Wait for it: Valuing natural capital when management is dominated by periods of inaction

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Abstract

Valuing natural assets is important for tracking management performance and for wealth accounting sustainability assessments. Measuring the value of service flows from ecosystems is also important for environmental income and product accounting and benefit-cost analysis of specific projects. Developments in valuing natural capital have focused on implicit intertemporal exchange revealed by management feedback rules that map the state of the system in continuous fashion onto a management response. However, the management of many real assets, including many natural capital assets, is best described as doing nothing with punctuated adjustment – an important type of non-convexity. We extend the current theory of natural capital asset valuation for such cases. In so doing, we develop an approach for measuring revealed non-use value. We develop a case study for Oregon Douglas fir forests managed by clear cutting, where forest may provide an amenity flow while standing and timber at harvest. We find that the non-use, "amenity," flow value of the forests is positive and depends on site class, and that wealth held in Oregon Douglas fir forests increased over the first decade of the 21st century.

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Introduction

Natural resources provide a broad portfolio of real assets to society. These natural assets store opportunities for society, and human decisions, with respect to how and when to access those opportunities, generate value in much the same way as traditional assets (e.g. real estate, financial assets, or machinery), and provide gains or losses depending on management. Natural capital is an important component of society's productivity base and allows management challenges to be framed as a form of "portfolio management" across capitals, where tradeoffs are evaluated across capital stocks using a common currency (Fenichel et al. 2016). Management can be evaluated broadly based on asset's or a portfolio's performance in the context of inclusive (or comprehensive) wealth accounting (Lange et al. 2018; Managi 2018), which measures natural capital in a manner compatible with traditional capital assets. Yet, for many real assets, and especially natural capital, markets are non-existent. The challenge of a general and practical methodology for calculating asset prices for valuing natural capital stocks continues to limit implementation of wealth-based sustainability measures despite much attention paid to the importance of natural capital and general agreement around its importance.

This paper adapts the capital asset pricing for nature (CAPN) approach, developed in Fenichel and Abbott (2014), to natural resource systems dominated by human inaction with punctuated non-marginal actions, e.g., standing forest. For example, forest provides market goods (e.g., timber products) and non-market goods (e.g., wildlife species habitat, water purification, and carbon sequestration). Timber is extracted and traded commercially as a market good. The marginal net benefit from an increase in the standing timber is received only when stand growth is terminated by harvest and can be measured as the effect of the change in the stock on net revenue from timber sales. The marginal timber value can be connected to the observed market value; that is uncontroversial. However, non-market goods and services can flow from forests while the trees are alive, but market transactions seldom reveal these net benefits. Since these services impact the shadow price of live forests, non-market valuation techniques are required. In this sense forests are representative of many forms of natural capital.

Non-market valuation techniques face limitations, especially for non-use services. Stated preference techniques, which are often considered the only option for non-use flows, are especially vulnerable to criticisms (Obst and Vardon 2014) despite the rigorous designs and validation that has gone into stated preference (Johnston et al. 2017). Furthermore, current practice in non-market valuation principally informs the benefit-cost analysis of specific programs by valuing the willingness to pay (accept) *for a change in the program* rather than tracking the value of a specific asset *given a pre-determined program*. Both questions are important but are different. Non-market valuation for benefit-cost analysis uses prices to help make forward looking decisions about specific alternative programs. Wealth and environmental accounting provide broad indicators about past performance, management institutions, and reflect what society is actually doing to provide a common set of facts for discussing future management.

The capital asset pricing for nature approach, when combined with an assumption about individual rational behavior, provides a “revealed” approach to estimating marginal non-use value for tracking changes in a specific asset given a pre-determined program. Such marginal values are appropriate accounting prices that reflect the actual decisions made by society. Revealed behavior reflects existing institutional frameworks and any unobserved contributing factors that lead to decision. This makes such valuation directly comparable with actual market prices, which are also subject to imperfections (Nordhaus 2006; Muller et al. 2011). The asset prices derived from the real-world example capture the extent to which society behaves as if the asset has a given value, and is therefore the best estimates of the appropriate accounting price (sensu Dasgupta and Mäler 2000). Asset prices are revealed through intertemporal allocation decisions, and such asset prices are subject to prevailing institutions.

We price natural capital in a manner fully consistent with Jorgenson’s classic capital asset pricing approach (Jorgenson 1963; Hulten 2006). As shown in Fenichel and Abbott (2014), Jorgenson’s asset pricing equation does not rely on the assumption of an optimizing economy, even for nonmarket natural capital. The framework provides the revealed marginal social benefit from holding an additional unit of natural capital. It can be applied to a range of natural capital stocks including groundwater (Fenichel et al. 2016; Addicott and Fenichel 2019), fisheries (Yun et al. 2017), and coastal wetlands (Bond 2017) but relies on continuous and differentiable management adjustments to the natural resources. However, natural resource management often involves doing nothing most of the time – waiting. Indeed, Stokey (2008) begins her seminal book by writing “In situation where action entails a fixed cost, optimal policies involve doing nothing most of the time and exercising control only occasionally," which aptly describes many problems with the intertemporal allocation of real assets, including natural capital, even for cases when economies fail to optimize. This introduces a non-convexity into the problem.

We contribute to the literature by generalizing Fenichel and Abbott's capital asset pricing for nature (CAPN) approach for cases when the economic program involves inaction punctuated by bursts of adjustment. In so doing, we also develop an approach to measuring the value of that accrue during periods of inaction, or non-use value. Discrete decisions provide different information than decisions that can be considered as continuous. We develop a case study for privately owned industrial Douglas fir forest in western Oregon, which is typically managed through clear cutting with little intermediate silvicultural treatment (e.g., thinning). Therefore, the industrial Douglas fir system is a good application to test the extension of natural capital asset approach to a wait-act choice environment. We use data on revealed harvest timing based on the harvest notification record submitted to the Oregon Department of Forestry. We contrast this with a hypothetical harvest program that only values timber, i.e., the Faustmann rotation period. The difference between what owners actually cut and the profit or use-value they could have earned as profit maximizers only interested in timber services, is attributed to unobserved preferences of landowners, such as non-use values, but also to idiosyncratic characteristics such as imperfect knowledge, different discount rate, or price expectation. Whatever the cause of departure is, the observed harvest decisions reflect a tradeoff between monetary benefits from immediate harvest and non-monetary amenity benefits. The difference is the timber revenue forgone by the owner's explicit decision to harvest later. It allows us to calculate a lower bound on the revealed non-use value from standing forest.

Theory of Natural Capital Pricing

Individuals reveal the value of natural capital through intertemporal allocation, i.e., savings and consumption, choices that are formalized as an economic program or resource allocation mechanism, $x(t)$ (reviewed in Fenichel and Hashida 2019).[1] These choices reflect the societal or

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[1] $x(t)$ is often vector valued, but the focus in this article is on cases where $x(t)$ takes only discrete values. Therefore, it is sufficient for exposition to consider $x(t)$ a scale response.

institutional constraints and shortcomings such as inefficient management institutions, cultural constraints, and that governments and institutions may be kakatopic and do not necessarily optimize social welfare (Fenichel and Abbott 2014).[2] Let Ω represent the set of prevailing and fixed institutions and $s(t)$ represent the state of natural capital at time t. The prevailing institutions Ω could be inefficient and might not operate optimally reflecting the institutional and societal constraints. Nonetheless, individuals may make their resource use and conservation decisions in a rational manner that provides them with the greatest private net benefits, subject to constraints they face and in response to the incentives set out in Ω. Then, define the economic program as $M: (s, \Omega) \mapsto x(s(t);\ \Omega)$, a mapping describing a feedback rule from the state of the stock to behaviors that is conditioned on institutional constraints. The economic program or resource allocation mechanism, $x(s)$, is modeled as a feedback rule that results from the deliberate actions of individual forest landowners within a given institutional context.[3] Treating the dependence on societal constraints as implicit, Fenichel and Abbott (2014) represent the economic program as $x\big(s(t)\big) \in \mathbb{C}^1$.[4]

Given $x\big(s(t)\big)$, the state of natural capital changes according to

$$\dot{s} = G(s) - f\big(s, x(s)\big) \tag{1}$$

where $G \in \mathbb{C}^1$ is an ecological growth function, and $f \in \mathbb{C}^1$ is an anthropogenic impact function that describes the effects of human actions on the natural capital stock (Fenichel et al. 2016). We follow convention in subtracting f, but the function could have a negative sign for some kinds of human actions.

The intertemporal welfare at time t as function of stock $s(t)$, $V\big(s(t)\big)$, is the net present value of the stream of social net benefits or real income (Fenichel et al. 2018; Hulten 2006), $W\big(s(t), x(t)\big)$ discounted at rate δ, where the dynamics of s are given in Eq (1). Formally,

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[2] Kakatopia is "at best, not-so-good society," (Dasgupta 2007).

[3] We suppress the dependence on institutional arrangements because we assume they are held fixed.

[4] Fenichel and Abbott (2014) originally suggest that all functions must be twice continuous rather than $\mathbb{C}^1$, but Fenichel et al. (2016) and Fenichel, Abbott, and Yun (2018) relax this original assertion so that functions only need to be continuous and differentiable.

(2) $$V(s(t)) = \int_t^{\infty} e^{-\delta(\tau-t)}\, W\left(s(\tau), x\left(s(\tau)\right)\right) d\tau$$

Following Dasgupta and Mäler (2000) and Arrow et al. (2003) define the (revealed) shadow price or accounting price of the asset as

(3) $$p\left(s(t)\right) \equiv \partial V\left(s(t)\right)/\partial s(t).$$

We suppress the dependency on time when doing so does not cause confusion. Fenichel and Abbott (2014) show that the relationship

(4) $$\delta V\left(s(t)\right) = W\left(s(t), x\left(s(t)\right)\right) + p\left(s(t)\right)\dot{s}(t) = H(s, x, p) = H^{*}(s, p)$$

does not require that the feedback rule $x(s)$ optimizes Eq (2) (also see Stokey 2008). Differentiating Eq (4) with respect to s, using the shadow price definition, and rearranging yields

(5) $$\frac{\partial V}{\partial s} = p(s) = \frac{W_S + \dot{p}}{\delta - G_S + f_S}$$

The shadow price of a natural capital asset measures the social value of the marginal unit of the asset. When markets for an asset are imperfect or completely absent, the actual individual decisions made in the society in terms of investment or depletion nevertheless affect the present value accorded by the last increment of capital, revealing an implicit shadow price or accounting price. Incorporating how individuals and societies decided to use the natural asset – the economic program – reveals the implicit valuation of natural capital. Functional approximations techniques applied to Eq (4) or Eq (5) can be used to approximate asset prices for natural capital for stock levels within a bound set (Fenichel et al. 2018; Yun et al. 2017).

Discrete decisions and non-convexity

The literature that builds on Fenichel and Abbott (2014) maintains that f and x are continuous and differentiable. But, natural resource management is investment in real capital, and investment in real capital often involves fix cost investment, which leads to periods of inaction punctuated by discrete decisions (Dixit and Pindyck 1994; Stokey 2008). For example, managed forests are an important natural asset, but many forests are managed through clear cutting, which is process of doing nothing punctuated with non-marginal action.[5] Other such cases, include conservation set asides, endangered species listing and delistings, and restoration activities. Such

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[5] Forest management is often thought to be a repeat "optimal" stopping program. However, we do not impose the social optimality of the decision here. Rather, we take the actions as revealing the marginal value of standing forest.

"go don't go" behavior can reflected in $x\left(s(t)\right)$ when it is a Heaviside function that only takes values of zero during periods of inaction, and one during periods of action. In the case of clear cutting a forest $f = x\left(s(t)\right)s(t)$, more generally f maybe equal $x\left(s(t)\right)Q\left(s(t)\right)$, where Q is some function that may depend on the stock. In this setting $f \notin \mathbb{C}^1$.

Binary decisions create an important type of non-convexity (Davidson and Harris 1981). Non-convexity poses challenges in identifying the *optimal* program because it potentially creates multiple equilibria (Starrett 1972; Tahvonen and Salo 1996), the problem does not occur in recovering *realized* shadow prices based on the real-world economic program because the economic program is determined ex-ante (Fenichel et al. 2018). Below, we implement the theory posited by Fenichel et al. (2018) for cases of revealed shadow prices in the presence of non-convexity. We focus on the concrete example of a discrete harvest decision. This also enables us recover the non-use benefits implicit in forest landowners' harvest decisions and extend Fenichel et al. (2018) ideas to cases where there are net benefits received during the inaction period.

The derivation of Eq (5) depends on the $\partial x(s)/\partial s$, which is based on the assumption of a behavior equilibrium feedback rule that is continuous and differentiable (Fenichel et al. 2018). By the chain rule, the behavioral equilibrium implies that $f_s\left(s, x(s)\right) = \frac{\partial f}{\partial s} + \frac{\partial f}{\partial x}\frac{\partial x}{\partial s}$. The behavioral equilibrium assumption is still valid if $x(s)$ is a Heaviside function, but chain rule implementation is unhelpful. Therefore, the direct application of Fenichel and Abbott's approach is not applicable to a large set of important natural capital assets and liabilities, e.g., forests, managed beach width (Gopalakrishnan et al. 2017), pests liabilities (Marten and Moore 2011; Fenichel et al. 2014), and the establishment of protected areas. Nevertheless, Eq (4) is still valid for the non-optimized system when $x(s)$ is a Heaviside function (Stokey 2008).

Revealed shadow prices must be continuous in the stock even if the in the case on a non-convexity. Fenichel et al. (2018), argue that it is possible to recover the piece-wise shadow prices for resources that involve any extraction decisions including discrete decisions. Consider a period of inaction or waiting so that the stock is not withdrawn, $x(s) = 0$, when $s \leq \bar{s}$ with some economic program of harvest $x(s) = 1$.

$$(6) \qquad x(s) = \begin{cases} 1 \; if \; s \geq \bar{s} \\ 0 \; if \; s < \bar{s} \end{cases}$$

This is classic problem of managing a system within a boundary space (Stokey 2008), and is indeed the way many ecosystems are managed. For example, a forest manager decides about when to harvest a forest stand, keeping the forest under a maximum age or volume boundary. In the deterministic setting, such as the classical Faustmann (Conrad 2010) or Hartman (1976) model, it is common to frame the decision in terms of the waiting period or rotation length, T^*, but in the deterministic setting, there is a one-to-one relationship between $\bar{s}$ and T^*.[6]

If the economic program is described by Eq (6), then the stock dynamics must be modified to

$$(7) \qquad s(t) = \begin{cases} Z(s)\ if\ s \geq \bar{s} \\ G(s)\ if\ s < \bar{s} \end{cases}$$

where $Z(s)$ describes the state of the world following the action implied by $x(s) = 1$. In classical forest harvesting problems or beach nourishment problems (e.g., Goplakrishnan et al. 2017) $Z(s) = s(0)$, where $s(0)$ is a constant, e.g., a planting volume in a forestry system or beach height in the nourishment problem. We will focus on these sorts of dynamics where the "reset" point of s is determined exogenously to be $s(0)$. However, other cases that could be considered include $Z(s) = ks(t)$, e.g., Fenichel, Richards, and Shanafelt (2014) or $Z(s, y(t))$ e.g., Fenichel et al. (2019), where y is another capital stock. The stock dynamics in Eq (7) represent a discrete jump in the stock at the point in time when the stock reaches $\bar{s}$. This leads to a non-convexity in V and to the two denominator terms in Eq (5) being undefined when $s = \bar{s}$.

Fenichel et al. (2018) develop a shadow pricing function for discontinuity in f assuming that there are no direct flow benefits from the stock itself, so that there are no benefits received during the inaction period. This is akin to an asset growth period where the "owner" of the asset only receives "paper" capital gains, which implies Eq [4] reduces to $\delta V(s(t)) = V_s(s(t))\dot{s}(t) = p(s)\dot{s}$. When $0 < s < \bar{s}$ there is some transitional interval $\tau(s)$ that goes to zero as s approaches $\bar{s}$. The derivative $-\tau'(s) = t'(s)$ is the inverse function of $\dot{s}$ conditional on $f(x(s)) = 0$, i.e., the function $G^{-1}(s)$. Therefore, the current period shadow price is $V_s(s(t)) = t'(s)\delta V(s(t))$.

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[6] Focusing on a state-based threshold is more general, since it can also apply in a stochastic setting (Miranda and Fackler 2002). Abbott, Fenichel, and Yun (2018) address measuring natural capital asset prices in a stochastic setting when $x(s) \in \mathbb{C}^1$, there results are extendable to the cases addressed in the present article. Stokey (2008) discusses such mappings in stochastic systems.

Fenichel et al. (2018) state that if $V(\bar{s})$ is known, then $V(s|s < \bar{s}) = e^{-\delta\tau(s)}V(\bar{s})$, so that $V_s\big(s(t)\big) = t'(s)\delta V(\bar{s})e^{-\delta\tau(s)}$.[7]

Define the $V\big(s(t)\big)$ for cases in the action region, where $s(t) \geq \bar{s}$ as

(8) $M(s) = V(s|s \geq \bar{s})$

Requiring that the revealed shadow price be continuous leads to the "smooth pasting" condition. This condition is a generalization of Eq [5], and is not an optimality condition. It simply states that the shadow price is defined as in Eq [5] in regions in the region where discrete actions are not taken as the partial derivative of Eq [8] in regions where such actions are taken, i.e., when $x(s) \notin \mathbb{C}^1$.

(9) $$p\big(s(t)\big) = \begin{cases} \frac{\partial M}{\partial s(t)} \; if \; s \geq \bar{s} \\ \frac{W_s + \dot{p}}{\delta - G_s + f_s} \; if \; s < \bar{s} \end{cases}$$

Some analogy to optimization may be helpful. The smooth pasting condition ensures that $p(s) = V_s(s)$ is continuous in s (Stokey 2008). The smooth pasting condition is the adjoint (or co-state) condition when $x(s) \in \mathbb{C}^1$. In optimization problems with free-endpoint transversality condition and a scrap value the smooth pasting condition is just a restatement that shadow price prior to the stopping period is the marginal value of scrap value, which could be interpreted as $M\big(s(t)\big)$. Indeed, this is one interpretation of revealed shadow price. It is the marginal value of stock just before a predetermined "scrap program" is implemented. That program may or may not be the socially optimal program.

We take $\bar{s}$ has given, unlike in an optimization problem, where the task is to find the optimal $\bar{s}$. This raises the question, what criterion do we expect the decision maker to use to choose $\bar{s}$. A rational or goal seeking decision maker would choose $\bar{s}$ so that he or she is privately different

(10) $M(s) = V\big(s(0)\big) + \pi(s|s \geq \bar{s})$

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[7] This is an error in Fenichel et al.'s (2018) eq (10), it has an extra term that should not be there.

where $\pi(s|s \geq \bar{s})$ is the net adjustment benefit (which may be negative and a cost) of moving the stock from the current value of s to $s(0)$. In optimization this is called the value matching condition. The case when there are no benefits or costs of waiting other than the opportunity cost of waiting -- no non-use value, Eq (10) can be expressed as

(11) $M(\bar{s}) = e^{-\delta\tau(s(0))}\, V(\bar{s}) + \pi(\bar{s}).$

Therefore, knowing the net benefits of adjusting the stock (e.g., harvest), the discount rate, and the inverse of the stocks growth function is sufficient to calculate the shadow price of the stock.

Non-use value

By definition periods of inaction represent are periods of non-use, but many forms of natural capital "pay dividends" during such periods. Non-use value is important but difficult to measure (Krutilla 1967; Freeman 2003).[8] In the case of forests Hartman (1976) called this amenity value.

Assume that $W\big(s, x(s)\big)$ can be decomposed into two additively separable parts: the real income received when $x(s) = 1$ or use value, $\pi\left(s, f\big(x(s)\big)\right)$, and the real income received when $x(s) = 0$ or non-use value, $w\big(s, x(s)\big)$. Assume that term $\pi\left(s, f\big(x(s)\big)\right)$ is observable and measurable from market-related data. In the forestry case this is net harvesting. The term $w\big(s, x(s)\big)$ can be considered as non-use or amenity value as in Hartman (1976) and not directly measurable. In this case, Eq (11) does not hold. There is an extra degree of freedom gained through the assumption implied by Eq (10) – that $\bar{s}$ is chosen at an indifference point. Moreover, some criteria for choosing $\bar{s}$ must be assumed to locate V_s, which is only measurable up to an

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[8] Fenichel, Abbott, and Yun (2018) argue that such a direct amenity flow is not necessarily a non-use type value but may simply be a reduced form approximation to a complex production processes that is heavily dependent on stock s, e.g., water filtration from a forest. In the context of forests, Scarpa et al. (2000) find that average non-industrial private forest owner of Maple-birch forests in Wisconsin is willing to forgo $24 ha-1 year-1 ($9.7 acre-1 year-1) in timber revenue for the non-timber amenities by delaying harvests. Raunikar and Buongiorno (2006) examine revealed willingness to pay of non-industrial private forest owners for the amenities of mixed age and mixed species forests instead of the less natural, but more profitable, even-aged loblolly plantations managed by the forest industry.

affine transformation otherwise. In the previous cases, this identification was achieved through a dynamic equilibrium, when $\dot{s} = 0$ (Fenichel et al. 2018). The choice the boundary plays, $\bar{s}$, plays a similar role.

If the analyst knew the value of π and that $w(s) = 0\ \forall s$ with certainty, then knowledge of condition Eq. (11) and the observation of $\bar{s}$ is sufficient to measure the revealed shadow price. However, when the analyst is confronted with the revealed behaviors that appear to violate Eq (11), which is isomorphic to failing to maximize V, then the analyst is left with three options: (i) assume the decision maker makes decisions that do not align with his or her preferences conditional on the constraints and incentives he or she faces,[9] (ii) assume the deviation results from measurement error in π, stock dynamics, or the discount rate, or (iii) allow $w(s) \neq 0$ to reflect a direct, but unobserved, amenity flow from the stock.[10] Option (iii) seems most likely with respect to a complex natural capital asset like forests given the vast environmental economics literature suggesting such services exist. Though option (ii) is possible in some cases, and the analyst should be especially concerned about measurement errors and the choice of discount rate.[11]

Option (iii) suggests treating $w(s)$ as a residual that is used to ensure that the revealed shadow price is continuous in the stock condition on the observed $\bar{s}$. This enables an approach to measure the revealed non-use value without relying on stated preference. In this case $w(s)$, must be chosen so that Eq (10) holds when $s = \bar{s}$. When $W(s) = \pi\left(s, f\big(x(s)\big)\right) + w(s)$, then $M_s\big(s(t)\big) = \pi_s(s)$, and the revealed shadow price is $V_s(s) = t'(s)\big(\delta V(s) - w(s)\big)$, which must equal $= \pi_s(s)$ when $s = \bar{s}$. However, for problems where management is inaction up to the boundary $\bar{s}$, only one data point is available so $w(s)$ is only recoverable up to a first-order approximation, $w(s) = ws$. By mathematic induction, if the economic program were continuous,

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[9] Using behavioral errors to explain such decisions is trendy. However, Gilboa (2009) argues these results are often dependent on the *analyst* misunderstanding the decision maker's constraints or low stakes coupled with the mental cost of solving the problem. Ketcham, Kuminoff, and Powers (2016) formally test these ideas and show rational behavior, associated with a self-defined objective, cannot generally be rejected for important health insurance decisions.

[10] Wilen and Homans (1998) take a similar approach conceptually to (iii) determine fishery manager's objectives, while Quaas et al. (2012) take an approach similar conceptually to (ii).

[11] We analyze alternative hypotheses about the discount rate in the empirical section. However, for social wealth accounts established social discount rates should be used.

then we could think of the boundary $\bar{s}$ as being state dependent, and it would be possible to identify the *function* of $w\big(s, x(s)\big)$ along the entire observed path of $s(t)$ using the first order condition of the maximum principle to identify the use value. This is most straightforward when the use value involves market transactions, but revealed preference estimates could also be used for non-market use value.

Option (iii) suggests an approach to measuring non-use value without relying on stated preference data. However, option (ii) cannot be ruled out, especially with respect to the discount rate. If the amenity flow is approximated with a first order approximation, then there is a one-to-one mapping between the marginal non-use value, w, and the discount rate so that the revealed shadow price is invariant to changes in the discount rate. The current (though not present) value revealed shadow price is invariant to the discount rate, conditional on observed behavior because of the free parameter w. The implicit function theorem implies $\frac{\partial w}{\partial \delta} = -\frac{\partial V}{\partial \delta}$. The reason for this result is that, conditional on an observed harvest volume, increases (decreases) in the discount rate shift the share of asset value from amenity (timber) to timber (amenity) when $\bar{s}$ exceeds the stock harvest value that satisfies the Faustmann-Hartman rotation.

Fenichel, Abbott, and Yun (2018) review the numerical techniques based on a functional approximation for measuring revealed shadow prices. The intertemporal welfare function can be approximated using functional approximation techniques (Judd 1998; Miranda and Fackler 2002) so that the Hamilton-Jacobi-Bellman relationship, Eq. (4), holds with zero or minimal error at a series of approximation nodes in the space of s (Fenichel et al. 2018). However, these techniques need to be adjusted when $x(s) \notin \mathbb{C}^1$. We focus on the case when $x(s)$ is a "stopping" decision so that $x = 0$ is waiting and $x = 1$ is stopping. The stopping threshold, $\bar{s}$, is empirically observable and forecastable based on a model of the economic program. The functional approximation approach is applied to Eq (4), which simplifies to $\delta V(s(t)) = w\big(s(t)\big) + V_s \dot{s}$, when $s < \bar{s}$, and the functional approximation approach is applied to Eq (8) when $s \geq \bar{s}$. See Appendix A1 for the details of the approximation steps to solve for the shadow prices, $p(s)$, and marginal non-use value flow, w.

Application of Natural Capital Pricing to Forestry: Private Douglas-fir Forest in Western Oregon

We assume that the policy-induced stopping or harvesting decisions reveals the marginal value that forest owners assign to the tradeoff between the non-use net benefits from extending the rotation and the foregone timber revenues. The economic program, $x(s)$, is a binary clear-cut decision (remove all stands) that leads to a harvest $f\left(x(s)\right) = s$ when $x(s) = 1$. We assume that clearcutting is the only source of Douglas-fir mortality on private forest lands.[12] Personal preference or institutional constraints can lead land managers to choose a harvest program that deviates from the Faustmann rotation. Unobserved carrying costs, e.g., pest management or fire risk, may cause managers to shorten the rotation relative to the Faustmann rotation.[13] Alternatively, land managers may have other objectives besides timber production these could range from aesthetics and personal enjoyment of the woods (a la Krutilla 1967) to the ability to sell recreational access. Furthermore, institutional constraints and incentives that reflect a social preference for standing forest, arrived at through public choice and collective action, can also lead deviations from the Faustmann rotation. For example, the qualified forestland owners are taxed at a special reduced property tax rate in Oregon. [14] The timber harvested from any land are taxed except for the first 25,000 board feet by any owner each year.

Holding a forest longer than the Faustmann rotation indicates that managers behave as if the marginal value of standing forest exceeds the marginal value associated with the Faustmann rotation. Programs like Oregon’s special tax program could effectively compensate the forest owner to internalize some of the “ecosystem service” value provided by standing forest and wait longer to harvest. Such tax programs are basically a form of payments for ecosystem services. At the same time, the empirical harvest behavior could reveal unobserved idiosyncratic preferences that contribute to the harvest timing to deviate from the Faustmann rotation. We use this revealed preference for lengthening/shortening the rotation at the expense of lower timber revenue as an approximation for the non-market value of forest. Our objective is to approximate jointly the revealed shadow price of the forest natural capital asset and amenity income flows revealed by the landowners’ behavior.

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[12] In Oregon, where we apply our framework to private forestlands, as well as Washington and California, for example, harvesting accounts for between 71 and 87 percent of mortality on non-federal land.

[13] Reed (1989) shows that when these risks are exogenous and arrive as Poisson process, then the risk effectively increases the discount rate. We investigate required changes in discount rates later in this paper.

[14] Under the forestland program (ORS 321.27-390), the landowners apply to the county assessor to be qualified as “highest and best use” and “designated” forestlands.

We observe clear-cut decisions at the plot level between 1990 and 2014 using data from the Oregon Department of Forestry (ODF). The data consists of individual records submitted by the harvesters as they are required to notify ODF of clear-cut operation at least 15 days before starting the operation. Each notification record includes acres harvested, volume harvested, location of activity at the quarter (40-acre block) level, ownership types (e.g., non-industrial or industrial), owner's names, total acres under the management, and year of activity. We calculate the per-acre harvested volume in thousand board feet for each unit of observation, which is each notification record for the contiguous forest plots. We use the data of per-acre harvested volume to non-parametrically estimate the shadow prices and amenity flow under a 7 percent discount rate. A 7 percent discount rate is the suggested real discount rate by the US Office of Management and Budget (OMB) for investments in the private sector (Office of Management and Budget 2003). It is the low end of the weighted average cost of capital for publicly held forest products firms operating in Oregon.[15] We also estimate the shadow prices imposing 3 percent discount rate, which is recommended by OMB as a constant, conservative, consumption rate when a "social rate of time preference" is required.

In order to model the growth trajectory of a forest plot and estimate the value of forest conditional on the observed harvest timing, we need tree growth yield curves that link stand volume and corresponding age. Since ODF data does not include stand age, we use the Forest Service's Forest Inventory Analysis (FIA)[16] that surveys plot-level stand volume and age. Using the empirical stand volume and age from the FIA data, following Chang (1984), the relationship between the stand volume for each site productivity group, s_{sc}, and age are estimated as

$$(12)\qquad s_{sc}(age) = \exp(\beta_{1\,sc} * \frac{1}{age} + \beta_{2\,sc} * \frac{1}{age^2})$$

Once we fit the yield curves for each group, we fit the growth function per thousand board feet, $\dot{s}(s)$, as a function of volume to map from the time to state domain.

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[15] Bloomberg data. The WACC for Weyerhaeuser, CatchMark, and Rayonier are 8.1%, 6.0%, and 7.2%, respectively in the most recent quarterly report (4Q 2018). Their WACC ranges roughly between about 6 percent and 10 percent historically from 2008 to 2018.

[16] Annual inventory surveys record biophysical conditions and disturbance information at each plot for the forestland and timberland outside of national forests and most reserved areas. Our sample includes about 700 privately-owned Douglas fir plots located in western Oregon. FIA data is available at https://www.fs.fed.us/pnw/rma/fia-topics/inventory-data/index.php. The summary statistics of the FIA data are included in Appendix.

(13) $\dot{s}(s) = b_0 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4$

The purpose is to map a standard growth and yield equation into the state space in sample. We apply this general growth equations to the western Oregon data set. Appendix Fig. 1 and Fig. 2 present the relationships between the volume and age (Eq. 12) and growth and volume (Eq. 13) respectively.

We allow the timber prices to rise with increased board feet per acre because for a given site productivity class the price is positively correlated with larger trees. To incorporate the price premium for the larger size, the timber price is calculated based on the harvested volume by estimating the relationship between the 10-year average timber prices and the minimum stand volume required to meet the grade.

(14) $p = a * \ln(vol) + b + \epsilon$

where p is 10-year average timber price between 2005 and 2014 at each size grade, vol is the minimum volume required to meet each size grade across the harvest age of 30 and 50, a and b are parameter to recover, ϵ is a residual. The required minimum size is calculated with diameter requirement, tree height that corresponds to the diameter based on the growth table, and the growth equation that converts diameter and height to the stand volume per acre (Zhou and Hemstrom 2010; Oregon Forest Resources Institute 2018). Appendix A2 lists assumptions used in the price model.

In the raw ODF data, owners are divided into three groups: partnership, corporation, or industrial owners ("industrial owners" thereafter), which make up 63 percent of the raw data, private individual or non-industrial ("non-industrial"), which make up 36 percent of the raw data, and a small fraction (1.5%) is non-profit owners. For this analysis we focus on industrial owners because harvesting likely has nontrivial income effects for non-industrial owners, which adds the challenge of needing to account for the marginal utility of income in decision making. Additional data cleaning process includes omitting owners with less than 10 acres and dropping the greatest and smallest half percentage of harvest volumes to remove the most extreme values. Summary statistics for our final data are presented in Table 1. Most industrial owners own more than 5,000 acres of forestland and their harvest age is roughly normally distributed (Fig.1).

Results

For each owner type by site productivity group, we estimate the revealed shadow prices of a standing volume per acre based on the "average behavior" rotation choice. The shadow price is a function of observed harvest volume (the economic program), tree growth, which varies across site productivity groups with greater site productivity groups being associated with greater yield at age, and timber prices, which also vary across site groups because of the mapping between volume and timber price as well as across plots, since timber price is endogenously determined in our estimation based on the harvest volume (Eq. 14).

To illustrate results, consider industrial owners on site class 3, the medium to poor sites. All industrial owners follow rotations longer than a Faustmann rotation, assuming a 7 percent discount rate (Table 2), but on site class 3 the difference is most pronounced and easiest to see. The revealed shadow price associated with the average behavior on these sites initially declines with volume and then increases (Fig 2, blue solid curve). The shadow price, or marginal value of standing volume, associated with the Faustmann rotation follows a qualitatively similar pattern (Fig 2, black solid curve).[17] The smooth pasting condition requires that the shadow price intersects the marginal profit curve from below (Fig 2, grey curve) at the time of harvest. A feature of these shadow price curves is that they slope up as the predicted harvest volume approaches. This is because there is a one-to-one relationship between volume and the delay between the current period and harvest for timber, because of the waiting nature of the decision. All shadow prices are measured in the current period when the standing volume is of a given size, and the value of the capital increases as the harvest period approaches because of pure time effects and the lumpy nature of net benefits from harvest. Normalizing the time period to the one with the smallest volume shown, the present-value price curve (Fig 2, dashed curves) has the typical shape.[18]

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[17] Average behavior in this case refers to the average harvest volume across industrial owners in site class 3 during the entire sample period, 1990-2014. Fig.2 also shows two other revealed shadow prices, one based on the average harvest volume during the period 2001-2006 (brown curve) and another based on the average harvest volume during the period 2011-2014 (green).

[18] These curves are included only for illustrative purposes. The current period asset price is the one to focus on.

The representative site group three owner's 57-year rotation is 13 years longer than the 44-year Faustmann rotation with a 7 percent discount rate (Table 2). This implies a non-use "amenity" flow from standing forest that raises the instantaneous measure of the intertemporal welfare function above the value associated with the timber harvest alone (Fig 3). Owners of these stands are acting as if the stands provide substantial ecosystem services flows that capitalize into the live forest volume. This additional value should be accounted for in changes in the intertemporal welfare function. Indeed, an amenity flow of $15 per unit standing volume is required to satisfy the asset pricing conditions (Table 2). An alternative explanation for the observed behavior would be that the industrial owners are following a Faustmann rotation but with a discount rate less than 7 percent. The discount rate that rationalizes the observed rotation as a Faustmann rotation is 3.7 percent (Table 2) – much lower than the reported weighted average cost of capital of publicly traded forestry firms and close to the OMB's discount rate for public projects that affect private consumption only. Any program impacting forest affects private allocation of capital related to timber provision, but likely also affects private consumption related to amenity services. Therefore, which OMB rate should be used is unclear. On all site classes the representative industrial owner acts as is if there is a positive non-use amenity flow (Table 2).

It is possible to use the mean behavior and expected stand growth to get an approximate value for the flow of non-timber amenities from Oregon Douglas fir forests and to consider how wealth stored in Oregon Douglas changes through time. The former estimate is the appropriate service flow for use in ecosystem income and produces accounting, e.g., SEEA–EEA accounts. The latter asset price provides a change in wealth that is appropriate for use in wealth-base sustainability accounts such as the Inclusive Wealth Report or the Changing Wealth of Nations. To provide examples of both types of calculation we use FIA data that averages over the period 2001-2006 and over the period 2011-2016 (Palmer et al., 2018).[19]

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[19] We thank Andrew Gray at the Pacific Northwest station for conducting a custom query.

The value of amenity flows is the estimate of the per-standing volume amenity flow by the standing volume. The amenity flow value is distinct from the timber value and is associated with some service that the standing forest provides other than future timber harvest. The methodology only allows a first order approximation so that the amenity flow must be age or volume invariant. Therefore, we use the FIA data (Appendix Table 1) to summarize the standing volume by owner and site class in each of the two periods and multiply these by the owner type and site class specific amenity flow values based on the average observed harvest volume in each period. There is a shift in standing volume from non-industrial to industrial owners between the periods (Table 3). This alone would be expected to raise the value of the amenity flows or "ecosystem income." In addition, the forest owners harvest at the later time on average in the 2011-2016 period compared to the 2001-2006 period. In the 2001-2006 period, the behavior of forest managers reveals that standing forest provides a net benefit or amenity of $176 million, but changes between the periods lead to an increase in amenity flows on the order of $164 million in the latter period. This change in amenity value emphasizes the importance of management and the economic program in generating value. The revealed shadow prices associated with the average behavior in these two periods also indicate that the Douglas fir forest increased its asset value over time (Figure 2).

The results are similar for changes in wealth held in the form of the Douglas fir forest asset. Wealth is based in the acres of a given age on a given site class for an owner type. FIA data are available in the form of 10-year age classes. The FIA data classify forests based on a dominant type. Therefore, caution is required for interpreting results because forest type can change if Douglas fir trees become the dominant species because other pioneer species die back or because Douglas fir is overtaken by other species. Additionally, areas with less than 10 percent cover are not classified as Douglas fir. This is an issue for young stands. We assume all acres in the age class are at the mid-point of the age class and use the inverse of growth in volume equation, Eq (12), to map to volume. Then, we assign a value based on the intertemporal welfare function (which is the area under the price curve) based on the observed harvest volume in each 5-year period and calculate a change in wealth. This provides an approximate change in wealth since it does not track the changes in individual acres. The FIA data report 3.7 million acres in Douglas fir forest in the 2001 to 2006 period and 4.0 million acres in the 2011 to 2016 period. The difference may come from planting new stands, privatization or sales from owner types excluded from the data set, changes in stand classification, or measurement error. A combination of the age dynamics of stands and the relative prevalence of non-industrial to industrial ownership (and increase in amenity flow) leads to a change in wealth held in industrial Douglas fir forest on the order of $5.9 billion. This equates to about a $196 increase in value per acre per year on average. However, care is required when interpreting these results, because we only measure the changes in the value of the industrial Douglas fir holdings – not forests more broadly. Much like a fund, some assets may grow while others decline overtime. Therefore, our results are not sufficient to inform "sustainable forest management," but expansion of the methodology to cover all forest types could be. Over the period studied, one could infer that Douglas fir is sustainably managed, to the extent that the economic questions differ from simply asking "is there more Douglas fir in the 2011-2016 period relative to 2001-2006?" One reason these questions might differ is the ownership types can change implying different economic programs, another is that behavior of owners may change between periods. We account for the latter by differentiating harvest behaviors across the two periods while assuming the ownership types stay the same.

Discussion

Measuring revealed asset prices requires revealed behavior. This is a challenge when the management of the majority of real assets, particularly natural capital, mostly involves doing nothing and the only occasion making a large adjustment to the stock, such as harvesting a forest. Nevertheless, capital theory provides a path to a revealed asset price, by requiring the price function to be continuous in the stock even when the decision rule is not.

Environmental and conservation policy concerns related to forests focus on amenity flows going unpriced, e.g., managers acting as if the value of the amenity flow is zero or negative. In the case of Oregon Douglas fir forests on average industrial owners act as if they derive some value from standing forests, though it may still may differ from the social optimum. Nevertheless, tracking these values through time is important because society may "do better or worse" even if management is never optimal.

The procedure for measuring amenity flows can be extended to forest areas that are never harvested but could have been, i.e., forests set aside for conservation. The Hartman (1976) model is clear that if w is sufficiently large, then it can be optimal to never harvest. Therefore, one could reasonably assume that for a conservation forest, the lower bound on the amenity flow is the Hartman amenity contribution that makes it optimal to forgo harvest. This amenity may be more reliable for the purposes of environmental or ecosystem accounting of forest that have been set aside in perpetuity than current efforts to construct amenity flows through ecological production functions and then summing multiple services. Furthermore, this sufficiently large value of w and a discount rate is sufficient to solve for a lower bound asset price for wealth accounting. An important feature of our measure of w is that it can provide a benchmark for "ecosystem service" valuation developed constructively from directly trying to map ecological production functions into marginal willingness to pay. For example, our model calculates that the average forest landowner in site group 1 in western Oregon would forgo harvest permanently if the amenity flow were at least \$68 per thousand board feet. The minimum amenity flow that is required for the owners in other site groups to give up harvest is \$62, \$53, and \$22 (all per thousand board feet) for site group 2, 3, and 4, respectively. These values could be used as lower bounds for valuation of preserved forest not included in our analysis.

Private and commercial forest owners reveal positive shadow prices for standing timber despite the large difference in the valuation of amenity flows. This is not surprising because a substantial share of the asset value is related to timber harvest, which is a private good. Nevertheless, it is clear that some future non-use amenity flow value is capitalizing into the revealed shadow price for Douglas fir forests in Oregon on some sites. Moreover, these shadow prices, their associated intertemporal welfare measures (the integral under the shadow price curve), and forest inventory can be used to provide a reasonable approximation for the revealed change in welfare associated with changes in a forest type. Those changes in welfare can be interpreted as changes in wealth. We find that the wealth stored in Douglas increased between approximately 2003 and 2013.

The difference in the results between amenity flow and asset prices highlights the difference between sustainability and efficiency. The asset price results broadly suggest the forest is being managed sustainably, while the amenity flow results suggest that the forest is probably not being managed efficiently – especially if there are substantial non-timber services from these forests. In order to increase efficiency and maintain sustainability, it appears substantial monetary transfers to non-industrial forest owners may be necessary in order to alter behavior.

Asset prices are fundamentally forward looking. Fenichel and Abbott (2014) and Fenichel et al (2018) show examining the actual conservation and utilization behaviors in society reveal the implied asset prices. Yet, prior approaches relied on observing behavior that continuously responded to changes in the stock. Most natural resource management involves doing nothing with punctuated periods of action. These punctuated actions are sufficient to measure asset prices for stocks of natural capital. Furthermore, such discrete changes in stocks may be easier to observe and model than the continuous adjustments that were previously analyzed. This opens up more types of natural capital assets to pricing, which should help society better assess whether collective wealth is declining – whether society is passing a sustainability test. This is true even when resources are inefficiently allocated.

Figures and Tables

Table 1: Summary statistics of industrial owners by site class.

site class*	1	2	3	4	All
Harvest units	6,668	8,806	2,030	427	17,931
% harvest units	37.2%	49.1%	11.3%	2.4%	NA
Total acres owned	220	215	52	12	499
10 -99 acres					
100 - 499 acres	190	265	56	23	534
500 - 99 acres	296	398	219	105	1,018
1,000 - 4,999 acres	194	343	79	26	642
> 5,000 acres	5,768	7,585	1,624	261	15,238
% 10 -99 acres	3.3%	2.4%	2.6%	2.8%	2.8%
% 100 - 499 acres	2.8%	3.0%	2.8%	5.4%	3.0%
% 500 - 99 acres	4.4%	4.5%	10.8%	24.6%	5.7%
% 1,000 - 4,999 acres	2.9%	3.9%	3.9%	6.1%	3.6%
% > 5,000 acres	86.5%	86.1%	80.0%	61.1%	85.0%
mean acres	53.3	54.0	55.7	53.0	53.9
mean harvest price	702.2	683.1	635.8	518.5	680.9

* Each site productivity group indicates a different productivity of the plot, ranging from the most productive (1) to the least productive (4). The grouping is based on seven classifications of "site class", a classification of forest land in terms of inherent capacity to grow crops of industrial wood expressed in cubic feet/acre/year. Site productivity group 1 includes site class 1 and 2, group 2 includes site class 3, group 3 includes site class 4, and group 4 includes site class 5. We dropped the lowest productivity group (site class 6 and 7) as it constitutes only 1% of the sample. Since the harvest notification records do not specify the site productivity group for each site, we overlaid the locations of the plots on the USDA Forest Service Forest Inventory Analysis data to assign the site productivity class.

Table 2: Results of Faustmann analysis, economic program description, amenity values and discount rates.

	Site class			
	1	2	3	4
Faustmann harvest age 7% discount rate	36.8	37.6	43.8	33.1
Faustmann harvest age 3% discount rate	52.7	52.7	62.9	45.1
Faustmann harvest volume 7% discount rate	23.5	18.8	11.3	7.5
Faustmann harvest volume 3% discount rate	43.8	33.4	21.8	11.4
Mean observed harvest age	37.4	41.7	57.0	47.1
Mean observed harvest volume	24.5	22.8	18.7	11.2
Amenity value per MBF				
7% discount rate	$0.91	$7.04	$15.40	$13.77
3% discount rate	-$17.39	-$11.29	-$2.88	$0.22
Discount rate rational behavior as a Faustmann rotation for industrial owners	6.6%	5.4%	3.7%	2.9%

Table 3: Results related to standing volume and amenity flow.

Site Class	Volume 2001 to 2006 (1,000s MBF)	Volume 2011 to 2016 (1,000s MBF)	Change in Volume (1,000s MBF)	Amenity value 2001-2006 in millions	Amenity value 2011 - 2016 in millions
1	15,139	19,323	4,184	$20.43	$92.45
2	12,998	15,146	2,148	$91.50	$165.44
3	3,459	3,865	405	$54.88	$68.26
4	837	1,165	328	$9.91	$14.55
Totals	**32,433**	**39,498**	**7,065**	**$176.72**	**$340.70**

Corresponding change in volume for the non-industrial owners

Site Class	Volume 2001 to 2006 (1,000s MBF)	Volume 2011 to 2016 (1,000s MBF)	Change in Volume (1,000s MBF)
1	7,532	4,686	-2,845
2	6,436	4,486	-1,951
3	1,673	1,587	-86
4	804	1,443	639
Totals	**16,445**	**12,202**	**-4,243**

Note: MBF = thousand board feet in Scribner scale.

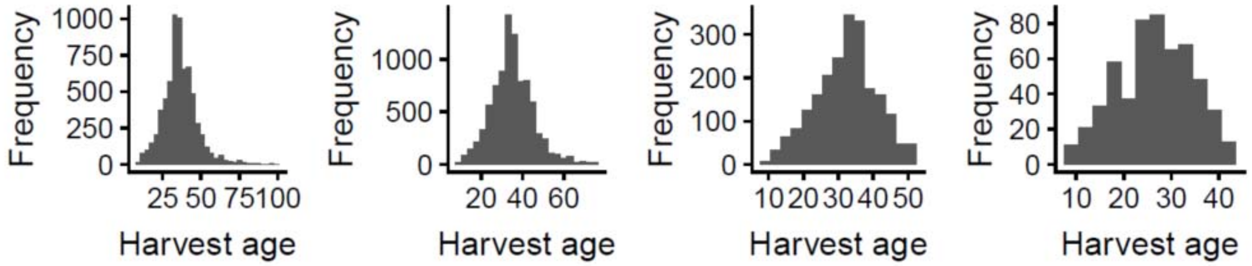

Figure 1: Age at harvest by site group. Site quality goes from 1 to 4, left to right.

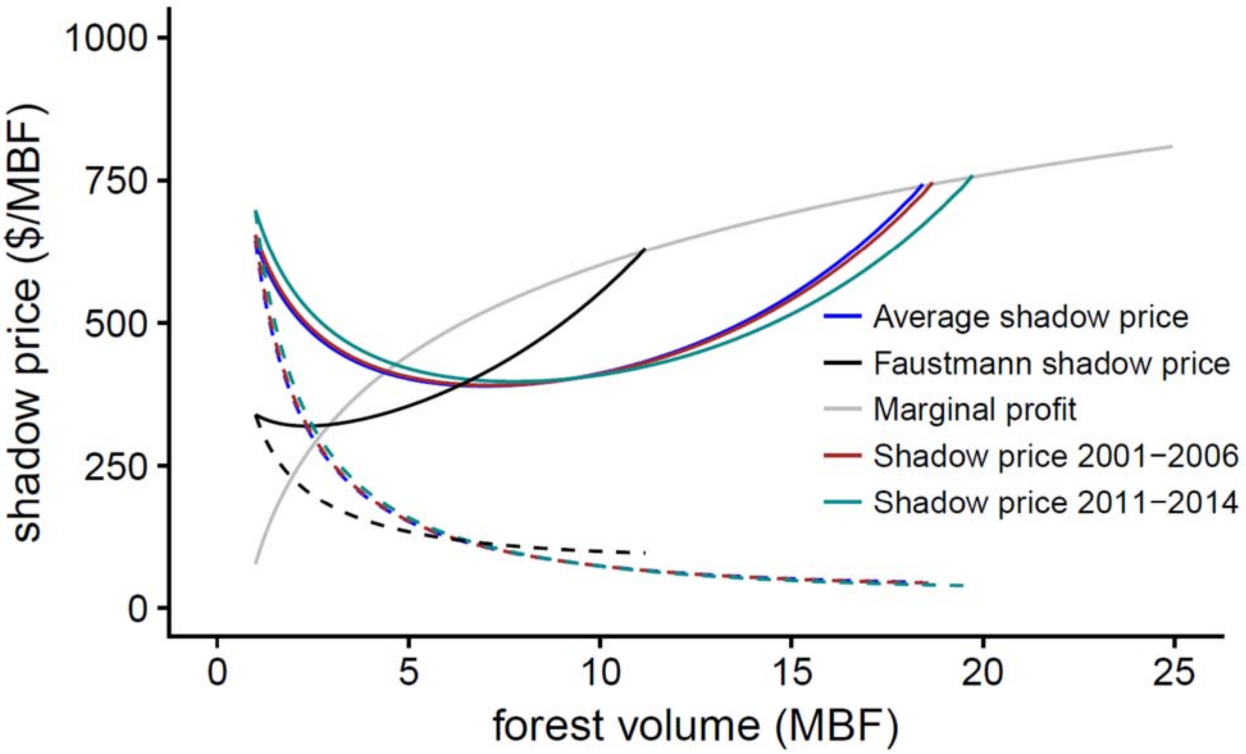

Figure 2: Shadow price of the average (years 1990-2014) industrial owners in the low-medium site classes (site group 3) with 7% discount rate. Blue curves represent current (solid) and present (dashed) shadow prices based on the observed average harvest volume during the entire period (1990-2014). Black curves show the current (solid) and present (dashed) shadow prices associated with the Faustmann rotation with a 7% discount rate. The gray curve is the marginal profit curve, which defines the smooth pasting condition. Red curves and green curves represent current (solid) and present (dashed) shadow prices based on the observed average harvest volume in 2001-2006 and in 2011-2014, respectively.

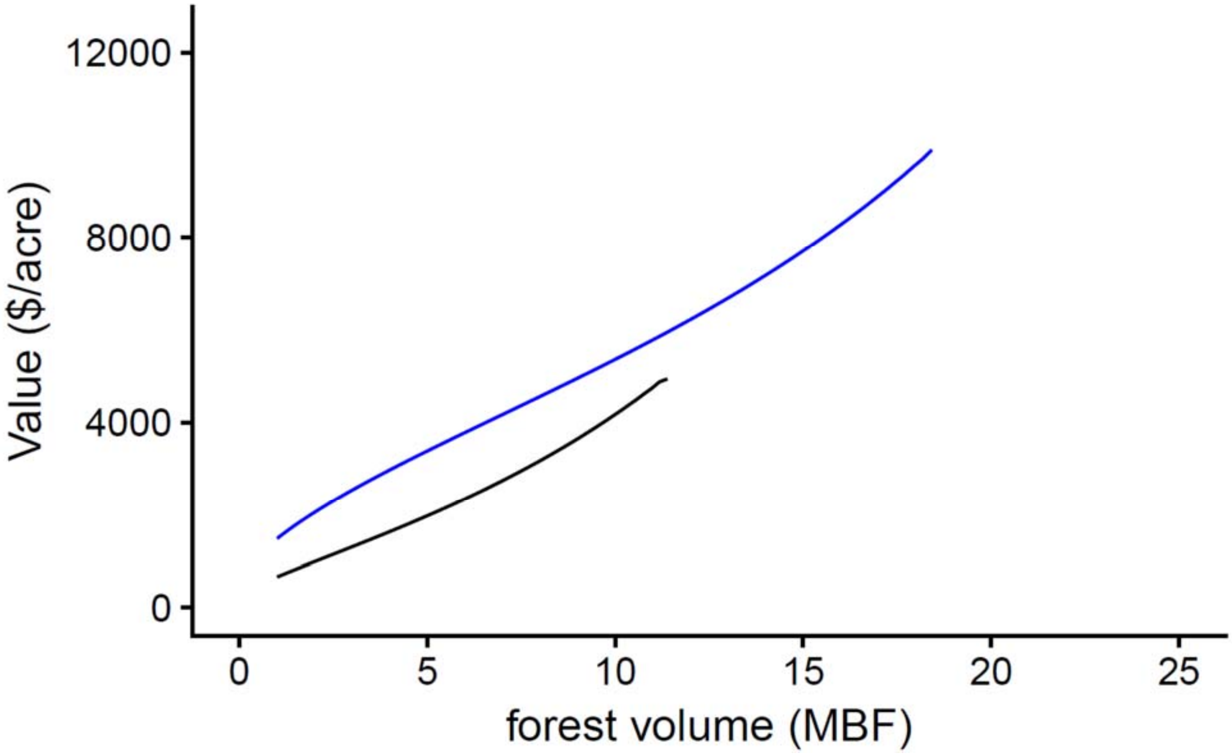

Figure 3: The value of intertemporal welfare function by standing stock volume for industrial owners in the site group 3 with 7% discount rate.

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Appendix

A1. Functional approximation

The marginal benefit right before the stopping is

(A1.1) $V_s(\bar{s}) = \pi_s(\bar{s}) = t'(\bar{s})\big(\delta V(\bar{s}) - w(\bar{s})\big)$

Solving for w_s yields

(A1.2) $w_s(\bar{s}) = \delta V(\bar{s}) - \dot{s}(\bar{s})(V_s(0) + \pi_s(\bar{s}))$

We solve for the w_s such that the Eq. (A2) holds, then approximate the intertemporal welfare function and marginal benefit function using functional approximation: $V(s) \approx \mu(s)\beta$ and $V_s(s) = p(s) \approx \mu_s(s)\beta$. We approximate the function at N nodes, where a node is a point in the space of forest stock. At each of N approximation nodes, s, let $\mu(s)\beta$ approximate an unknown function. Here, $\mu(s)$ is and $N \times K$ matrix of k distinct basis functions and β is a $k \times 1$ vector of coefficients that weight the individual basis functions.

(A1.3) $\delta V(s) = \pi(s)\sigma(s) + w(s) + diag(\dot{s})V_s(s) \approx \delta\mu(s)\beta = \pi(s)\sigma(s) + w(s) + diag(\dot{s})\mu_s(s)\beta$

where $diag(\dot{s})$ is the diagonal matrix operator having the elements of $\dot{s}$ on the diagonal and zeros elsewhere. This implies

(A1.4) $(\delta\mu(s) - diag(\dot{s})\mu_s(s))\beta - \pi(s)\sigma(s) - w(s) = 0$

which yields

(A1.5) $\beta = \big(\delta\mu(s) - diag(\dot{s})\mu_s(s)\big)^{-1}(\pi(s)\sigma(s) + w(s))$

The value $\mu_s\beta$ at given s provides the approximate value of $p(s)$. The value of income flows, $\pi(s) + w(s)$, and the rate of change of forest stock $\dot{s}$, each of which are the results of economic program, are the critical pieces of data for the reliable identification of the coefficients.

A2. Assumptions used in timber price modeling

In Eq. (14), the timber prices are estimated using the average timber prices between 2005 and 2014 and the minimum volume required to meet each size grade (Table A2.1), assuming that harvest occurs between 30 and 50 years of time, totaling 27 data points. The minimum volume is calculated from the minimal diameter at breast height (DBH) and expected height at the harvest using the following relationship between the volume, DBH, and height (HT) for Douglas fir (Zhou and Hemstrom 2010):

(A2.1) $CVTS =$

$$10^{-3.21809+0.04948*\log(HT)*\log(DBH)-0.15664*(\log(DBH))^2+2.02132*\log(DBH)+1.63408*\log(HT)-0.16185*(\log(HT))^2}$$

where CVTS is cubic-foot volume of total stem, ground to tip.

CVTS is then converted to thousand board feet. Since it is a volume at the stand, we expand it to per-acre measurement by assuming that the landowner harvest 70 stands per acre.

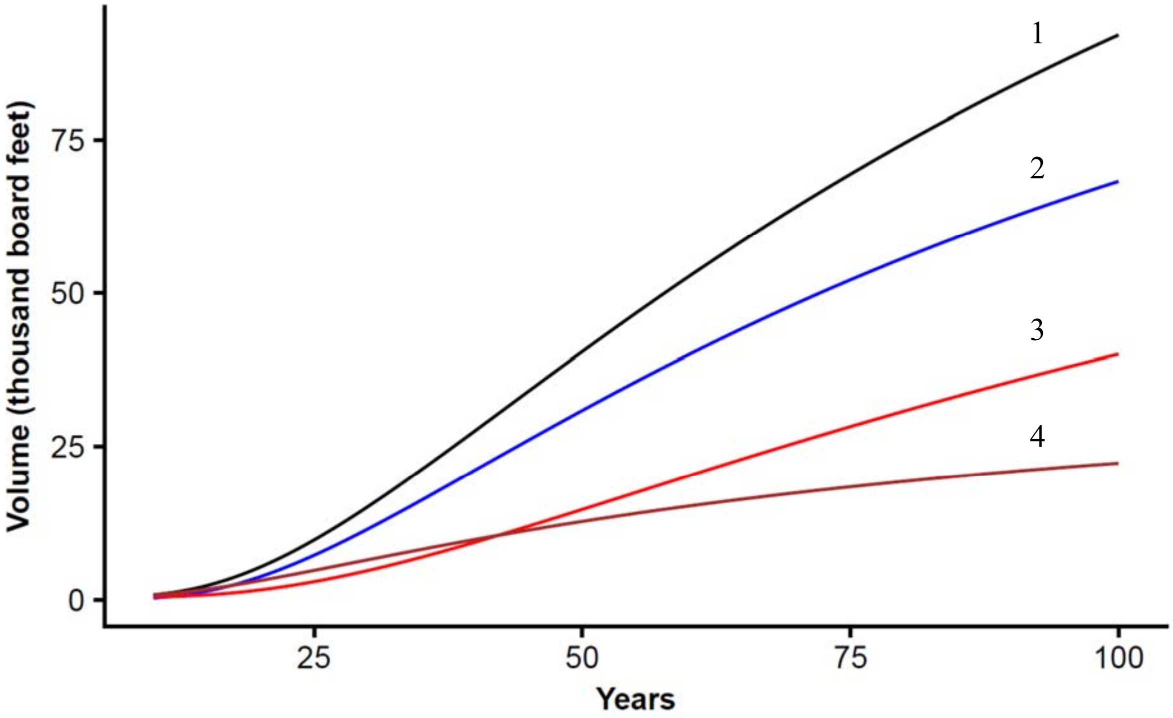
Table A2.1

Grade	Avg. price 2005-2014 (2014 $)	Minimal DBH (inch)
1P	$754	30
2P	$732	24
3P	$705	20
SM	$647	18
2S	$592	16
3S	$550	12
4S	$524	10
SC	$175	8
Pulp/Utility	$102	6

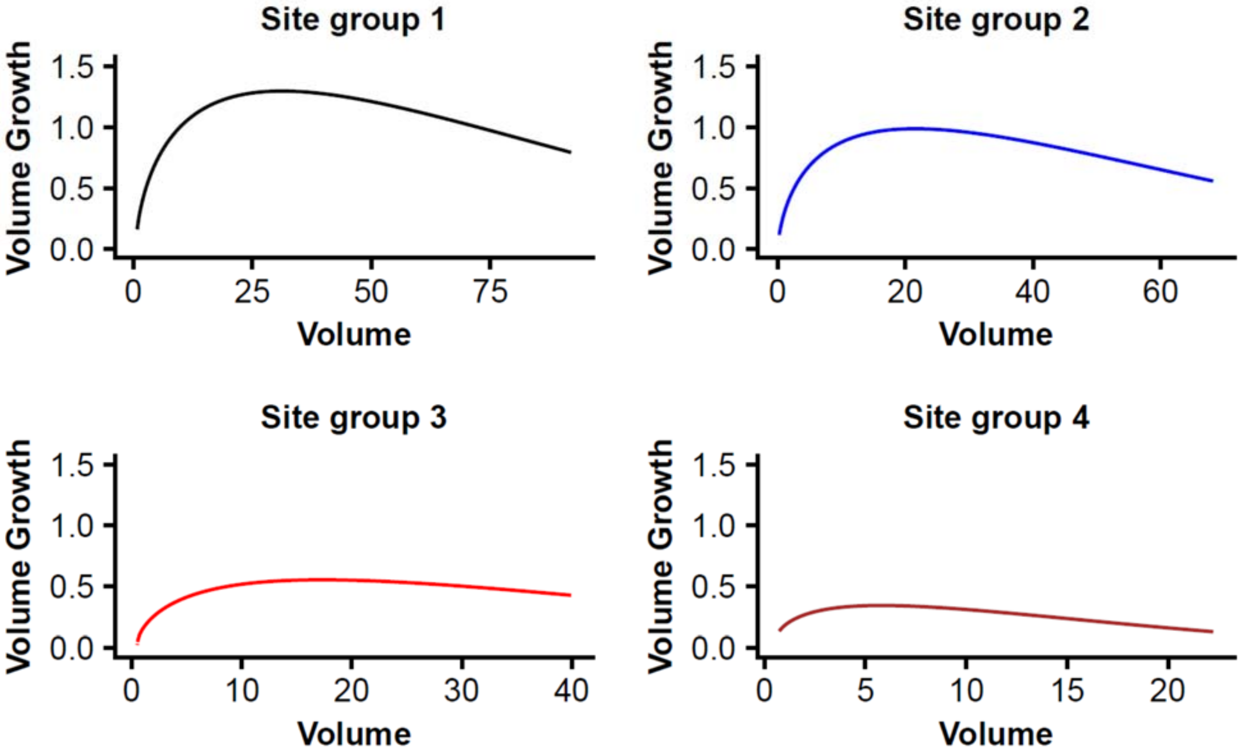
Appendix Table 1: Summary statistics of FIA data (private plots only) used in estimating yield curves.

FIA site class*	1	2	3	4	5	All
Count	11	246	343	103	43	760
Acres						
Mean	6,151	5,123	5,120	4,861	4,487	5,059
Std. dev.	907	1,879	1,918	1,963	2,134	1,921
Per-acre volume (MBF)						
Mean	9	18	13	10	11	14
Std. dev.	13	24	18	12	9	19
Stand age (years)						
Mean	23	33	36	43	60	38
Std. dev	18	23	57	24	30	43
Elevation (feet)						
Mean	845	1,090	1,460	1,828	1,942	1,418
Std. dev	537	642	916	1,105	1,027	923

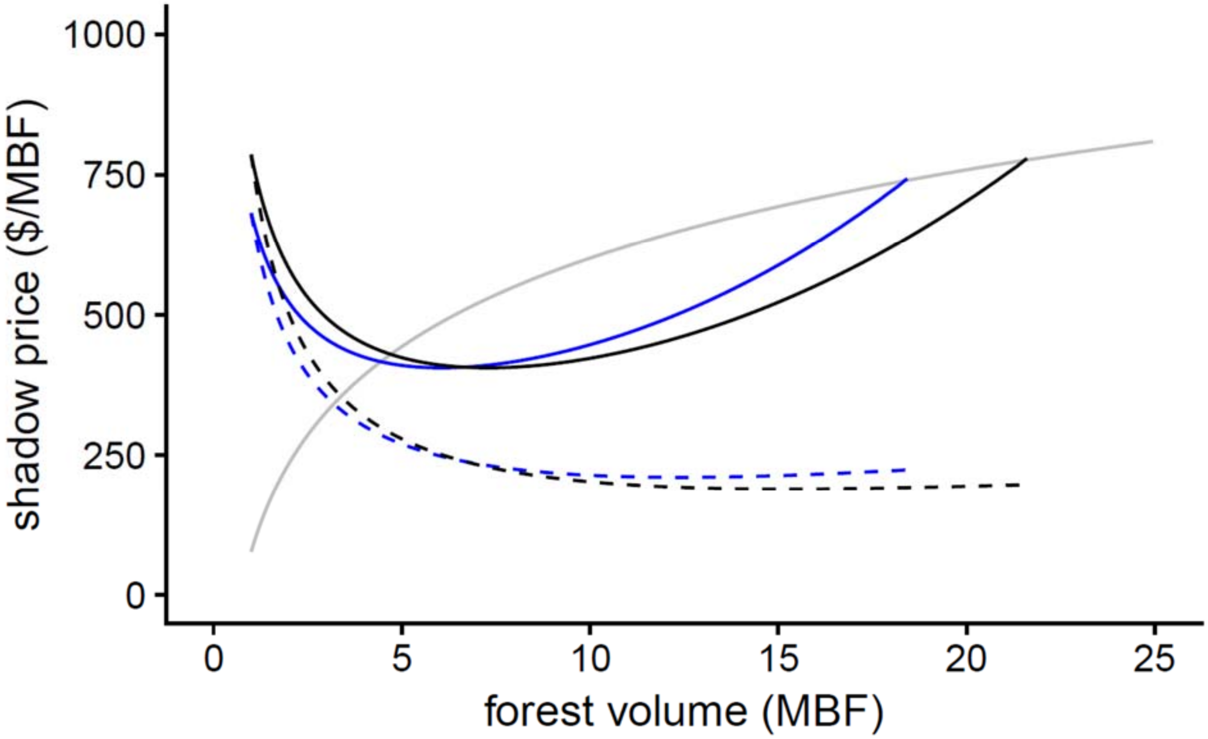
*In our analysis, we use FIA site class 1 and 2 to estimate yield curve for our site group 1, FIA site class 3 for our site group 2, FIA site class 4 for our site group 3, and FIA site class 5 for our site group 4.



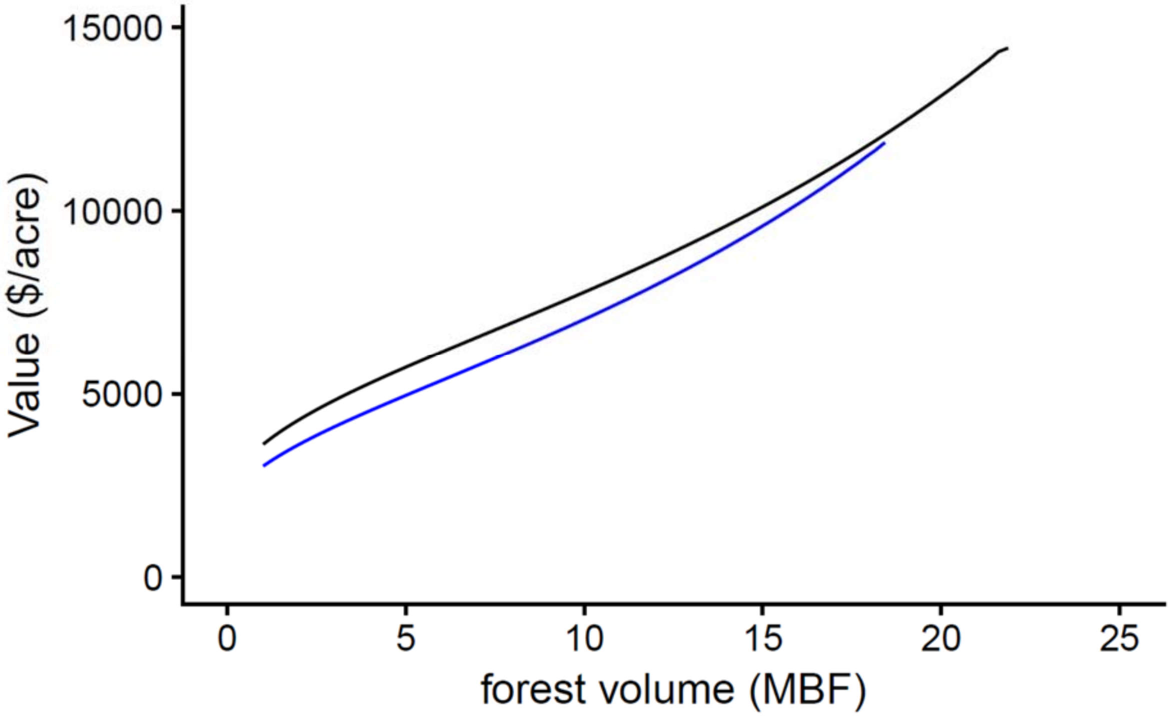
Appendix Figure 1: Volume as a function of age calculated with the FIA data (Eq. 12). The curves correspond to our site group 1, 2, 3, and 4 from the highest to the lowest.



Appendix Figure 2: Volume growth as a function of volume (Eq. 13) by site group.



Appendix Figure 3: Shadow price of the average industrial owners in the low-medium site classes (site group 3) with 3% discount rate.



Appendix Figure 4: The value of intertemporal welfare function by standing stock volume for industrial owners in the site group 3 with 3% discount rate.