilde{c}) \frac{(N-k)(1-F(\tilde{c}))^{N-k-1}f(\tilde{c})}{(1-F(c))^{N-k}} d\tilde{c}. \tag{A4}$$

where

$$G_k(c) = \begin{pmatrix} N-1 \\ k-1 \end{pmatrix} (1-F(c))^{N-k}(F(c))^{k-1}. \tag{A5}$$

Differentiating w.r.t. c on both sides,

$$\begin{aligned}
\sum_{k=1}^{M} \frac{dG_k(c)}{dc} c &= -s(c)\frac{f(c)}{1-F(c)} \sum_{k=1}^{M} (N-k) G_k(c) \\
&+ \sum_{k=1}^{M} \frac{\frac{dG_k(c)}{dc}(1-F(c))^{N-k} + G_k(c)(N-K)(1-F(c))^{N-k-1})f(c)}{(1-F(c))^{2N-2k}} \int_c^{\bar{c}} s(\tilde{c})(N-k)(1-F(\tilde{c}))^{N-k-1} f(\tilde{c}) d\tilde{c} \\
&= -s(c)\frac{f(c)}{1-F(c)} \sum_{k=1}^{M} (N-k) G_k(c) \\
&+ \sum_{k=1}^{M} \left(\frac{dG_k(c)}{dc} \frac{N-k}{(1-F(c))^{N-k}} + G_k(c) \frac{(N-k)^2 f(c)}{(1-F(c))^{N-k+1}} \right) \int_c^{\bar{c}} s(\tilde{c})(1-F(\tilde{c}))^{N-k-1} f(\tilde{c}) d\tilde{c}
\end{aligned} \tag{A6}$$

Differentiating equation (A5),

$$\begin{aligned}
\frac{\partial G_k(c)}{\partial c} &= \begin{pmatrix} N-1 \\ k-1 \end{pmatrix} f(c)[1-F(c))^{N-k-1} F^{k-2}(c)[(k-1)(1-F(c)) - (N-k)F(c)] \\
&= \frac{G_k(c)}{(1-F(c))F(c)} f(c)[(k-1)(1-F(c)) - (N-k)F(c)] \\
&= (k-1)\frac{f(c)}{F(c)} G_k(c) - (N-k)\frac{f(c)}{1-F(c)} G_k(c)
\end{aligned} \tag{A7}$$

Combining equations (A6) and (A7),

$$\begin{aligned}
& \sum_{k=1}^{M} \left((k-1)\frac{f(c)}{F(c)} G_k(c)c - (N-k)\frac{f(c)}{1-F(c)} G_k(c)c \right) \\
=\ & -s(c)\frac{f(c)}{1-F(c)} \sum_{k=1}^{M} (N-k) G_k(c) \\
+\ & \sum_{k=1}^{M} \left((k-1)(N-k)\frac{f(c)}{F(c)(1-F(c))^{N-k}} G(c) \right) \int_c^{\bar{c}} s(\tilde{c})(1-F(\tilde{c}))^{N-k-1} f(\tilde{c}) d\tilde{c}
\end{aligned} \tag{A8}$$

Rewrite the above equation,

$$\begin{aligned}
& \sum_{k=1}^{M} (N-k)\frac{f(c)}{1-F(c)} G_k(c)(s(c)-c) \\
=\ & -\sum_{k=1}^{M} (k-1)\frac{f(c)}{F(c)} G_k(c)c \\
+\ & \sum_{k=1}^{M} \left((k-1)(N-k)\frac{f(c)}{F(c)(1-F(c))^{N-k}} G(c) \right) \int_c^{\bar{c}} s(\tilde{c})(1-F(\tilde{c}))^{N-k-1} f(\tilde{c}) d\tilde{c}.
\end{aligned} \tag{A9}$$

Therefore,

$$s(c) = c + \sum_{k=1}^{M} \gamma_s(c) \int_c^{\bar{c}} (c + s(\tilde{c}))(1-F(\tilde{c}))^{N-k-1} f(\tilde{c}) d\tilde{c}. \tag{A10}$$

where

$$r_s(c) = \frac{(k-1)(N-k)\frac{G_s(c)}{F(c)(1-F(c))^{N-k-1}}}{\sum_{t=1}^{M}(N-t)G_t(c)}. \tag{A11}$$

Specifically, when $N = 5$, $M = 2$, equation (A10) implies

$$\frac{(s(c)-c)(4G_1(c)+3G_2(c))F(c)(1-F(c))^2}{3G_2(c)} = \int_c^{\bar{c}} (c+s(\tilde{c}))(1-F(\tilde{c}))^2 f(\tilde{c})d\tilde{c}. \tag{A12}$$

or

$$(s(c)-c)\frac{(1-F(c))^2(1+2F(c))}{3} = \int_c^{\bar{c}} (c+s(\tilde{c}))(1-F(\tilde{c}))^2 f(\tilde{c})d\tilde{c}. \tag{A13}$$

$$\int_c^{\bar{c}} (c+s(\tilde{c}))(20-\tilde{c})^2 d\tilde{c} = (s(c)-c)\frac{(20-c)^2(c+5)}{3}. \tag{A14}$$

Rewrite the above equation as

$$s(c) = c + \frac{3\int_c^{\bar{c}}(c+s(\tilde{c}))(20-\tilde{c})^2 d\tilde{c}}{(20-c)^2(c+5)}. \tag{A15}$$

Non-existence of Symmetric Equilibrium Take the first order derivative with respect to c,

$$s'(c) = 1 + \frac{3((20-c)-(c+s(c))(20-c)^2)-(s(c)-c)(3c^2-7c+200)}{(20-c)^2(c+5)}. \tag{A16}$$

When $s(c) > 0$ and $c \in [5, 20]$, according to equation (A16), we can show that $s' < 0$, which violates the requirement that $s'(c)$ must be strictly increasing for an efficient equilibrium, and a symmetric equilibrium does not exist in our case (Caragiannis et al., 2011; Gomes and Sweeney, 2014).

B Additional Regression Results and Figures

Table A1: Regression Results, Last 25 Periods, Quadratic Cost

	(1) R.E.	(2) R.E.	(3) R.E.	(4) R.E.
cost	0.510***	0.961***	0.572***	0.691***
	(0.0381)	(0.00705)	(0.0376)	(0.0456)
cost^2	0.0159***		0.0156***	0.0108***
	(0.00151)		(0.00148)	(0.00181)
FPR	1.675***	7.147***	3.706***	5.667***
	(0.196)	(0.427)	(0.248)	(0.492)
FPR×cost		-0.792***	-0.163***	-0.522***
		(0.0651)	(0.0122)	(0.0789)
FPR×cost^2		0.0253***		0.0145***
		(0.00258)		(0.00314)
Constant	3.211***	0.359**	2.495***	1.839***
	(0.246)	(0.143)	(0.248)	(0.285)
N	4500	4500	4500	4500
log-likelihood	-8786.6	-8706.2	-8698.9	-8688.4

Notes: Standard errors in parentheses. Results are based on the bids from last 25 periods and the Uniform Price and Generalized Second Price Reverse Auction. Regression models (1) to (4) are based on a two-factor random effects model. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A2: Regression Results, Efficiency Level, Buyer Value 15

	(1) Efficiency All 50 Periods	(2) Efficiency All 50 Periods	(3) Efficiency Last 25 Periods	(4) Efficiency Last 25 Periods
FPR	-0.00489***	-0.00489***	-0.0108***	-0.0108***
	(0.00178)	(0.00178)	(0.00297)	(0.00296)
GSPR	0.000174	0.000174	0.000273	0.000273
	(0.00178)	(0.00178)	(0.00297)	(0.00296)
Constant	0.991***	0.993***	0.994***	0.994***
	(0.00550)	(0.00227)	(0.00685)	(0.00366)
N	1800	1800	900	900
log-likelihood	N/A	3697.7	N/A	1699.9
Adjusted R^2	0.0283	N/A	0.0601	N/A

Notes: Standard errors in parentheses. The dependent variable efficiency is calculated at the group level. Regression models (1) and (3) control for the Session and Period fixed effects. Models (2) and (4) are based on two-factor random effects model. Model (1) and (2) are based on the data from all periods. Model (3) and (4) are based on the data from the last 25 periods. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A3: Regression Results, Efficiency Level, Buyer Value 25

	(1) Efficiency All 50 Periods	(2) Efficiency All 50 Periods	(3) Efficiency Last 25 Periods	(4) Efficiency Last 25 Periods
FPR	-0.00264***	-0.00264***	-0.00587***	-0.00587***
	(0.000962)	(0.000961)	(0.00159)	(0.00159)
GSPR	0.0000909	0.0000909	0.000132	0.000132
	(0.000962)	(0.000961)	(0.00159)	(0.00159)
Constant	0.995***	0.996***	0.997***	0.997***
	(0.00296)	(0.00122)	(0.00368)	(0.00196)
N	1800	1800	900	900
log-likelihood	N/A	4810.4	N/A	2258.5
Adjusted R^2	0.0279	N/A	0.0600	N/A

Notes: Standard errors in parentheses. The dependent variable efficiency is calculated at the group level. Regression models (1) and (3) control for the Session and Period fixed effects. Models (2) and (4) are based on two-factor random effects model. Model (1) and (2) are based on the data from all periods. Model (3) and (4) are based on the data from the last 25 periods. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

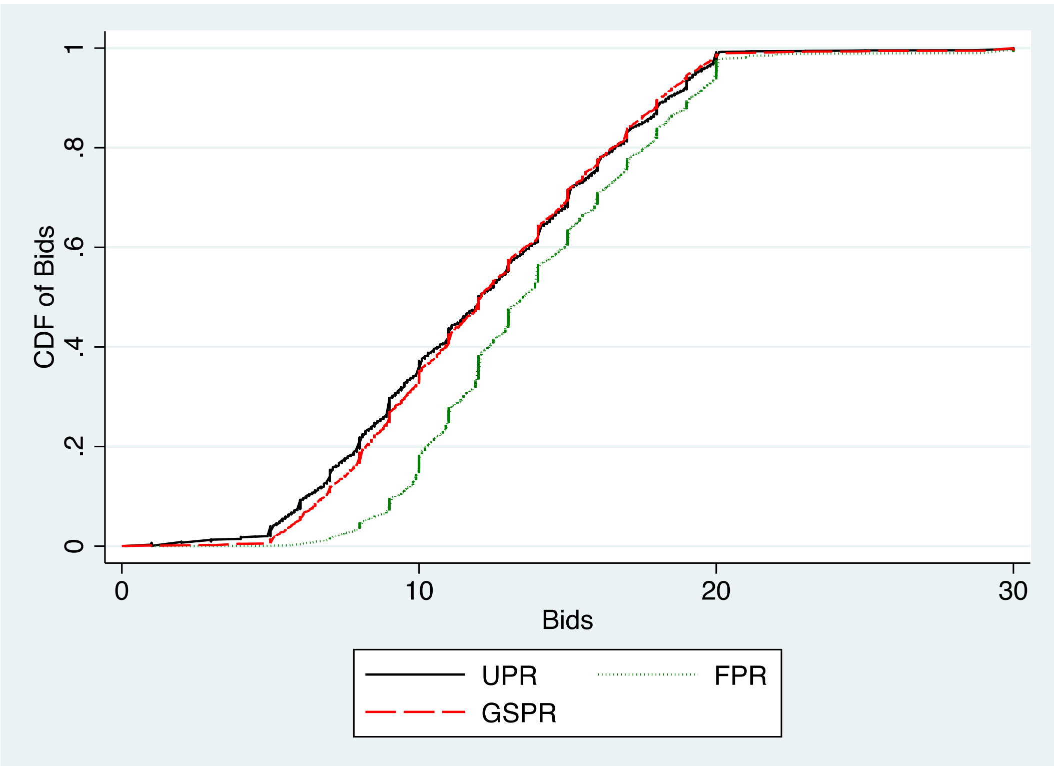


Figure A1: Cumulative Bids Distribution under Different Reverse Auction Mechanisms.

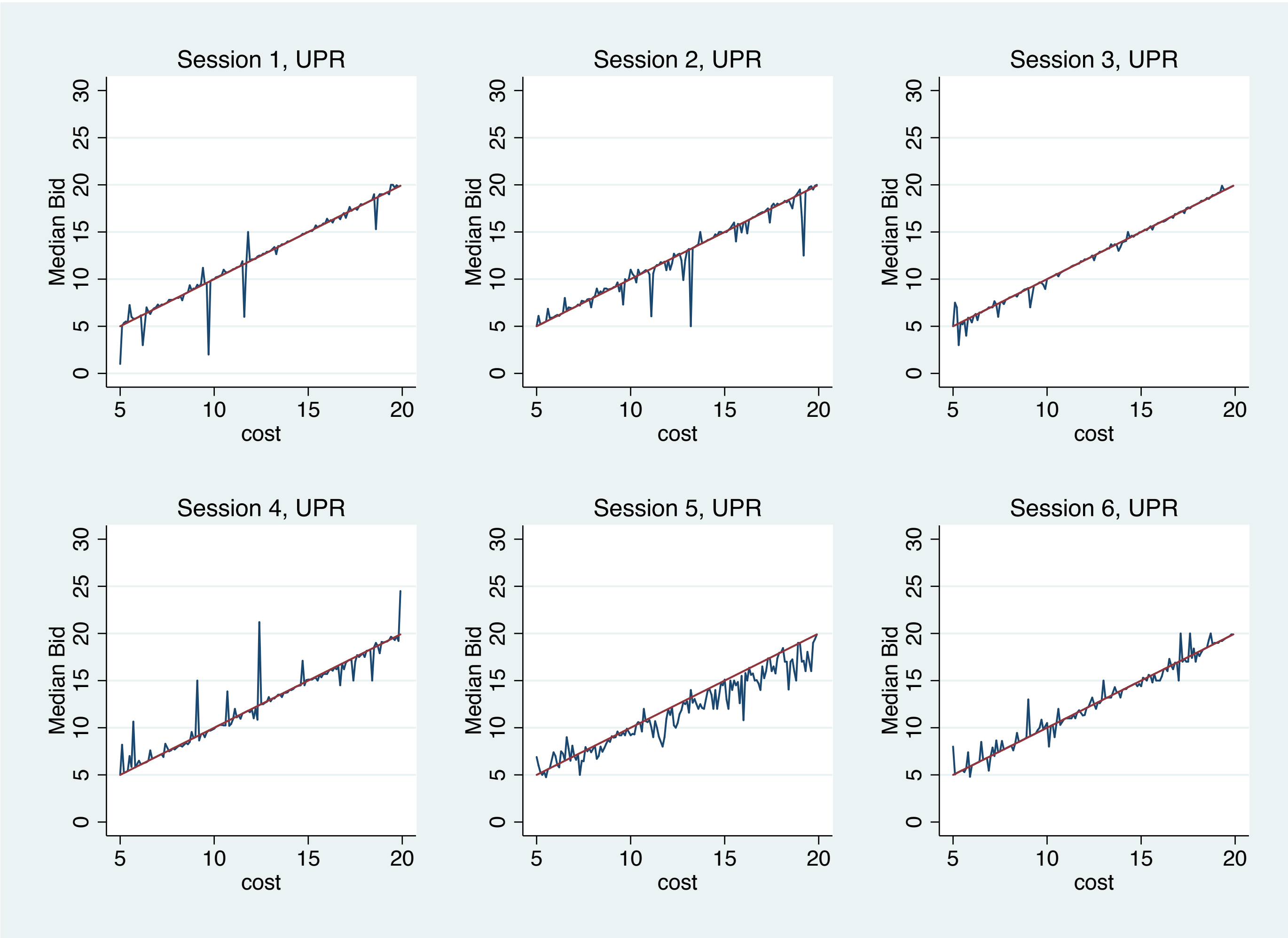


Figure A2: Session Specific Bid Median, UPR

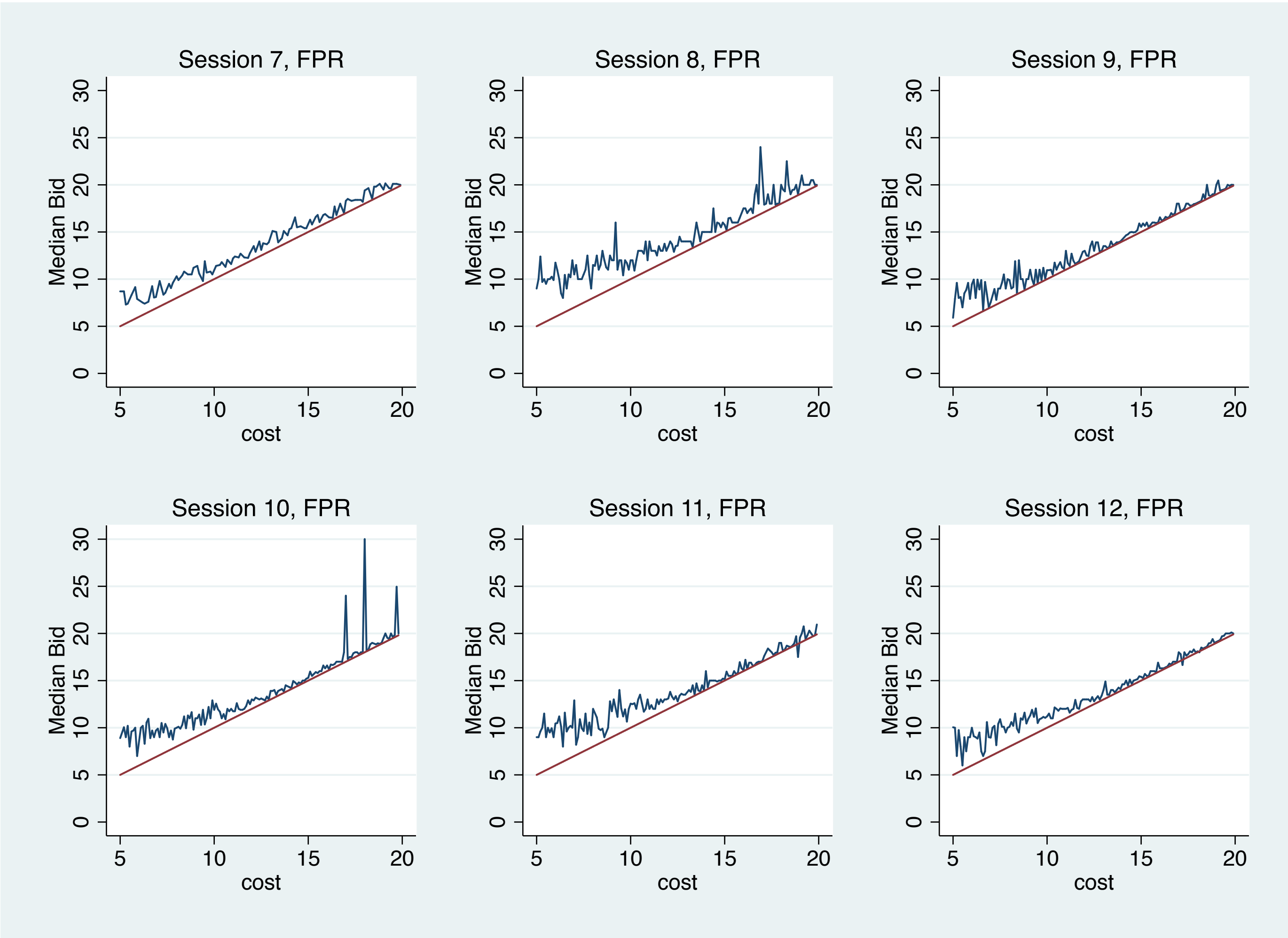


Figure A3: Session Specific Bid Median, FPR

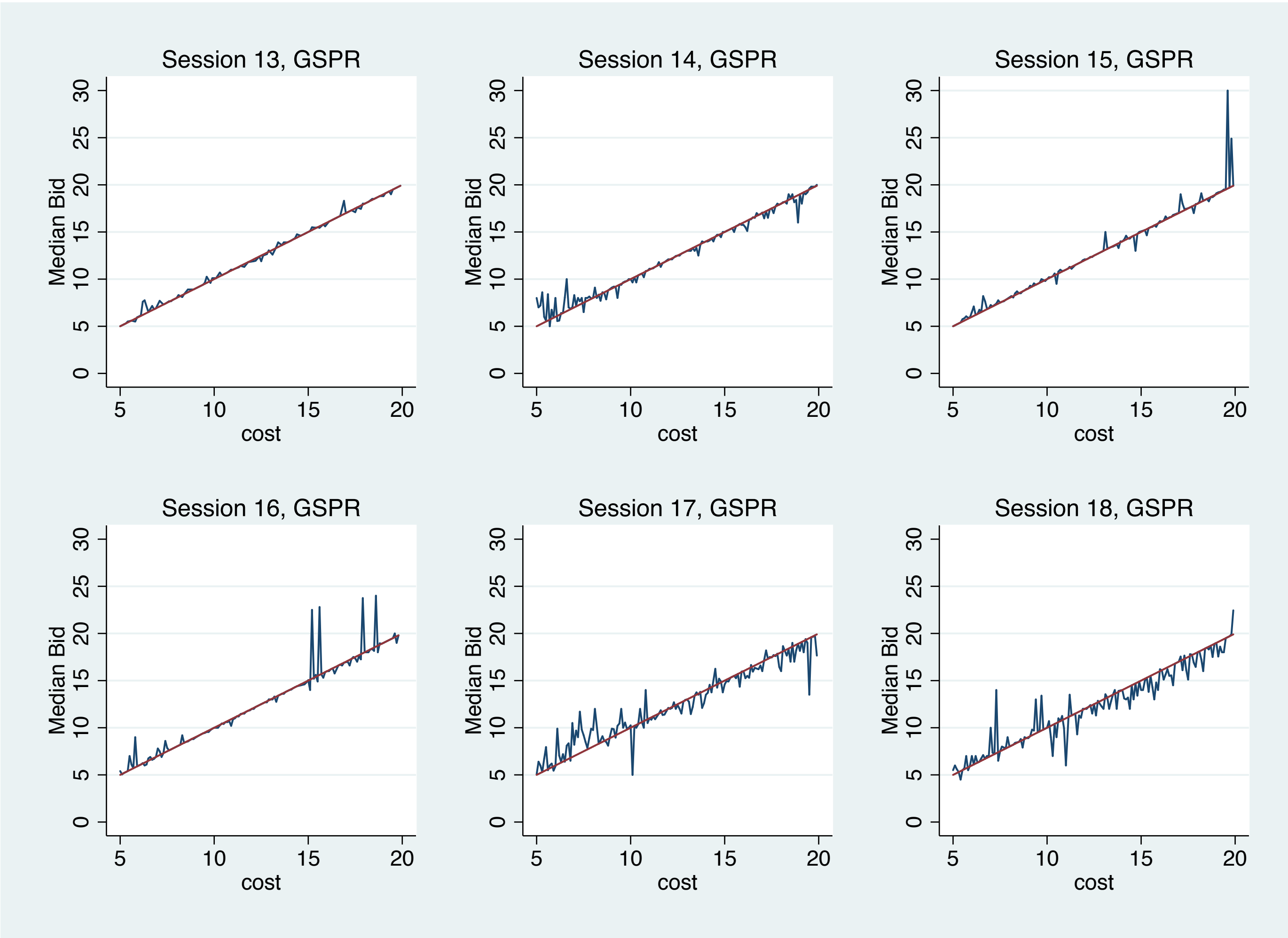
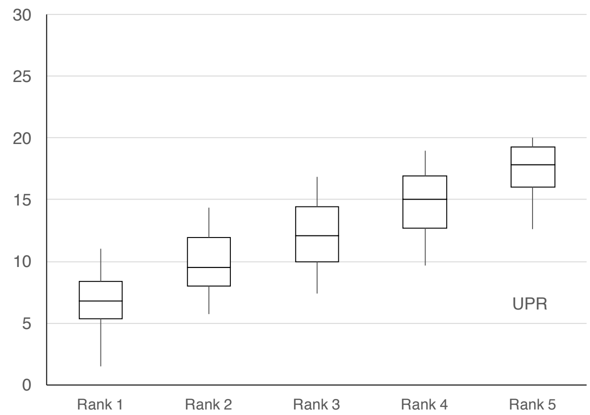
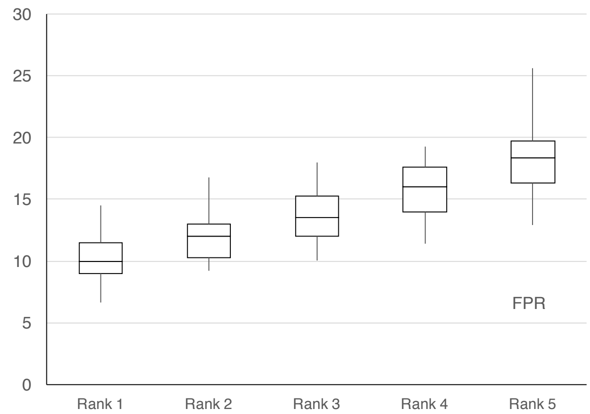


Figure A4: Session Specific Bid Median, GSPR

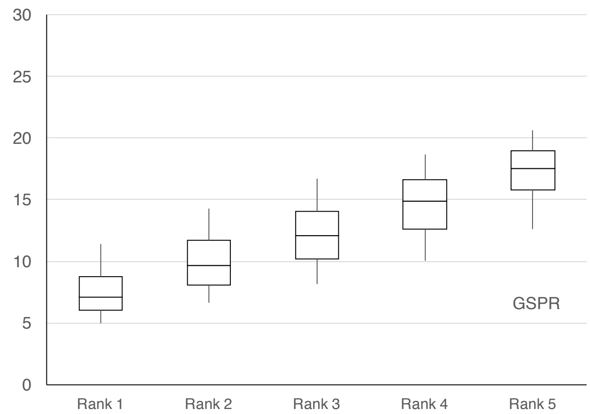
Figure A5: Bids Distribution by Relative Rank in a Group, Last 25 Periods



(a) Bids Distribution by Relative Rank in a Group, UPR, Last 25 Periods



(b) Bids Distribution by Relative Rank in a Group, FPR, Last 25 Periods



(c) Bids Distribution by Relative Rank in a Group, GSPR, Last 25 Periods

Notes: Figures are based on the data collected from the last 25 periods. The black-lined boxes show the interquartile range, the line in the box is the median, and the vertical line segments stretch to 5% and 95% percentile.