Lobbying and environmental policy instruments*

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May 6, 2019

Abstract

The choice of environmental policy instruments is analysed when the regulated firms have an option to join a lobby group that is able to influence the level of the chosen instrument. The choice of the instrument level is modeled with a three-stage game, where the firms decide to join the lobby group, the regulator decides the instrument level under the influence of the formed lobby, and finally, the firms individually choose their emissions. Hence the number of lobbying firms is endogenous, and the model characterizes the equilibrium number of lobbyists, instrument level and emissions. The results show that lobbying causes the aggregate emissions to be greater than in the social optimum. Although the aggregate emissions differ between the instruments, the regulator turns out to be indifferent between the instruments.

Keywords: Environmental policy; Emission tax; Lobbying; Political economy; Quantities

JEL codes: D72; H23; Q50

1 Introduction

Environmental policy instrument choice is modeled in a political economy framework in which the polluting firms can form a lobby group in order to influence the instrument choice. In the model the lobby group, if formed, influences the level of the given pollution control instrument, for example the level of the emission tax, but not directly regulator's decision which instrument to use. The political economy framework that is used here is based on the menu auction model of Bernheim and Whinston (1986), which has been applied in numerous papers in different fields. One of the first applications was the trade model of Grossman and Helpman (1994). Since then the menu

*__Acknowledgements:__ I acknowledge the funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie Grant Agreement N° 748066.

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auction or the common agency model has been applied to a wide array of topics including extensions to international trade models (Mitra, 1999), international agreements (Habla and Winkler, 2013; Marchiori *et al.*, 2017), forest policy (Eerola, 2004), environmental policy (Finkelshtain and Kislev, 1997) and public goods (Persson and Tabellini, 1994).[1] Most of these models presuppose the existence of the organised lobby groups.

The present model includes a pre-lobbying stage, where the firms choose to participate the lobby group. Only a handful of studies have investigated the endogenous formation of lobby groups using the common agency model. The first one is to the author's knowledge the trade model of Mitra (1999). In his model each sector can form a lobby group in order to influence sector's own product price and the prices of the other sectors. In principle, his model is a version of the model in Grossman and Helpman (1994), where a preliminary stage is added. At this stage every sector chooses to form or not a lobby group. Forming a lobby is costly, and every sector has a different cost. He lets the number of sectors and hence lobby groups be continuous, and the equilibrium number of lobby groups is defined as the one where the benefit for the sector of forming a lobby group is equal to the cost of forming the group. Contrary to Mitra (1999), in the present model there is only one sector and the number of firms who may join a single lobby group is a continuous variable.

Also Magee (2002), Le Breton and Salanie (2003), Krishna and Mitra (2005), Drazen *et al.* (2007), Bombardini (2008) and Kawahara (2014) have analysed lobbying with endogenous lobby formation. Bombardini (2008) is perhaps the most relevant for the present model, and she analyses the formation of lobby groups in a trade model that is also based on Grossman and Helpman (1994). Contrary to Mitra (1999), she lets the firms in each sector to choose whether or not to participate in a lobby group of that sector. The criterion for the equilibrium number of firms in a lobby group is that a firm is included in the group if (and only if) the joint payoff of the group is larger with that firm than without it. A similar criterion for the equilibrium number (or more precisely for the mass) of lobbyists is applied in the present model.

The focus here is not on trade policy, like in Mitra (1999) and Bombardini (2008) among others, but on the choice of environmental policy instruments. Environmental

[1] For a recent survey, see Mallard (2014).

policy has been analysed in a political economy framework by many authors including Aidt (1998), Fredriksson (1997), Damania (1999), Damania and Fredriksson (2000), Fredriksson and Svensson (2003), Fredriksson *et al.* (2005), Finkelshtain and Kislev (1997), Yu (2005), Grey (2018) and Bramoullé and Orset (2018).[2] Of these, Finkelshtain and Kislev (1997) is of particular relevance. They apply the common agency framework to analyse the choice of two environmental policy instruments, an emission tax and non-tradable quantities/quotas, when the firms have already formed a single lobby group that aims to influence regulator's choice of the instrument level. Hence the present model is much influenced by their work. They show that the tax level under lobbying is lower than without lobbying, and that the level of non-tradable quantities the firm receives is increased due to lobby group's efforts. However, they take the size of the lobby group as given, and, in fact, they assume that some firms may free-ride on the efforts of the lobbying firms when tax is used, but that all the firms participate in the lobby group when quotas are applied.

The analyzed instruments are emission tax, emission quota and auctioned emission permits. The lobby formation, the choice of the environmental policy instrument level and the choice of the emissions are made in the following game with three stages:[3]

1: Each firm individually decides whether to participate in the lobby group to influence the policy instrument level. Participation in the lobby group is costly.

2: Lobby group presents to the regulator a contribution schedule that is contingent on the level of the policy instrument. The regulator cares about the contribution and the social welfare, and decides the instrument level that maximizes his payoff.

3: Firms make their emission choices individually given the level of the policy instrument.

[2]Damania and Fredriksson (2000) analyse a polluting duopoly both of which may give political contributions to the regulator in exchange for a change in the emission tax. They too analyse lobby group formation, but the analysis is based on the assumption that "*if both firms decide to do so* [offer contributions], *an industry lobby group is organized*". However, this implies that also Grossman and Helpman (1994) would contain an analysis on lobby formation: lobby is formed if all sectors offer positive contributions in equilibrium.

[3]There is an implicit Stage 0, where the regulator announces which instrument is used. The firms are not able to lobby in favor of different instruments.

Stage 2, or the policy-game, is the critical part of the model. This stage is modeled as a menu auction (Bernheim and Whinston, 1986), where the principals (firms) have a common agent (regulator). The regulator weights the contribution and the social welfare differently using weight parameter on social welfare as in Grossman and Helpman (1994). It is assumed that there is a continuum of firms, and that, following Bombardini (2008), the equilibrium lobby size at Stage 1 is the one that maximizes the joint payoff of the lobby group. Firms are identical except they all have different cost of joining the lobby.

It is shown that the equilibrium number of lobbyists differs between the instruments, except between emission tax and auctioned permits, which are in equilibrium equivalent instruments as expected. The last firm to join the lobby group makes a zero net profit, but the net profit that the firm can obtain differ between the instruments, and hence does the equilibrium number of lobbyists. The aggregate emissions are greater than the socially optimal emissions with any of the instruments, and the regulator obtains the same payoff with each instrument that equals the social welfare at the socially optimal emissions (multiplied by the preference parameter).

The study is organized as follows. Section 2 presents the model, results and analysis starting with an emission tax (and auctioned permits) and continuing with quotas. Last section offers further discussion and possibilities for extensions.

2 Model and results

2.1 Emission tax (and auctioned permits)

Suppose that there is a continuum of polluting firms, who are identical except that each has a different cost of joining the lobby. The firms are indexed with m and the emissions of firm m are denoted with e_m. The mass of firms is 1. It is assumed that the cost of joining the lobby group is increasing in m so that it is plausible for the first firms to join the lobby.[4] Denote the cost of joining the group with $F(m)$ and let $F'(m) > 0$ for all m. Let the interval of firms who join the lobby be $[0, m_L] \subset [0, 1]$. Then, given an emission

[4] This assumption is similar to the trade model of Mitra (1999). In there the sectors who each can form a lobby group are ordered according to the fixed cost of lobby formation.

tax t, the aggregate emissions and the emissions for the lobby group are

$$E(t) := \int_0^1 e_m(t)\ \mathrm{d}m, \quad \text{and} \quad E_L(t) := \int_0^{m_L} e_m(t)\ \mathrm{d}m, \tag{1}$$

respectively.

Firm's profit is $\pi(e_m)$ with $\pi'(e_m) > 0$ and $\pi''(e_m) < 0$, and therefore for a given emission tax t the aggregate profits for the whole regulated industry and for the lobby group are

$$\Pi(t) := \int_0^1 \pi(e_m(t))\ \mathrm{d}m, \quad \text{and} \quad \Pi_L(t) := \int_0^{m_L} \pi(e_m(t))\ \mathrm{d}m. \tag{2}$$

Aggregate pollution causes damages, which are captured by the damage function D, whose value is strictly increasing and convex in E. More precisely, it is assumed that $D'(E) > 0$ and $D''(E) \geq 0$. The social welfare and the net profits for the lobby group are

$$W(t) := \Pi(t) - D(E(t)), \quad \text{and} \quad W_L(t) := \Pi_L(t) - tE_L(t). \tag{3}$$

Next the socially optimal emission tax is analysed and after that the tax policy under lobbying is investigated.

Social optimum: Given an emission tax t, the problem for firm m is to choose its emissions to maximize net profits, that is, to $\max_{\{e_m\}}\{\pi(e_m) - te_m\}$. The interior solution satisfies equation $\pi'(e_m) = t$. Given this reaction by the firm, the regulator chooses the tax to maximize the social welfare, that is, the regulator solves the problem

$$\max_{\{t\}} \Big\{ \int_0^1 \pi(e_m(t))\ \mathrm{d}m - D\Big(\int_0^1 e_m(t)\ \mathrm{d}m \Big) \Big\}. \tag{4}$$

The solution satisfies equation

$$\int_0^1 \pi'(e_m(t)) e_m'(t)\ \mathrm{d}m - D'\Big(\int_0^1 e_m(t)\ \mathrm{d}m \Big) \int_0^1 e_m'(t)\ \mathrm{d}m = 0. \tag{5}$$

Therefore the socially optimal tax is the Pigouvian tax, $t^* = D'(E^*)$, as expected.

Tax under lobbying: The choice of the tax under lobbying is analysed using a game with three stages. First, every firm decides whether or not to join the lobby group and pay the cost of joining. Second, the emission tax and contribution schedules are chosen. Third, the firms choose their emissions.

In the third stage the emissions are chosen according to equations $\pi'(e_m) = t$ for every $m \in [0,1]$. As the firms are assumed to be identical, emission levels are the same for the firms, that is $e_m = \pi'^{-1}(t)$ for every m, and therefore emissions are denoted simply by e unless said otherwise. Note that although the marginal profits are equalized across firms, they are equalized at the wrong level from the social welfare point of view due to lobby group's activity to influence the tax level in the second stage of the game (if the group has formed at all at Stage 1).

At Stage 2, the lobby group with m_L firms presents the regulator a contribution schedule that is contingent on the tax level. This schedule is denoted with C. The regulator cares about the social welfare W and about the contribution, and the payoff for the regulator is defined as $C(t) + \alpha W(t)$, where $\alpha \geq 0$ is a preference parameter. The equilibrium in the policy-game of Stage 2 is a subgame-perfect Nash equilibrium characterized by Bernheim and Whinston (1986) and applied by many including Grossman and Helpman (1994). This characterization is repeated in the following definition:

Definition 1. An allocation (t^0, C^0) is a subgame-perfect Nash equilibrium in the policy-game of Stage 2 if and only if

1. $C^0 \geq 0$,
2. t^0 maximizes
$$C^0(t) + \alpha\Big(\Pi(t) - D(E(t)))\Big), \tag{6}$$
3. t^0 maximizes
$$\Pi_L(t) - tE_L(t) + \alpha\Big(\Pi(t) - D(E(t))\Big), \tag{7}$$
4. there exists a $\hat{t}$ that maximizes $C^0(t) + \alpha\Big(\Pi(t) - D(E(t))\Big)$ such that $C^0(\hat{t}) = 0$.

Part 1 means that that the equilibrium contribution is feasible and Part 2 that the equilibrium tax under lobbying maximizes the regulator's payoff. As there is only one lobby group, the contribution schedules have been deleted from the objective function in Part 3, where the sum of lobby group's and regulator's payoffs is maximized. Part 4 says that there exists a tax level that maximizes regulator's payoff such that the lobby group's contribution obtains the value zero. This means that this tax level is the Pigouvian tax, and therefore Part 2 and 4 imply together that the regulator's equilibrium payoff equals its payoff under the social optimum (multiplied by α).

Part 3 of Definition 1 implies that an interior solution satisfies equation

$$\Pi'_L(t) - E_L(t) - tE'_L(t) + \alpha\Big(\Pi'(t) - D'(E(t))E'(t)\Big) = 0. \tag{8}$$

After using Stage 3 optimality condition and the definition of aggregate profit functions, this equation simplifies to

$$- E_L(t) + \alpha\Big(\int_0^1 \pi'(e(t))e'(t)\ \mathrm{d}m - D'(E(t))E'(t)\Big) = 0, \tag{9}$$

which implies after reorganisation and dividing by t that

$$t - D'(E(t)) = \frac{tE_L(t)/E(t)}{\alpha \mathrm{El}_t E(t)} = \frac{tm_L}{\alpha \mathrm{El}_t E(t)}, \tag{10}$$

where $\mathrm{El}_t E(t)$ is the elasticity of the aggregate emissions with respect to the emission tax. This tax rule is the same as the one in Finkelshtain and Kislev (1997) apart from the fact that in their model α is the weight for the contribution (and therefore in their formula the right-side is multiplied by α) and that here there are continuum of identical firms.

Supposing, as is standard in the literature, that the contribution schedule is differentiable, Part 2 of Definition 1 implies that

$$\frac{\mathrm{d}C^0(t)}{\mathrm{d}t} + \alpha\Big(\Pi'(t) - D'(E(t))E'(t)\Big) = 0. \tag{11}$$

This and Equation (9) imply together that

$$\frac{\mathrm{d}C^0(t)}{\mathrm{d}t} = -E_L(t). \tag{12}$$

This equation holds at $t = t^0$, and it means that the contribution schedule is locally truthful (Grossman and Helpman, 1994). But as is typical in applications, including Grossman and Helpman (1994), this local property is extended to cover all prices and not just the ones "close" to t^0. In other words, as it is assumed here too that the contribution schedules are truthful, the contribution to the regulator for a small decrease in the price of emissions equals the change in the lobby's net benefits *for all prices*.

Hence the contribution schedule is strictly decreasing in the emission tax; as the regulator increases the price of emissions, the lobby decreases its contribution to the regulator. Equation (12) holds for all t for which the contribution schedule is non-zero, and therefore integration gives

$$C^0(t) = -\int_{t^+}^{t} E_L(\tau)\ \mathrm{d}\tau + k, \tag{13}$$

where k is a constant and t^+ is the smallest price at which the contribution is zero. Part 4 of Definition 1 says that $\hat{t} = t^*$ (the Pigouvian tax), and therefore

$$\alpha W(t^*) = C^0(t^0) + \alpha W(t^0), \tag{14}$$

that is, the payoff to the regulator from the Pigouvian tax, which is the tax level when the lobby group gives zero contribution, equals the payoff obtained by the regulator in equilibrium. Rewriting these gives

$$k = \alpha W(t^*) - \alpha W(t^0) + \int_{t^+}^{t^0} E_L(\tau)\ \mathrm{d}\tau, \tag{15}$$

and therefore the contribution schedule is

$$C^0(t;t^0) = \int_{t}^{t^0} E_L(\tau)\ \mathrm{d}\tau + \alpha W(t^*) - \alpha W(t^0). \tag{16}$$

Note that $C^0(t^0;t^0) = \alpha W(t^*) - \alpha W(t^0)$, which agrees with Example 1 in Grossman and Helpman (1994) and with Finkelshtain and Kislev (1997). These results in the policy-game are collected to the following proposition:

Proposition 1. *Suppose that the regulator applies an emission tax to control emissions and that a fraction m_L of the firms have formed a lobby group to influence the tax choice. Then, in a truthful equilibrium, firms' emissions, emission tax and lobby group's contribution satisfy the following conditions:*

(*i*) $\pi'_m(e^0_m) = t^0$ *for all* $m \in [0,1]$,

(*ii*)

$$t^0 - D'(E(t^0)) = \frac{t^0 E_L(t^0)/E(t^0)}{\alpha El_t E(t^0)} = \frac{t^0 m_L}{\alpha El_t E(t^0)}. \tag{17}$$

(*iii*) $C^0(t;t^0) = \int_t^{t^0} E_L(\tau)\ \mathrm{d}\tau + \alpha\Big(W(t^*) - W(t^0)\Big)$, *where* t *is any other tax level such that the contribution is feasible.*

Part (i) describes the standard trade-off a firm faces when the emissions are regulated with an emission tax. The tax rule in Part (ii) is (essentially) the same as the one presented in Finkelshtain and Kislev (1997) and states that the equilibrium tax is less than the marginal damages. In Finkelshtain and Kislev (1997) the contribution schedule is given only at the equilibrium tax level. Part (iii) contains the whole contribution schedule (when it is strictly positive), and it can be rewritten as

$$C^0(t;t^0) = W_L(t) - W_L(t^0) + \alpha\Big(W(t^*) - W(t^0)\Big). \tag{18}$$

The contribution at tax level t equals the sum of net benefit/cost to the lobby group and the compensation to the regulator that equals the difference between the welfare level at the social optimum and at the lobbying equilibrium (multiplied by the preference parameter α). Hence the equilibrium payoff for the regulator is the payoff it would receive when there is no lobbying, that is, the amount $\alpha W(t^*)$.

Example: When $\pi'(e) = a - be$ and $D'(E) = dE$, the Pigouvian tax is $t^* = ad/(b+d)$, and the tax under lobbying with m_L firms in the lobby group is

$$t = \frac{a(d - bm_L/\alpha)}{b + d - bm_L/\alpha}. \tag{19}$$

Note that, as it is implicitly assumed that $t > 0$, the tax in the example suggests that one must check ex-post (when one knows the equilibrium value of m_L) that $t > 0$.

The tax level depends on the size of the lobby group. Denote the equilibrium tax with m_L lobbyists with $t(m_L)$. It is easy to see that in the above example that

$$t'(m_L) < 0. \tag{20}$$

The same relationship between the number of lobbyists and the equilibrium tax holds also if one supposes that the sufficient second-order condition for a maximum holds. Furthermore, without this supposition, Strict Monotonicity Theorem of Edlin and Shannon (1998) implies that t is strictly decreasing in m_L: the right-side of Equation (9) can be written as $-e(t)m_L + \alpha\Big(te'(t) - D'(e(t))e'(t)\Big)$, which is, as a function of m_L, strictly decreasing, and therefore the optimal t is strictly decreasing in that variable. Hence, the tax rate decreases as more firms join the lobby as intuition suggests. In what follows, it is assumed that t is differentiable in m_L, so that inequality (20) holds.

The number or mass of lobbying firms is determined at Stage 1. The equilibrium number of lobbyists is taken here to be the number of lobbyists that maximizes the joint payoff of the lobby group. This assumption is similar to the one in Bombardini (2008). Hence, a marginal firm is allowed to join the group, if (and only if) the joint payoff of the group increases. When m_L firms have joined the lobby group, the group's payoff is

$$P(m_L) := \int_0^{m_L} \pi(e(t(m_L)))\ \mathrm{d}m - t(m_L)E_L(t(m_L)) - C^0(t(m_L)) - \int_0^{m_L} F(m)\ \mathrm{d}m. \quad (21)$$

Lemma 1. *The change in the lobby group's payoff from a marginal increase in the number of lobbyists is given by* $P'(m_L) = \pi(e(t(m_L))) - t(m_L)e(t(m_L)) - F(m_L)$.

Proof. See Appendix A.1. □

Hence the derivative of the group's payoff equals the profit of the last joined firm.[5] Hence, if the net profit of the last joined firm is strictly positive, more firms should be allowed to join. Next result shows the equilibrium mass of firms when joining the lobby group is costless.

Proposition 2. *If joining the lobby group is costless, then all firms join the group, that is, the equilibrium number (or mass) of lobbyists is* $m_L = 1$.

Proof. If joining is costless, $F \equiv 0$. Then $P'(m_L) = \pi(e(t(m_L))) - t(m_L)e(t(m_L))$, which is strictly positive for all m_L since $t(m_L) < t^*$ for all $m_L > 0$. □

Hence no firm free-rides on the other firms' contributions. This is contrary to the analysis of Finkelshtain and Kislev (1997), who argue that some firms may not join the group due to incentives to free-ride on the lobby group's effort to lower the emission price. Note, however, that the proposition is obtained by assuming identical firms. More generally, with costly lobby participation, the equilibrium number of lobbyists is characterized by the following result:

Proposition 3. *When participation in the lobby group is costly, the equilibrium number (or mass) of lobbyists is either zero, one or given by the solution to equation* $\pi(e(t(m_L))) - t(m_L)e(t(m_L)) = F(m_L)$.

[5] Note that the part of the contribution that the marginal firm pays is zero, as the "size" of a single firm is the differential $\mathrm{d}m$.

Proof. The value of m_L that maximizes P is either on the boundary of $[0,1]$, or satisfies equation $P'(m_L) = 0$. $\square$

The size of the lobby group depends, as expected, on the benefit and cost of joining the group. If the lobby group's payoff is maximised at an interior point, then the last firm to join the group receives exactly zero net profit, since equation $\pi(e(t(m_L))) - t(m_L)e(t(m_L)) = F(m_L)$ holds for this marginal firm.[6] As the firms are identical, every firm on the interval $[0, m_L)$ receives a strictly positive net profit. The equilibrium size of the group can also be zero or one. Intuitively, none of the firms join the group if the cost is too high, and every firm joins if the cost is small enough for every firm.

The above analysis applies also for the case where the emissions are allocated using auctioning. The necessary modifications include for example that the regulator decides the amount of auctioned permits and that the emission tax is is replaced by the equilibrium permit price in Proposition 1. The formula for the contribution schedule in Part (*iii*) would be different, but the equilibrium contribution would be the same and equal to the difference between the welfare level at the socially optimal emissions and at the lobbying equilibrium emissions (multiplied by the preference parameter α).

2.2 Non-tradable quantities

In this section non-tradable quantities under lobbying are analyzed, when the firms can form a single lobby group to influence policy. Non-tradable quantities with a continuum of firms means an emission quota for each firm m that the firm is not allowed to trade. Hence the regulator chooses an emission function, which is defined on $[0,1]$ to maximise his payoff. The class of piecewise continuous functions is chosen for the set of feasible emission functions. Similarly to the game with the emission tax as the policy instrument, the quota is also set in a game with three stages. However, the third stage of the game involves little choice by the firms: every firm respects the quota set by the regulator, and chooses its emissions according to it.

The non-tradable quantities are set at Stage 2. Denote by e an emission function from $[0,1]$ to the real numbers. Its value at some point m, e_m, gives the emissions for

[6] Note that if m_L is an interior maximum of P, then $P''(m_L) = -t'(m_L)e(t(m_L)) - F'(m_L) \leq 0$ must hold. That is, the participation cost must rise fast enough.

firm m. As will be seen, it is useful to define a restriction of this function that describes the quota given to the lobbying firms. Therefore, denote the restriction of the emission function to interval $[0, m_L]$ with e_L. As in the analysis of the emission tax, suppose that m_L firms have participated in the lobby group. The emissions are valued by the firms, regulator and the rest of the society in the same way as with the tax. Let

$$\Pi(e) := \int_0^1 \pi(e_m)\ \mathrm{d}m, \quad \Pi_L(e) := \int_0^{m_L} \pi(e_m)\ \mathrm{d}m, \quad \text{and} \ \ E := \int_0^1 e_m\ \mathrm{d}m, \tag{22}$$

be the total profit for the firms, the total profit for the lobby group and the aggregate emissions, respectively. The payoff for the regulator is $C(e) + \alpha(\Pi(e) - D(E))$. It can be shown that if firms are not allowed to lobby, then the emission quota is the same for every firm and given by the solution to equation $\pi'(e_m) = D'(E) = D'(e_m)$[7], which is the (constant) emission function that maximises the social welfare defined with $W(e) := \Pi(e) - D(E)$. The solution to this equation is the socially optimal amount of emissions for firm m and it is denoted with e^*.

The characterization of a subgame-perfect Nash equilibrium from Bernheim and Whinston (1986) is adapted to the present context and it is the following:

Definition 2. An allocation (e^0, C^0) is a subgame-perfect Nash equilibrium in the policy-game of Stage 2 if and only if

1. $C^0 \geq 0$,

2. e^0 maximizes
$$C^0(e_L) + \alpha\Big(\Pi(e) - D(E))\Big), \tag{23}$$

3. e^0 maximizes
$$\Pi_L(e) + \alpha\Big(\Pi(e) - D(E)\Big), \tag{24}$$

4. there exists a $\hat{e}$ that maximizes $C^0(e_L) + \alpha\Big(\Pi(e) - D(E)\Big)$ such that $C^0(\hat{e}_L) = 0$.

The interpretation of these conditions is not repeated here, but note that $\hat{e}_L \equiv e^*$, that is, $\hat{e}_L$ equals the socially optimal emission quota. As can be expected, equilibrium emission levels are not the same for the firms under lobbying as shown in the next result,

[7]The formal proof of this is a special case of the proof of Proposition 4.

which has similarities with the results in Finkelshtain and Kislev (1997). Denote the equilibrium aggregate emissions with $E^0 := \int_0^1 e_m^0 \, \mathrm{d}m$. "Pointwise-like maximisation" shows the following:

Proposition 4. *Suppose that the regulator applies non-tradable quantities to control emissions and that a fraction m_L of the firms have formed a lobby group to influence the quantity choice. Then, in a truthful equilibrium, firms' emissions and lobby group's contribution satisfy the following conditions:*

(*i*) *Quantities are given by*

$$\pi'(e_m^0) - D'(E^0) = -\frac{1}{\alpha}\pi'(e_m^0), \quad \textit{for all } m \in [0, m_L], \tag{25}$$

and $\pi'(e_m^0) - D'(E^0) = 0$ *for all* $m \in (m_L, 1]$,

(*ii*) *Contribution is given by*

$$C^0(e_m; e_m^0) = -\int_{e_m}^{e_m^0} m_L \pi'(\tau) \, \mathrm{d}\tau + \alpha\Big(W(e^*) - W(e^0)\Big), \tag{26}$$

where e_m^0 is the constant given by Equation (25) in Part (i), and e_m is any other emission level for the lobbying firms such that C^0 is feasible (that is, $C^0(e_m) \geq 0$).

Proof. See Appendix A.2. □

For the firms in the lobby group, the difference between the marginal profit and marginal social damage is negative, but for those outside to lobby group it is zero. All the firms in the lobby group have the same level of emissions as do the firms outside the group, but the firms in the group are allowed to pollute more than the outside firms due to their lobbying effort. Hence the equilibrium emission function e is piecewise constant with a discontinuity at m_L. As expected, the equilibrium amount of contribution from the lobby to the regulator equals the difference between the social welfare and the welfare under lobbying (multiplied by the preference parameter α).

The effect of increasing the amount of lobbyists on the equilibrium quota for a lobbying firm and for a non-lobbying firm are different. Denote the equilibrium quota for a lobbying firm with e_L^0 and for a non-lobbying firm with e_{-L}^0. Then, as $e_L^0 > e_{-L}^0$ and $E^0 =$

$m_L e^0_L + (1 - m_L)e^0_{-L}$, implicit function theorem applied to the equations in Part (i) of Proposition 4 gives after some calculations

$$\frac{\mathrm{d}e^0_L}{\mathrm{d}m_L} = \frac{\pi''(e^0_{-L})D''(E^0)(e^0_L - e^0_{-L})}{\Phi} < 0, \tag{27}$$

and

$$\frac{\mathrm{d}e^0_{-L}}{\mathrm{d}m_L} = \frac{\Big(\pi''(e^0_L) + (1/\alpha)\pi''(e^0_L)\Big)D''(E^0)(e^0_L - e^0_{-L})}{\Phi} < 0, \tag{28}$$

where

$$\Phi = D''(E^0)\Big((m_L - 1)(\pi''(e^0_L) + (1/\alpha)\pi''(e^0_L)) - m_L\pi''(e^0_{-L})\Big) \tag{29}$$

$$+ \pi''(e^0_{-L})\Big(\pi''(e^0_L) + (1/\alpha)\pi''(e^0_L)\Big) > 0. \tag{30}$$

That is, as the size of the lobby group increases the equilibrium quotas strictly decrease. In addition, if the profit function π is quadratic, then an increase in the size of the lobby group results in a greater decrease in the quota for non-lobbyists than for the lobbyists.

The size of the lobby group, or the threshold firm, m_L is determined at Stage 1. Denote the amount of quota given to the lobbying firm with $e(m_L)$. As with the analysis of the emission tax, the equilibrium amount of lobbying firms is defined as the mass that maximizes the joint payoff of the lobby group. The joint payoff as a function of m_L is defined as

$$P(m_L) := \int_0^{m_L} \pi(e(m_L))\ \mathrm{d}m - C^0(e(m_L)) - \int_0^{m_L} F(m)\ \mathrm{d}m. \tag{31}$$

Note that this function is different from the lobby group payoff function with an emission tax, but to simplify the notation this difference is left out from the notation of P.

Lemma 2. *The change in the lobby group's payoff from a marginal increase in the number of lobbyists is given by* $P'(m_L) = \pi(e(m_L)) - F(m_L)$.

Proof. See Appendix A.3. □

Similar arguments as with the emission tax show the following:

Proposition 5. *If joining the lobby group is costless, then all firms join the group, that is, the equilibrium number (or mass) of lobbyists is* $m_L = 1$. *When participation in the lobby group is costly, the equilibrium number (or mass) of lobbyists is either zero, one or given by the solution to equation* $\pi(e(m_L)) = F(m_L)$, *where* $e(m_L)$ *is the quota set to the lobbying firms.*

3 Additional discussion and further research

Propositions 1 and 4 show that the lobby group must give the regulator a contribution that equals in equilibrium the difference between what the regulator receives without lobbying and what he receives when some of the firms lobby. This is as expected.[8] That is, the regulator receives as a payoff the social welfare evaluated at the social optimum (and multiplied by the preference parameter). This means that the regulator would be indifferent between the instruments if the game would involve an additional stage, where at the beginning of the game, the regulator decides the instrument that will be used to control emissions. However, the emissions levels between instruments vary, and therefore also the contribution levels of the lobby group and the payoffs of the firms. It is left for further work to analyse how the firms would lobby if allowed to influence the choice between instruments. Also, the welfare comparison of the instruments under endogenous lobbying is left for further work.[9]

In addition, the above model does not consider permits that are allocated for free to the firms. With a free allocation, the lobby group can presumably lobby for a higher aggregate allocation, or like in the above non-tradable quantities situation, for a higher initial allocations just for the lobbying firms. It can be expected that the ability of the firms to lobby would make the lobbyists permit sellers in equilibrium, but the detailed analysis of this case is left as a future extension of the above model.

[8]See Example 1 in Grossman and Helpman (1994).

[9]Some recent welfare comparisons include Ambec and Coria (2013) and Baldursson and von der Fehr (2004), but they do not consider lobbying. Miyamoto (2014) has analyzed lobbying and the welfare comparison, but without lobby formation.

A Appendix

A.1 Proof of Lemma 1

To calculate the derivative, note that $\int_0^{m_L} \pi(e(t(m_L)))\,\mathrm{d}m = \pi(e(t(m_L)))m_L$, $E_L(t(m_L)) = e(t(m_L))m_L$ and $C^0(t(m_L)) = \alpha\Big(W(t^*) - W(t(m_L))\Big)$. Furthermore

$$\begin{aligned} W(t(m_L)) &= \Pi(t(m_L)) - D(E(t(m_L))) && \text{(A.1)}\\ &= \int_0^1 \pi(e(t(m_L)))\,\mathrm{d}m - D\Big(\int_0^1 e(t(m_L))\,\mathrm{d}m\Big) && \text{(A.2)}\\ &= \pi(e(t(m_L))) - D(e(t(m_L))). && \text{(A.3)} \end{aligned}$$

Hence the group's payoff is

$$\begin{aligned} P(m_L) = \Big(\pi(e(t(m_L))) - t(m_L)e(t(m_L))\Big)m_L - \alpha W(t^*) && \text{(A.4)}\\ + \alpha\Big(\pi(e(t(m_L))) - D(e(t(m_L)))\Big) - \int_0^{m_L} F(m)\,\mathrm{d}m. && \text{(A.5)} \end{aligned}$$

The derivative is

$$\begin{aligned} P'(m_L) = \Big(\pi'(\cdot)e'(\cdot)t'(m_L) - t'(m_L)e(\cdot) - t(m_L)e'(\cdot)t'(m_L)\Big)m_L && \text{(A.6)}\\ + \Big(\pi(e(t(m_L))) - t(m_L)e(t(m_L))\Big) && \text{(A.7)}\\ + \alpha\Big(\pi'(\cdot)e'(\cdot)t'(m_L) - D'(\cdot)e'(\cdot)t'(m_L)\Big) - F(m_L). && \text{(A.8)} \end{aligned}$$

As equation $\pi'(\cdot) = t(m_L)$ holds, this derivative becomes after reorganisation

$$\begin{aligned} P'(m_L) = -t'(m_L)e(\cdot)m_L + \alpha\Big(\pi'(\cdot)e'(\cdot)t'(m_L) - D'(\cdot)e'(\cdot)t'(m_L)\Big) && \text{(A.9)}\\ + \pi(e(t(m_L))) - t(m_L)e(t(m_L)) - F(m_L). && \text{(A.10)} \end{aligned}$$

Because $E_L(t(m_L)) = e(t(m_L))m_L$ and $E'(t) = e'(t)$, this equals

$$\begin{aligned} P'(m_L) = t'(m_L)\Big(- E_L(t(m_L)) + \alpha\Big(\pi'(\cdot)E'(\cdot) - D'(\cdot)E'(\cdot)\Big)\Big) && \text{(A.11)}\\ \pi(e(t(m_L))) - t(m_L)e(t(m_L)) - F(m_L). && \text{(A.12)} \end{aligned}$$

Given the tax rule (17), this can be written as

$$P'(m_L) = \pi(e(t(m_L))) - t(m_L)e(t(m_L)) - F(m_L). \qquad \text{(A.13)}$$

A.2 Proof of Proposition 4

Part (i): Suppose that function e^0 is a piecewise continuous solution to

$$\max_{e_m} \Big\{ \int_0^{m_L} \pi(e_m)\ \mathrm{d}m + \alpha\Big(\int_0^1 \pi(e_m)\ \mathrm{d}m - D\Big(\int_0^1 e_m\ \mathrm{d}m\Big)\Big)\Big\}, \tag{A.14}$$

which is the problem in Part 3 of Definition 2 (denote the functional in the curly brackets with J). It is shown that e^0_m solves, for every fixed $m \in (0, m_L]$, the problem

$$\max_{e_m} \Big\{\pi(e_m) + \alpha\Big(\pi(e_m) - D'\Big(\int_0^1 e^0_m\ \mathrm{d}m\Big)e_m\Big)\Big\}, \tag{A.15}$$

and, for every fixed $m \in (m_L, 1]$, the problem

$$\max_{e_m} \Big\{\pi(e_m) - D'\Big(\int_0^1 e^0_m\ \mathrm{d}m\Big)e_m\Big\}. \tag{A.16}$$

The standard methods are applied to show these.Consider a small perturbation in e^0 that occurs at a point of continuity of e^0 either on $[0, m_L]$ or on $(m_L, 1]$, and which is defined as

$$e_m = \begin{cases} \overline{e}, & m \in [s-\delta, s) =: I, \\ e^0_m, & m \in [0,1] \setminus I, \end{cases} \tag{A.17}$$

for small δ and for any $\overline{e}$. Define the difference between functional J under the perturbation and under the optimum with $\Delta J(\delta)$, and note that for $s \in (0, m_L)$

$$\Delta J(\delta) = (1+\alpha)\int_{s-\delta}^{s} \pi(\overline{e}) - \pi(e^0_m)\ \mathrm{d}m - \alpha\Big(D\Big(\int_0^1 e_m\ \mathrm{d}m\Big) - D\Big(\int_0^1 e^0_m\ \mathrm{d}m\Big)\Big), \tag{A.18}$$

where $\int_0^1 e_m\ \mathrm{d}m = \int_0^{s-\delta} e^0_m\ \mathrm{d}m + \int_{s-\delta}^{s} \overline{e}\ \mathrm{d}m + \int_s^1 e^0_m\ \mathrm{d}m$. Since $\Delta J(\delta) \approx \Delta J'(0)\delta$, $\Delta J'(0) \leq 0$ for small enough δ. The derivative $\Delta J'(\delta)$ is

$$\Delta J'(\delta) = (1+\alpha)\Big(\pi(\overline{e}) - \pi(e^0_{s-\delta})\Big) - \alpha\Big(D'\Big(\int_0^1 e_m\ \mathrm{d}m\Big)(-e^0_{s-\delta} + \overline{e})\Big). \tag{A.19}$$

Evaluating this at $\delta = 0$, gives inequality

$$(1+\alpha)\Big(\pi(\overline{e}) - \pi(e^0_s)\Big) - \alpha\Big(D'\Big(\int_0^1 e^0_m\ \mathrm{d}m\Big)(-e^0_s + \overline{e})\Big) \leq 0. \tag{A.20}$$

That is, for any $\overline{e}$,

$$(1+\alpha)\pi(\overline{e}) - \alpha D'\Big(\int_0^1 e^0_m\ \mathrm{d}m\Big)\overline{e} \leq \tag{A.21}$$

$$(1+\alpha)\pi(e^0_s) - \alpha D'\Big(\int_0^1 e^0_m\ \mathrm{d}m\Big)e^0_s, \quad \text{for all} \ \ s \in (0, m_L), \tag{A.22}$$

and hence e^0_m solves (A.15) is verified. Clearly, case (A.16) is true also. Part (i) of the proposition follows from these maximisation problems.

Part (ii): If e^0 a piecewise continuous solution to the problem in Part 2 of Definition 2, then, for every $m \in (m_L, 1]$, e^0_m solves the problem (A.16); define this emission level as z. Furthermore, for every $m \in (0, m_L]$, e^0_m solves the problem (note that, by Part (i), e^0_m is the same for all lobbying firms)

$$\max_{e_m} \Big\{ C(e_m) + \alpha \Big(m_L \pi(e_m) + (1 - m_L)\pi(z) - D(m_L e_m + (1 - m_L)z) \Big) \Big\}. \tag{A.23}$$

Therefore, and again for every $m \in (0, m_L]$, $C^{0\prime}(e^0_m) = m_L \pi'(e^0_m)$, where e^0_m is constant in m. Hence, for truthful schedules,

$$C^0(e_m) = \int_{e^+_m}^{e_m} m_L \pi'(\tau) \, \mathrm{d}\tau + k, \tag{A.24}$$

where k is a constant and e^+_m is the greatest emission quantity for which the contribution is zero. By Part 4 of Definition 2, $\alpha W(e^*) = C^0(e^0_m) + \alpha W(e^0)$. Solving for k gives the result.

A.3 Proof of Lemma 2

The joint payoff of the lobby group can be written as

$$P(m_L) := \pi(e(m_L))m_L - C^0(e(m_L)) - \int_0^{m_L} F(m) \, \mathrm{d}m. \tag{A.25}$$

The derivative of P is $P'(m_L) = \pi'(e(m_L))e'(m_L)m_L + \pi(e(m_L)) - C^{0\prime}(e(m_L)))e'(m_L) - F(m_L)$. Proof of Proposition 4 showed that $C^{0\prime}(e(m_L)) = m_L \pi'(e(m_L))$, which implies that $P'(m_L) = \pi(e(m_L)) - F(m_L)$, as required.

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