Discounting, inclusive wealth and sustainability

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Abstract

Capital approach to sustainability focuses on whether wealth as an aggregate of capital assets is not on the decline over time. Although sustainability hinges on how we frame intergenerational ethics, the role of discounting in this sustainability assessment has not been extensively studied yet. This paper rebuilds the produced, human, and natural capital framework, in which the role of discounting in shadow prices of capital assets is clarified. We uncover how relevant parameters — such as the pure rate of time preference, elasticity of marginal utility, consumption and natural capital growth rates, marginal regeneration of natural capital — affect human and natural capital income discount rates, shadow prices, and the level and change in inclusive wealth. Numerical examples for selected countries demonstrate that, among other results, under a plausible set of parameters and assumptions, human capital income discount rates are likely higher than forest capital income discount rates.

Keywords: genuine savings; inclusive wealth; sustainable development; discounting; shadow prices

JEL codes: C21; C22; O43; Q20; Q30; Q56

1 Introduction

How have economies been doing in terms of sustainability? Among a number of indicators, capital approach to sustainability focuses on a relevant set of capital assets in economies (Pearce and Atkinson 1993; Hamilton and Clemens 1999; Asheim 2000; Dasgupta and Mäler 2000; World Bank 2011; UNU-IHDP and UNEP 2012; 2014). The basic idea is that, as long as that set of capital assets are not on the decline, productive base to meet the needs of future generations is secured. The relevant list of capital assets, when aggregated in one measure, is called inclusive or comprehensive wealth, whose change is cosistent with the idea of genuine savings.

In assessing sustainability of an economy by its capital assets, the notion of shadow prices has been reinvoked to play a major role in attaching weights to individual capital assets. The shadow price of a given capital asset tells us how much value it would add to intergenerational well-being were it to increase on the margin. By appealing to analogy with financial assets, the shadow price should signify the present value of the stream of service flow the studied capital is expected to yield. This reminds us that intertemporal discounting should make a vital role in the determination of shadow prices, as discounting directly affects how much capital service at a time in the future is valued from the viewpoint of the present generation.

Intuition suggests that higher discounting means less imporantce is attached to well-being of future generations, and thus less care is taken about the decline in forest stock, for instance. To quote a seminal work in wealth accounting (Arrow et al., 2012), "if the conception of intergenerational wellbeing involves the use of high discount rates on the wellbeing of future generations (i.e., if δ is large), the influence on today's shadow prices of future scarcities would be attenuated. Intergenerational ethics plays an important role in the structure of shadow prices."

Despite the alleged importance, and in contrast to a continuing and heated debated on the role of discounting in the econmics of climate change, few, if any, previous studies have investigated the role of discounting on capital-based sustainability in a consistent manner. Indeed, the role of discounting has been casually discussed in sensitivity analysis in applied work (Hamilton and Liu, 2017; Lange et al. 2018). Arrow et al. (2012), for example, show such a sensitivity analysis with regard to the discount rate applied to additional years of life for 3%, 5%, and 7%, although their results have been shown not so senstive to discounting. Hamilton and Liu (2017) show sensitivity analysis of 4% and 5% to inclusive wealth which includes human and natural capital. As expected, and as will be formally shown in the current study, applying higher discounting results with lower wealth.

In the current study, we re-build the wealth accounting framework of produced, human, and natural capital, in which the role of discounting is studied. We first examine how discounting plays out in the theory of wealth and well-being. In the wealth accounting framework, the discount rate appears in either the shadow price of the capital in question.[1] In particular, we are interested in what constitute *capital income discount rates*, and how the constituent parameters affect shadow prices. Moreover, in the final analysis, they affect the level of inclusive wealth, and more critically to sustainability, the change of inclusive wealth. For example, we can confirm that a rise in the pure rate of time preference and inequality aversion would translate into a decrease in social well-being and wealth, by lowering human and natural capital shadow prices (as long as the weighted sum of consumption growth and natural capital growth is positive). Along with theory and practice, the empirical magnitude of the sensitivity with regard to discounting is also of interest.

In our analysis, we show how we can tap into recent developments of social discounting for climate change (e.g., Stern 2006; Hoel and Sterner 2007; Dasgupta 2008; Gollier 2010; Traeger 2011; Drupp et al. 2018), as well as natural capital accounting (Arrow et al. 2012; Fenichel and Abbott 2014). In climate change policy, what matters is the damage cost imposed on future generations, which is frequently assumed to be proportional to GDP (Nordhaus, 2008). The basic idea behind climate change discounting is that, as long as future generations are expected to be richer than the present generation, the damage cost should be discounted. What distinguishes our study from the climate change discounting is that our focus is not on consumption goods but on human and natural capital income, so that obviously we are in the realm of dual (or more) discounting, as different capital income should be discounted at differentiated rates. As a result, the consequences of higher or lower discounting on sustainability are more unclear than on climate policy.

The rest of the current paper is organized as follows. In Section 2, the basic framwork of inclusive wealth accounting and sustainability assessment, followed by capital dynamics equations we assume, is presented. In Section 3, utility and production function forms are specified to derive capital income discount rates to be employed in capital shadow prices. The effect of critical parameters, including

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[1] We can think of a setting where discounting appears in the net investment of the capital. In fact, health capital is formalized in Arrow et al. (2012) in such a way that discounting is embodied in the remaining life years, not the shadow price of that remaining years.

the pure rate of time preference, is also examined. In Section 4, we examine the extent to which inclusive wealth accounts are sensitive to discount rates using selected countries data. Concluding remarks are made in Section 5.

2 Analytical framework

2.1 Inclusive wealth accounting and sustainability assessment

Following and extending Arrow et al. (2012) and others in the wealth accounting literature, denote social well-being at t by

$$V(t) = \int_t^\infty U(C(\tau), S(\tau))e^{-\delta(\tau-t)}ds, \tag{1}$$

where $U(C(\tau), S(\tau))$ is utility flow at τ, $C(s)$ is a vector of consumption flows at τ, and $\delta \geq 0$ is the utility discount rate. The addition of natural capital, $S(\tau)$, in utility function is not standard in the theory of wealth accounting, and thus requires a justification. As we will discuss further later, the shadow price of natural capital now includes non-consumption, amenity values, in harmony with the well-established empirical literature of ecosystem services.[2] Produced and human captial are not arguments of utility, as their shadow prices in practice do not have amenity values. Let $\mathbf{K}(\tau)$ denote the vector of capital assets at τ, which are relevant to utility.

The entire future course of capital assets and consumption is determined, by projecting capital assets from the present to the future in an iterative manner, using specific dynamic equations of motion of capital assets. We call it an economic forecast made at t for a pair of the functions $\{c(\tau), \mathbf{K}(\tau)\}_t^\infty$. Then, economic development is defined to be sustainable at t if $dV/dt \geq 0$. With the economic forecast given, $V(t)$ is determined. In particular, it is determined solely by the capital assets in the future. This enables us to write

$$V(t) = V(\mathbf{K}(t), t), \tag{2}$$

implying the equivalence between inclusive wealth and social well-being.

Assuming the differentiability of V with regard to $\mathbf{K}$, take the time derivative of social well-being:

$$\frac{dV(t)}{dt} = \sum_i \frac{\partial V}{\partial K_i}\frac{dK_i}{dt} \equiv \sum_i p_i \frac{dK_i}{dt}, \tag{3}$$

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[2] The correspondence of ecosystem service income to natural capital stock is stressed in Fenichel et al. (2018).

where p_i is the shadow price of capital asset i. As Arrow et al. (2012) highlights, a capital's shadow price is defined as its contribution to social well-being through production and direct enjoyment, which reflects relative scarcity of all capital assets both today and all future dates. Also following Arrow et al. (2012), we define inclusive wealth at t as

$$W(t) \equiv \sum_i p_i(t) K_i(t). \tag{4}$$

Note that the weighting factors attached to capital assets continue to be the marginal shadow prices we have assumed above. Since the marginal shadow price changes as the level of capital changes, this implies that the value of W does not have any economic significance; only the marginal change in W does.

Another relevant equivalence result that Arrow et al. (2012) arrived at is the change in wealth and the change in well-being, holding constant shadow prices. The latter assumption of constant shadow prices is justified by imagining a small perturbation or project in the economy. To be specific, consider a small perturbation Δ to the economy in question. Then it can be shown that

$$\Delta V(t) = \sum_i p_i(t) \Delta K_i(t). \tag{5}$$

This is the foundation of wealth accounting and sustainability assessment (Hamilton and Clemens 1999; World Bank 2006; UNU-IHDP and UNEP 2012): aggregate all the change in capital assets across the economy, and if it is increasing (decreasing), the economy is accumulating (decumulating) productive base from which current and future generations are to enjoy utility.

2.2 Capital dynamics and equations of motion

In the previous section, capital dynamics was not spelled out to outline the basic framework, in the tradition of Arrow et al. (2012). This was a sufficient treatment, as long as we recognize the fact that future consumption and capital paths can be determined by assuming some capital dynamics, however they will evolve. To flesh out what shadow prices imply in the framework of inclusive wealth, we now specify the dynamics of $\mathbf{K} = (K, H, S)$, following but slightly adjusting Dasgupta (2009). In what follows, time subscripts are omitted to save on notations. Produced capital K is assumed, as usual, to be accumulated as output net of consumption and educational investment:

$$\dot{K} = F(K, H, R) - C - E, \tag{6}$$

where R is the provisioning service from natural capital, S. The production function F is a non-decreasing, twice differentiable function of each of its arguments. H stands for human capital due to educational investment, E. Natural capital is assumed to have regeneration, $G(S)$, which depends on the level of the stock, and to be subject to resource extraction:

$$\dot{S} = G(S) - R. \tag{7}$$

Imagine also that human capital evolves in line with the function of educational investment:

$$\dot{H} = \psi(E). \tag{8}$$

where E is not assumed to depend on the number of people, something like a fixed cost of investing in facilities, or national system.

We follow Hamilton and Clemens (1999) and others to consider necessary conditions for optimality for now. Static efficiency conditions are

$$U_C = \lambda_K, \tag{9}$$

$$\lambda_K F_R = \lambda_S, \tag{10}$$

$$\lambda_K = \lambda_H \psi'. \tag{11}$$

Dynamic conditions for optimality include

$$-\lambda_K F_K = \dot{\lambda}_K - \delta \lambda_K, \tag{12}$$

$$-U_S - \lambda_S G_S = \dot{\lambda}_S - \delta \lambda_S, \tag{13}$$

$$-\lambda_K F_H = \dot{\lambda}_H - \delta \lambda_H. \tag{14}$$

Equations (9) and (12) immediately show that

$$F_K = \rho_C \equiv \delta - \dot{U}_C / U_C = \delta + \eta_{CC} g_C - \eta_{CS} g_S, \tag{15}$$

where $\eta_{CC} \equiv -U_{CC} C / U_C > 0$ is the elasticity of marginal utility of consumption, and $\eta_{CS} \equiv U_{CS} S / U_C \geq 0$ is the cross elasticity of marginal utility. The growth rate of a variable i is expressed as g_i. (15) is the well-known Keynes-Ramsey rule (extended to include natural capital), which is an artefact of assuming efficiency. In the climate change discounting context, using the LHS of (15), F_K, represents the descriptive approach, which is proxied by the opportunity cost of produced capital observed in the market rate of interest, whereas using the RHS, ρ_C, represents the prescriptive approach, which is often called the Ramsey formula for consumption discount rate.[3]

As practical wealth accounting adopts consumption (dollars) as numeraire (World Bank, 2006; 2011; UNU-IHDP and UNEP, 2012; 2014), we redefine a shadow price of each capital asset relative to produced capital.[4] Shadow prices for human and natural capital in terms of produced capital, which are marginal rates of substitution, are $q_H \equiv \lambda_H/\lambda_K = 1/\psi'$ and $q_S \equiv \lambda_S/\lambda_K = F_R$. From (9) - (14), motions of shadow prices of human and natural capital can be described by

$$\dot{q}_H = q_H F_K - F_H, \tag{16}$$

$$\dot{q}_S = q_S(F_K - G_S) - \frac{U_S}{U_C}. \tag{17}$$

In applied work, they are frequently proxied by resource price and wage rate currently observed in the market (Hamilton and Clemens, 1999). Here instead, simple integration from t to ∞ yields human and natural capital shadow prices as

$$q_H = \int_t^\infty F_H \exp\left(-\int_t^\tau F_K dv\right) d\tau, \tag{18}$$

$$q_S = \int_t^\infty \frac{U_S}{U_C} \exp\left(-\int_t^\tau (F_K - G_S) dv\right) d\tau. \tag{19}$$

The pair of equations tells us that human and natural capital shadow prices can be obtained by summing future marginal productivity and marginal utility, respectively, where effective discount rates should reflect the productivity of the numeraire (i.e., produced capital). In the case of natural capital, where intrinsic growth is assumed, the productivity of produced capital relative to natural capital should enter the discounting formula. In fact, Fenichel and Abbott (2014) explicitly show that the effective discount rate is the difference between the utility discount rate and the derivative of the regeneration function of natural capital in a non-optimizing framework. As natural capital gets scarcer, the effective discount rate becomes smaller, thereby increasing the spot shadow price.

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[3] Alternatively, we can assume away efficiency and suppose that the marginal productivity of produced capital does not equate with the consumption discount rate. In which case, consumption discount rate is substituted by investment shadow price in the ensuing analysis. This approach would not be able to unveil the roles of decomposed parameters.

[4] This is a simplification assuming efficiency, as consumption and investment are not necessarily optimally allocated. Thus, their shadow prices should differ in general (Dasgupta et al., 1972).

This channel on the production side of natural capital is distinict from the relative price change through natural capital as an argument of utility. As the relative price of the environment gets higher, the annual benefit from natural capital, U_S/U_C becomes larger. This is pronounced notably in the literature of discounting and climate change by Hoel and Sterner (2007). Limited substitutability of natural capital suggests that this term can rise non-monotonically (Traeger, 2011).

3 Shadow prices and capital income discount rates

3.1 Specifications

To bring forward the framework to accounting in practice, it is useful to specify utility and production. Following Hoel and Sterner (2007), let the utility function of consumption and natural capital be of constant elasticity of substitution, constant relative risk aversion type:

$$U(C,S) = \frac{1}{1-\eta}\left[(1-\gamma)C^{1-\frac{1}{\sigma}} + \gamma S^{1-\frac{1}{\sigma}}\right]^{(1-\eta)\frac{\sigma}{\sigma-1}}, \tag{20}$$

where $\eta, \sigma > 0$ and $0 < \gamma < 1$ are the elasticity of marginal utility, coefficients of elasticity of substitution, and relative weight of natural capital in utility.

It is then straightforward to check for (15) that

$$\eta_{CC} = -U_{CC}C/U_C = (1-\gamma^*)\eta + \gamma^*\frac{1}{\sigma}, \tag{21}$$

$$\eta_{CS} = U_{CS}S/U_C = \gamma^*\left(\frac{1}{\sigma} - \eta\right) \tag{22}$$

where $\gamma^* \equiv U_S S/(U_C C + U_S S)$ is the value share of the natural capital. Also, the relative "price" of natural capital with regard to consumption goods works out to be

$$\frac{U_S}{U_C} = \frac{\gamma}{1-\gamma}\left(\frac{C}{S}\right)^{1/\sigma} \tag{23}$$

which is an increasing function of C/S under the assumption of limited substitutability, $\sigma < 1$.

On the technology side, let the production function be a Cobb-Douglas type:

$$F(K,H,R) = K^{\alpha}H^{\beta}R^{\phi}, \tag{24}$$

where $0 < \alpha,\ \beta,\ \phi < 1$. Since the elasticity of substitution is unity in a Cobb-Douglas function, we are not assuming the changing scarcity in natural vs. produced capital, for example.

For the production of natural capital, the most typical regeneration function is the quadratic type (e.g., Conrad 1990; Dasgupta 2009; Fenichel and Abbott 2014):

$$G(S) = -b + mS\left(1 - \frac{S}{Q}\right) \tag{25}$$

for $S > 0$, and $G(S) = 0$ for $S = 0$, where parameters are chosen so that $Q > 4b/m$.

3.2 Capital income discount rates

In what follows, we are led by the climate change discounting literature to use the predicted growth rate approach to discounting.[5] Instead of analytically solving for the problem, we are focused on choosing appropriate discount rates based on average predicted growth rates of variables for the future.[6] Assume also that C_t, R_t, K_t, H_t, S_t, and w_t are the given consumption, natural capital use, produced capital, human capital, natural capital, and real wage at the current period, t, and g_i continue to represent the growth rates of variable, i.

From (18), the shadow price of human capital can be expressed by

$$q_H = w_t\frac{1}{\rho_H} = \beta K_t^{\alpha} H_t^{\beta-1} R_t^{\phi}\frac{1}{\rho_H} \tag{26}$$

where the *human capital income discount rate* is

$$\rho_H \equiv F_K - g_w \tag{27}$$

where g_w is the growth rate of the real wage.[7] (27) tells us that the human capital income is discounted at the rate of marginal productivity of produced capital net

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[5] This approach of assuming a constant consumption and natural capital growth has limitations, as permanent growth of consumption and natural capital is impossible. Smulders (2012) and Withagen (2018) point to the paradox of working on shadow prices based on predictions.

[6] In the economic growth literature, it is customary to solve for the steady state variables. However, steady state analysis is not so relevant for sustainability analysis, as in steady state, everything should be sustainable by definition.

[7] Capital income discount rate can be defined as the marginal change rate of the income (in terms of the numeraire) that the capital under study yields.

of the growth rate of human capital income, much like standard financial asset pricing. This is in line with human capital accounting adopted in World Bank as well (Lange et al., 2018). Using the Euler equation (15) and specified production function (24), this can be reexpressed as

$$\begin{aligned}\rho_H &= \delta + \eta_{CC} g_C - \eta_{CS} g_S - \alpha g_K + (1-\beta) g_H - \phi g_R, \\ &= \delta + \left[(1-\gamma^*)\eta + \gamma^* \frac{1}{\sigma}\right] g_C - \left[\gamma^* \left(\frac{1}{\sigma} - \eta\right)\right] g_S - \alpha g_K + (1-\beta) g_H - \phi g_R\end{aligned} \tag{28}$$

which decomposes the discount rate into the growth rates of key variables with their elasticities. Note in particular that the growth rate of human capital lowers its scarcity, thereby raising the discount rate of its income, while the growth rates in other factor inputs works in the opposite direction. In addition, an increase (decrease) in the capital income share, α, with growing produced capital means linearly lower (higher) human capital income discount rate, if other things are equal. Unlike utility, where we have assumed a CES type, our specification of the production function does not reflect changing relative scarcity, so thatthe effect from the capital income share is linear.

By the same token, from (19) and (23), the shadow price of natural capital can be expressed by

$$q_S = \frac{\gamma}{1-\gamma} \left(\frac{C_t}{S_t}\right)^{1/\sigma} \frac{1}{\rho_S} \tag{29}$$

where the *natural capital income discount rate* is

$$\rho_S \equiv F_K - G_S + \frac{1}{\sigma}(g_S - g_C), \tag{30}$$

which just says that the discount rate to be applied to compute the natural capital shadow price is the wealth accounting numeraire discount rate (F_K) relative to the natural capital productivity (G_S) plus the increase rate of natural capital relative to consumption goods, weighted by the inverse of the elasticity of substitution. Using the Euler equation (15) and specified regeneration function (25), we obtain

$$\begin{aligned}\rho_S &= \delta + \left(\eta_{CC} - \frac{1}{\sigma}\right) g_C + \left(\frac{1}{\sigma} - \eta_{CS}\right) g_S - m\left(1 - \frac{2S}{Q}\right) \\ &= \delta + (1-\gamma^*)\left(\eta - \frac{1}{\sigma}\right) g_C + \left[\frac{1}{\sigma} - \gamma^* \left(\frac{1}{\sigma} - \eta\right)\right] g_S - m\left(1 - \frac{2S}{Q}\right)\end{aligned} \tag{31}$$

3.3 The effect of the pure rate of time preference

To summarize, inclusive wealth in total value (4) divided by the produced capital shadow price can be written as

$$\frac{W}{p_K} = K + q_H H + q_S S \; = K + \frac{w_t}{\rho_H} H + \frac{\gamma}{1-\gamma}\left(\frac{C_t}{S_t}\right)^{\frac{1}{\sigma}} \frac{1}{\rho_S} S. \tag{32}$$

Likewise, the change in inclusive wealth (3) relative to the produced capital shadow price works out to be

$$\frac{dV/dt}{p_K} = \dot{K} + q_H \dot{H} + q_S \dot{S} \; = \dot{K} + \frac{w_t}{\rho_H} \dot{H} + \frac{\gamma}{1-\gamma}\left(\frac{C_t}{S_t}\right)^{\frac{1}{\sigma}} \frac{1}{\rho_S} \dot{S} \tag{33}$$

As Arrow et al. (2012) noted, it is particularly helpful to see how the pure rate of time preference affects shadow prices and inclusive wealth. It can be shown from (26)-(30) that

$$\frac{\partial q_H}{\partial \delta} = -w_t \frac{\partial \rho_H / \partial \delta}{(\rho_H)^2} = -\frac{w_t}{(\rho_H)^2} < 0, \tag{34}$$

$$\frac{\partial q_H}{\partial \delta} = -\frac{\gamma}{1-\gamma}\left(\frac{C_t}{S_t}\right)^{1/\sigma} \frac{\partial \rho_S / \partial \delta}{(\rho_S)^2} = -\frac{\gamma}{1-\gamma}\left(\frac{C_t}{S_t}\right)^{1/\sigma} \frac{1}{(\rho_S)^2} < 0. \tag{35}$$

All in all, one can see the effect of the pure rate of time preference, δ, on the inclusive wealth, as in

$$\frac{\partial}{\partial \delta} \frac{W}{p_K} = \frac{\partial q_H}{\partial \delta} H + \frac{\partial q_S}{\partial \delta} S \; < 0. \tag{36}$$

In fact, this negative effect of a larger pure rate of time preference can be directly confirmed from the definition of social well-being at the outset. From (1), it holds that

$$\frac{\partial V}{\partial \delta} = -\int_t^{\infty} (\tau - t) U(C(\tau), S(\tau)) e^{-\delta(\tau - t)} d\tau < 0. \tag{37}$$

Other things being equal, the less future well-being is weighted, the smaller the social well-being as well as wealth becomes. The effects of the pure rate of time preference on wealth and well-being are equivalent, from the construction of shadow prices in (3).[8]

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[8] Put intuitively, in an extreme case where only the present generation matters, wealth is useless from the ensuing period, so that it is only worth the present period income.

Our intergenerational ethics should also affect the *change* in wealth and well-being. On the well-being front, it can be shown from the identity $dV/dt = \delta V - U$ that

$$\frac{\partial(dV/dt)}{\partial\delta} = V > 0, \tag{38}$$

which suggests that a larger utility discount rate translates into more increase in well-being. This is another way of saying that the return on well-being increases as less weight is put on future generations. To sustain wealth, other things being equal, the present generation is permitted to consume more.

On the wealth front, things do not seem so straightforward, as in

$$\frac{\partial(dV/dt)}{\partial\delta} = \frac{\partial q_H}{\partial\delta}\dot{H} + \frac{\partial q_S}{\partial\delta}\dot{S}, \tag{39}$$

whose sign is ambiguous. The seeming disconnect between the signs is resolved once we recall that the increased return on well-being is allocated to consumption or investment.

3.4 Discounting and inclusive wealth

Of course, human and natural capital shadow prices hinge on the assumptions of other parameters as well, if capital income discount rates are assumed to be decomposed in the way we have shown. In the spirit of Hoel and Sterner (2007, Table 2), Table 1 summarizes the partial derivatives of capital income discount rates, shadow prices, and inclusive wealth, with respect to primary parameters. A full list of partial derivatives is shown in Appendix A.

There are some important immediate findings from Table 1. First, aside from the case of zero effects, all the parameters have an opposite effects on the discount rate (ρ_H and ρ_S) and the shadow price (q_H and q_S) of the capital in question. This inverse relationship is not a surprising result, as under simplifying assumptions, the shadow price of a capital is the net present value of future benefits with the capital income discount rate being applied.

Second, many parameters have opposite effects on the human and natural capital shadow prices (or on the discount rates, for that matter). Thus, exogenous changes in them have ambiguous consequences on the valuation of inclusive wealth (the sixth column in Table 1, to which we shall get back).

Two notable exceptions are the pure rate of time preference and the elasticity of marginal utility, implying that an increase (decrease) in them have an unambiguously negative (positive) effect on the total value of inclusive wealth, if the

Table 1: Sign of partial derivatives of capital income discount rates, shadow prices, inclusive wealth, and its change, with respect to various parameters

	ρ_H	q_H	ρ_S	q_S	V	$\dot{V}$
δ	+	−	+	−	−	N.A.
η	+	−	$+^*$	$-^*$	$-^*$	N.A.
σ	− (if $g_C > g_S$)	+ (if $g_C > g_S$)	+ (if $g_C > g_S$)	− (if $g_C > g_S$)	N.A.	+ (if $g_C > g_S$)
g_C	+	−	− (if $\eta\sigma < 1$) + (if $\eta\sigma > 1$)	+ (if $\eta\sigma < 1$) − (if $\eta\sigma > 1$)	N.A. (if $\eta\sigma < 1$) − (if $\eta\sigma > 1$)	− (if $\eta\sigma < 1$) N.A. (if $\eta\sigma > 1$)
g_S	− (if $\eta\sigma < 1$) + (if $\eta\sigma > 1$)	+ (if $\eta\sigma < 1$) − (if $\eta\sigma > 1$)	+	−	N.A. (if $\eta\sigma < 1$) − (if $\eta\sigma > 1$)	+ (if $\eta\sigma < 1$) N.A. (if $\eta\sigma < 1$)
α	− (if $g_K > 0$)	+ (if $g_K > -\rho_H \log K$)	0	0	+	+
β	− (if $g_H > 0$)	+ (if $g_H > -((1/\beta) + \log H)\rho_H$)	0	0	+	+
ϕ	− (if $g_R > 0$)	+ (if $g_R > -\rho_H \log R$)	0	0	+	+
g_K	−	+	0	0	+	+
g_H	+	−	0	0	−	−
g_R	−	+	0	0	+	+
m	0	0	$+^{**}$	$-^{**}$	$-^{**}$	$+^{**}$
Q	0	0	$+^{***}$	$-^{***}$	$-^{***}$	$+^{***}$

$*$ if $(1-\gamma^*)g_C + \gamma^* g_S > 0$

$**$ if $\frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta > 1$ and $S < Q$

$***$ if $\frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta > 2$

For the last column, it is assumed that human capital is increasing but natural capital is decreasing.

N.A. means the sign cannot be determined.

weighted growth rate of consumption and natural capital is positive. A rise in the pure rate of time preference (δ) means less value attached to future income of human or natural capital, as mentioned above. Regarding the elasticity of marginal utility (η), if the weighted consumption and natural capital growth rates are positive, then stronger intergenerational inequality aversion implies that capital income in the future is valued less. If the weighted growth rate of consumption and natural capital is negative, on the other hand, then the effect on inclusive wealth value cannot be spelled out, as only natural capital is valued more.

A rise in the elasticity of substitution (σ) means that the society is willing to substitute more natural capital upon a marginal increase in the relative price of natural capital. This pushes up the human capital shadow price and pushes down the natural capital shadow price, even in the face of the rising relative scarcity of natural capital ($g_C > g_S$). The overall effect on inclusive wealth depends on the relative endowment of human and natural capital.

The effect of consumption growth rate (g_C) on the value of inclusive wealth is clearly negative only when $\eta\sigma > 1$. Under the latter condition, growing consumption translates into valuing future capital income less, as either inequality aversion or the willingness to substitute natural capital is sufficiently large. When the above condition on the two parameters is met (i.e., $\eta\sigma > 1$), inclusive wealth is valued less in a growing economy, if other things are equal.

The same can be said about the growth of natural capital. Under the condition $\eta\sigma > 1$, more natural capital growth means that overall utility is expected to improve in the future, so that human capital income discount rate goes up (its shadow price goes down). Natural capital shadow price also goes down because of its abundance in the future. On the whole, inclusive wealth value declines.

Finally, how the parameters in the regeration function of natural capital affect inclusive wealth is also of interest. A rise in the intrinsic growth rate of natural capital, m, has two channels to affect discounting. On the one hand, it implies an inrease of expected natural capital growth, g_S, which raises natural capital income discount rate (if the weighted sum of inequality aversion and the inverse of elasticity of substitution is positive ($\frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta > 0$) and natural capital stock is within its carrying capacity ($S < Q$)). On the other hand, it implies a lower natural capital income discount rate when the stock size is relatively small to carrying capacity. The combined effect on natural capital shadow price is negative if $\frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta > 1$ and $S < Q$.

An increase in the carrying capacity, Q, also has two opposing effects. It raises the expected natural capital growth rate, which raises the associated discounting, if $\frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta > 0$. Along with that, it increases the marginal regeneration, lowering the natural capital discounting. As a whole, a larger carrying capacity increases the associated discounting if the former effect dominates (i.e., $\frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta > 2$). Thus, it implies a lower natural capital shadow price and a smaller inclusive wealth, other things being equal.

3.5 Discounting and sustainability

In the end, though, what matters for sustainability analysis is the *change* in inclusive wealth. Various effects of parameters on the change in inclusive wealth cannot be generalized. However, we can focus on some realistic and interesting scenarios. Suppose that human capital is on the rise but natural capital is on the decline in a given country, which seems to be very typical of many developed and developing nations in the world over decades.[9] The last column of Table 1

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[9] UNU-IHDP and UNEP (2014) report that natural capital may have been substituted by human capital, based on a cross-country observation of the correlation of the change rates of both capital for 140 countries, averaged over the period of 1990-2010.

signifies such a situation.

The pure rate of time preference and inequality aversion have undetermined effects on inclusive wealth change. In contrast, if society permits more substitution of natural capital by consumption goods (i.e., a rise in the elasticity of substitution, σ), inclusive wealth change as an indicator of sustainability naturally improves in an economy where consumption growth is higher than natural capital growth. A rise in the consumption growth rate (g_C) has a negative impact on inclusive wealth change if $\eta\sigma < 1$. This appeals to intuition, as the latter assumption suggests either small inequality aversion or limited elasticity of substitution. Increasing human capital is valued less (because of discounting), while decreasing natural capital is weighted more (because of limited substitutability), so that the overall consequence of sustainability is negative. Likewise, a decrease in natural capital growth (g_S) has a negative impact on inclusive wealth change under the assumption $\eta\sigma < 1$.[10]

4 Numerical examples

4.1 Prescriptive approach to capital income discounting

Since we have seen how each component of capital income discount rates affects shadow prices, inclusive wealth and their change, we are now ready to obtain the plausible order of the magnitude for capital income discount rates. Before putting our framework to specific figures for the parameters, it is worthwhile rationalizing country- or region-specific capital income discount rates. In climate economy modeling, there is a rationale for adopting a single consumption discount rate for the whole economy, as carbon damage is a global public bad, although marginal rates of substitution with consumption goods differ in regions. In contrast, capital income flows are context-specific in principle. In particular, capital income discount rates are effective rates involving growth forecasts of capital income, which are inherently context-specific. Practical accounting should adopt discount rates that are region- or country-specific, but not too disaggregated, as accounting could be complex and untransparent at a high level of disaggregation. Thus, in the following numerical examples, we pick up five countries and see the order of the magnitude in their specific contexts where available. The choice of these five

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[10] We mention a decrease in g_S, as this is consistent with the specific scenario of natural capital decline we suppose for the analysis of $\dot{V}$.

countries, representing a large share in the global economy, is due to Arrow et al. (2012).

In what follows, we discuss plausible values for each parameter in turn, although it is often hard to decide *the* value definitively. There has been a long debate on the pure rate of time preference. It is a consensus among economists that it should be positive but also very low in a discussion involving long-term consequences of policy making (e.g., Dasgupta and Heal 1979). In a recent expert survey, Drupp et al. (2018) suggest a mean value of 1.1% with the range of $[0, 8]\%$. However, in a normative approach to discounting in a long-run, influential studies recommend we set it somewhere around $\delta = 0.1\%$ per annum (Stern 2006; Dasgupta 2008).

This literature also suggests a global average consumption growth rate at the order of the magnitude of 1.3%. Recent climate scenario studies suggest similar consumption growth rates for the future as well. The Shared Socioeconomic Pathways SSP2 “Middle of the Road” scenario, for example, set the per capita income growth forecast as 2.0% for the 21st century (e.g., Leimbach et al. 2017). However, as shadow prices are forward-looking, we are advised to use consumption growth forecasts where available. We thus use a forecast of GDP per capita for the next two decades (OECD, 2014)[11].

The elasticity of marginal utility is also debatable. Common interpretations include inequality aversion and risk aversion under uncertainty; in view of (1), we are also inclined to interpret it as intergenerational inequality aversion. In the climate change literature, two influential studies apply $\eta = 1$, implying a logarithmic utility (Stern, 2006; Nordhaus, 2008). Society may exhibit more inequality or risk aversion, thus falling in the region of [1,4] (Dasgupta, 2008; Groom and Maddison, 2018). It could be a function of consumption per capita, but we do not find a credible estimate of the correspondence between income or consumption and η.

In a widely cited work, Jacobsen and Hanley (2009) suggest that the income elasticity of willingness to pay for ecosystem services falls in the range of $1/\sigma \approx 0.38 \pm 0.14$. This is still a narrow estimate interval compared to Meya et al. (2018), who review the literature and use lower and upper bounds for σ as 0.86 and 7.14, respectively.

The utility value share of the environment, γ, is set at 0.1 in Hoel and Sterner (2007) and Sterner and Persson (2008), and at 0.29 in Gollier (2010). In a paper

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[11] For want of a projection, the consumption growth rate for Venezuela is proxied by the past actual GDP growth rate per capita, 1990-2010.

that refers to some meta studies including Kopp et al. (2012), Meya et al. (2018) take a mean of 0.15 for environmental goods.

For natural capital growth rate, we can directly tap into past trends in forest growth.[12] Having the U.S. in mind, for example, forest stock has been on the deline with its growth rate being $g_S = -0.4\%$ per capita from 1990 to 2010 (UNU-IHDP and UNEP 2014). To fix ideas, take an example in temperate forest amenity, which is valued at USD 2,091/yr/ha, of which USD 171 is from provisioning services (van der Ploeg and de Groot 2010). We can interpret $\frac{U_S}{U_C} = \frac{\gamma}{1-\gamma}\left(\frac{C_0}{S_0}\right)^{\frac{1}{\sigma}}$ at the initial period as USD 1,920.

There are some estimates of capital and labor income shares (α and β), but natural capital income share data (ϕ) is scant. Thus, we have no choice but to use the wage growth rate in the numerical example. In particular, we use the past regional wage growth estimates by ILO (2018). Table 2 summarizes relevant parameters and growth parameters of selected countries.

Finally, it is challenging to determine a plausible intrinsic growth rate for forest, but we follow Brander and Taylor (1998) who assumed 4% per annum. For fishery resources, the growth rate is much higher than forest resources, and the evidence is less uncertain, which still varies from 0.025 to 0.75 depending on the species under investigation (Froese and Pauly 2009; Clark et al. 2010; Fenichel and Abbott 2014).

Table 2 sums up our choice of parameters, based on the literature and past data for natural capital growth rates, as well as projections for consumption growth. The bottom line figures for human capital income discount rate (ρ_H) and natural capital (forest) discount rate (ρ_S) and shadow price (q_S) are shown in the last three rows. Many observations can be made from here. First off, consumption growth and wage growth show similar figures for the five countries, as expected. From how human capital income discounting is constructed, these two effects are canceled out to a certain extent. The intuition is that, if future generations are richer, their wage will also be higher, but their value in terms of social well-being is also discounted more. From equations (27) and (28), if η_{CC} is unity and consumption and wage growth rates are equal, they exactly cancel off. When these conditions are met, we could even say that what matter to human capital income discount rate are the pure rate of time preference and the natural capital growth

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[12] In an applied work to reflect ecosystem service flow into social discounting, Baumgärtner et al. (2015) estimate the global average growth rate of ecosystem service as -0.52%, with a range from -1.28 to 0.08 %.

term.[13]

In additoin, forest capital has been on the decline at an alarming rate in all the countries, which lower natural capital income discounting, and to a lesser extent, human capital income discounting. Even so, the resultant natural capital discount rates are higher than human capital counterparts, partly due to the assumed substitutability of natural capital ($\sigma > 1$), low intrsinsc growth rate (m) and high capital-capacity rate (S/Q), as well as due to the canceling effect of consumption-wage for human capital we have mentioned above.

Along with the result of Arrow et al. (2012), Venezuela stands out from the rest of the countries, as human capital income discount rate is very low, often falling in the negative region. This is traceable, among others, to the stagnant consumption prospect (relative to wage) and the forest decrease rate which is even worse than the other nations.

Table 2: Construction of country-specific capital income discount rates and shadow prices

δ	0.001
η	1.5
σ	2
m	0.04
S/Q	0.8

	U.S.	China	Brazil	India	Venezuela
g_C	1.7%	5.0%	1.8%	4.7%	0.6%
g_S	-0.4%	-2.0%	-1.8%	-1.5%	-2.5%
g_w	0.4%	5.3%	1.1%	3.5%	1.1%
ρ_H	2.0%	1.7%	1.4%	3.1%	-0.4%
ρ_S	4.5%	7.9%	3.8%	7.8%	1.8%
q_S	42,851	24,193	50,607	24,732	107,684

For g_w, we use the annual growth of mean real monthly earnings of employees in ILO (2018).
The figures for U.S., China, Brazil, India, and Venezuela are taken from Northern America, Eastern Asia, Latin America and the Caribbean, Southern Asia, and Latin America and the Caribbean, 2000-2010.

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[13] Indeed, a similar case is being made about the social cost of carbon. Carbon damage is frequently assumed to be roughly proportional to GDP. A high consumption growth rate (g_C) thus implies higher consumption discount rate as well as higher carbon damage, which attenuate each other.

Extending our framework to a case of uncertainty is both interesting and meaningful. There are some previous studies looking to how sustainability rules would change under uncertainty (Agliardi, 2010; Mäler and Li, 2010; Fenichel et al., 2018b). Here we take a very simple route to get the feel. In Table 2, we posited that the past natural capital growth rate continue in the future. Suppose instead that the natural capital growth rate for the U.S., for example, can either be ±0.4% in an equiprobabilistic fashion, so that the expected growth rate is simply zero. Assuming a positive 0.4% would end up with natural capital income discount rate as $\rho_S = 5.0\%$, which would raise the shadow price to USD 38,705 per hectare. If the expected growth rate of 0% were simply applied, the natural capital income discount rate would become $\rho_S = 4.7\%$, with the shadow price of USD 40,673. This example reminds us that what directly matters for the shadow price is the discount factor, not a simple mean of the parameters. The emphasis on the discount factor under uncertainty in, e.g., Gollier and Weitzman (2010) carries on to natural capital income discounting.

4.2 Current practice: Descriptive approach to capital income discounting

In the climate change discounting literature, disputes on descriptive vs. prescriptive approaches have not been contained. From the Euler equation (15), using the marginal product of capital and consumption discount rate are equivalent in theory, which however does not apply to the real economy. The descriptive approach focuses on the opportunity cost of produced capital, suggesting we use the real interest rate observed in the market, whereas the prescriptive approach focuses on the consumption discount rate constructed from the Ramsey formula, using normatively plausible figures for relevant parameters (e.g., Heal 2009; Cropper et al. 2017).

At the current state of affiars, the wealth accounting literature takes this descriptive approach to discounting, as shown in Table 3. UNU-IHDP and UNEP (2012; 2014) employ capital income discount rates of 5% for renewable natural capital and 8.5% for human capital, the latter of which is harmonized with the average rate of retun on human capital (Klenow and Rodríguez-Clare 1997).

Likewise, discount rates for non-renewable natural capital income is set at 4% in Lange et al. (2018) from "the long-term (100 years or more) real return on financial assets globally, derived from Credit Suisse data" and "therefore represents the opportunity cost of holding wealth as fossil fuels rather than investing in financial assets" (p112).

Equation (30) reminds us that even in the descriptive approach, the opportunity cost of produced capital cannot be used directly as an effective discount rate of natural capital income. It has to be adjusted for the natural capital regenerative capacity (Fenichel and Abbott 2014) and the relative price change of natural capital (Hoel and Sterner 2007).

It is thus commendable that, in a recent report by Lange et al. (2018), some elaborations seem to have begun. For instance, the effective discount rate for human capital, which corresponds to (27), is assumed to be $F_K - g_w = 1.5\%$. Reflecting "efficiency improvement," this differs from the discount rate used for non-renewable resources (4%).

In addition, growth rates of cropland are assumed to be 1.94% and 0.97% for high-income countries and low- and middle-income countries, respectively. These figures can be interpreted as G_S in (30), but still do not reflect such factors as the changing scarcity of capital assets.

On the human capital front, Hamilton and Liu (2017) contain a discussion on the choice of the discount rate. They chose the effective human capital income discount rate ($\rho_H = F_K - g_w$) as $F_K = 4.58\%$ for all OECD countries, net of the annual real income growth rate (g_w) of individual countries, based on earlier work by Liu (2011). In fact, they point out the choice of 4.58% is due to Jorgenson and Fraumeni (1989), who set it based on the long-run rate of return in the private sector in the U.S.

4.3 Sensitivity analysis of inclusive wealth change

As we have mentioned, a few studies have looked into sensitivity of discounting in applied wealth accounting. However, their treatment is either only to a specific class of capital (Arrow et al., 2012) or to wealth (Hamilton and Liu, 2017; Lange et al., 2018), not the *change* in wealth. Thus, we examine how the pure rate of time preference (δ) affects the change in inclusive wealth. Our focus on the pure rate of time preference requires justification. As effective discount rates used in practical wealth accounting are based on the descriptive approach and thus often dissectible into individual parameters, as we have seen in Table 3, the effects of other parameters can be less informative and can be guessed from the case of the pure rate of time preference. Moreover, we have seen in the case of human capital income discounting, consumption growth and wage growth effects largely cancel off with each other.

Table 3: Discount rates currently used in wealth accounting

Source	What to discount	Figures
World Bank (2011)	Consumption (to compute social well-being)	%
Arrow et al. (2012)	Human capital income	8.5 %
	Years of life remaining	5%
	(to compute the value of statistical life year for health capital)	
UNU-IHDP and UNEP (2012; 2014)	Human capital income	8.5 %
	Forest income	5%
	Agricultural land income	5%
Hamilton and Liu (2014)	Human capital income	4.58% net of annual real inc
McLaughlin et al. (2014)	Human capital income	2.5%
	Natural capital income	2.5% ?
Lange et al. (2018)	Human capital income	1.5%
	Human capital income foregone by air pollution	1.5% [*]
	Non-renewable natural capital income	4%
	Forest income	4%
	Agricultural land income	4% net of productivity growt

[*] The discount rate and the annal income growth rate are assumed to be 4% and 2.5%, respectively.

[**] For cropland, 1.94% for low-and middle-income countries and 0.97% for high-income countries. For land for livestock products, 2.95% and 0.89%

5 Conclusion

We have shown in the inclusive wealth framework that previous literatures on climate change discounting and on natural capital valuation can be applied to the wealth accounting, and moreover, human and natural capital discount rates should be determined in a consistent framework. Table 1 basically extends the discounting debate to sustainability analysis, with many new insights. We do not have an intention to transplant the neverending debate on the "right" discounting for climate change to sustainability assessment. However, what we have called human and natural capital income discount rates turned out to be very different among selected countries.

It is also important to note that Table 1 is merely a partial equilibrium analysis. Interactions of parameters are not taken into account (with the effect of m on g_S being an exception). It remains to be seen that to what extent that the variations in the order of the magnitude for capital income discount rates can affect the level of inclusive wealth, and more importantnly, the change rate of inclusive wealth empirically. This constitutes our immediate empirical research item.

A Derivatives of capital income discount rates, shadow prices, and inclusive wealth

In what follows, we show partial derivatives of human and natural capital income discount rates with regard to relevant parameters, which makes for the basis of Table 1.

A.1 Derivatives of capital income discount rates

$$\frac{\partial \rho_H}{\partial \delta} = 1, \tag{40}$$

$$\frac{\partial \rho_H}{\partial \eta} = (1-\gamma^*)g_C + \gamma^* g_S > 0 \text{ if } g_C > g_S \geq 0. \tag{41}$$

$$\frac{\partial \rho_H}{\partial \sigma} = -\frac{\gamma^*}{\sigma^2}(g_C - g_S) < 0 \text{ if } g_C > g_S. \tag{42}$$

$$\frac{\partial \rho_H}{\partial g_C} = \eta_{CC} = (1-\gamma^*)\eta + \gamma^* \frac{1}{\sigma} > 0, \tag{43}$$

$$\frac{\partial \rho_H}{\partial g_S} = -\eta_{CS} = \gamma^* \left(\eta - \frac{1}{\sigma}\right) < 0 \text{ if } \eta\sigma < 1, \tag{44}$$

$$\frac{\partial \rho_H}{\partial \alpha} = -g_K < 0 \text{ if } g_K > 0, \tag{45}$$

$$\frac{\partial \rho_H}{\partial \beta} = -g_H < 0 \text{ if } g_H > 0, \tag{46}$$

$$\frac{\partial \rho_H}{\partial \phi} = -g_R < 0 \text{ if } g_R > 0, \tag{47}$$

$$\frac{\partial \rho_H}{\partial g_K} = -\alpha < 0, \tag{48}$$

$$\frac{\partial \rho_H}{\partial g_H} = 1 - \beta > 0, \tag{49}$$

$$\frac{\partial \rho_H}{\partial g_R} = -\phi < 0. \tag{50}$$

$$\frac{\partial \rho_S}{\partial \delta} = 1, \tag{51}$$

$$\frac{\partial \rho_S}{\partial \eta} = (1-\gamma^*)g_C + \gamma^* g_S, \tag{52}$$

$$\frac{\partial \rho_S}{\partial \sigma} = \frac{1}{\sigma^2}(1-\gamma^*)(g_C - g_S) > 0 \ \text{ if } \ g_C > g_S, \tag{53}$$

$$\frac{\partial \rho_S}{\partial g_C} = \eta_{CC} - \frac{1}{\sigma} = (1-\gamma^*)(\eta - \frac{1}{\sigma}) < 0 \ \text{ if } \ \eta\sigma < 1, \tag{54}$$

$$\frac{\partial \rho_S}{\partial g_S} = \frac{1}{\sigma} - \eta_{CS} = \frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta > 0, \tag{55}$$

$$\frac{\partial \rho_S}{\partial m} = \left(\frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta\right)\left(1 - \frac{S}{Q}\right) + \frac{S}{Q} > 0 \ \text{ if} \frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta > 1 \ \text{ and } \ S < Q, \tag{56}$$

$$\frac{\partial \rho_S}{\partial Q} = \left(\frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta - 2\right)\frac{mS}{Q^2} > 0 \ \text{ if } \ \frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta > 2. \tag{57}$$

A.2 Derivatives of shadow prices

$$\frac{\partial q_H}{\partial \delta} = -w/(\rho_H)^2 < 0, \tag{58}$$

$$\frac{\partial q_H}{\partial \eta} = -w\left((1-\gamma^*)g_C + \gamma^* g_S\right)/(\rho_H)^2 < 0, \tag{59}$$

$$\frac{\partial q_H}{\partial \sigma} = w\left(\frac{\gamma^*}{\sigma^2}(g_C - g_S)\right)/(\rho_H)^2 > 0, \tag{60}$$

$$\frac{\partial q_H}{\partial g_C} = -w\left((1-\gamma^*)\eta + \gamma^*\frac{1}{\sigma}\right)/(\rho_H)^2 > 0, \tag{61}$$

$$\frac{\partial q_H}{\partial g_S} = -w\left(\gamma^*\left(\eta - \frac{1}{\sigma}\right)\right)/(\rho_H)^2 > 0 \ \text{ if } \ \eta\sigma < 1, \tag{62}$$

$$\frac{\partial q_H}{\partial \alpha} = (\rho_H \log K_t + g_K)w/(\rho_H)^2 > 0 \ \text{ if } \ g_K > -\rho_H \log K_t, \tag{63}$$

$$\frac{\partial q_H}{\partial \beta} = (((1/\beta) + \log H_t)\rho_H + g_H)w/(\rho_H)^2 > 0 \ \text{ if } \ g_H > -((1/\beta) + \log H_t)\rho_H, \tag{64}$$

$$\frac{\partial q_H}{\partial \phi} = (\rho_H \log R_t + g_R)w/(\rho_H)^2 > 0 \ \text{ if } \ g_R > -\rho_H \log R_t, \tag{65}$$

$$\frac{\partial q_H}{\partial g_K} = \alpha w_t/(\rho_H)^2 > 0, \tag{66}$$

$$\frac{\partial q_H}{\partial g_H} = -(1-\beta)w_t/(\rho_H)^2 < 0, \tag{67}$$

$$\frac{\partial q_H}{\partial g_R} = \phi w_t/(\rho_H)^2 > 0. \tag{68}$$

$$\frac{\partial q_S}{\partial \delta} = -\frac{\gamma}{1-\gamma}\left(\frac{C_t}{S_t}\right)^{\frac{1}{\sigma}} /(\rho_S)^2 < 0, \tag{69}$$

$$\frac{\partial q_S}{\partial \eta} = -\frac{\gamma}{1-\gamma}\left(\frac{C_t}{S_t}\right)^{\frac{1}{\sigma}} ((1-\gamma^*)g_C + \gamma^* g_S)/(\rho_S)^2 < 0 \text{ if } (1-\gamma^*)g_C + \gamma^* g_S > 0, \tag{70}$$

$$\frac{\partial q_S}{\partial \sigma} = -\frac{\gamma}{1-\gamma}\left(\frac{C_t}{S_t}\right)^{\frac{1}{\sigma}} \left(\log\left(\frac{C_t}{S_t}\right)\rho_S + (1-\gamma^*)(g_C - g_S)\right)/(\sigma\rho_S)^2 < 0 \text{ if } g_C > g_S, \tag{71}$$

$$\frac{\partial q_S}{\partial g_C} = -\frac{\gamma}{1-\gamma}\left(\frac{C_t}{S_t}\right)^{\frac{1}{\sigma}} (1-\gamma^*)(\eta - \frac{1}{\sigma})/(\rho_S)^2 > 0 \text{ if } \eta\sigma < 1, \tag{72}$$

$$\frac{\partial q_S}{\partial g_S} = -\frac{\gamma}{1-\gamma}\left(\frac{C_t}{S_t}\right)^{\frac{1}{\sigma}} \left(\frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta\right)/(\rho_S)^2 < 0, \tag{73}$$

$$\frac{\partial q_S}{\partial m} = -\frac{\gamma}{1-\gamma}\left(\frac{C_t}{S_t}\right)^{\frac{1}{\sigma}} \left(\left(\frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta\right)\left(1 - \frac{S}{Q}\right) + \frac{S}{Q}\right)/(\rho_S)^2 < 0 \text{ if} \frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta > 1 \text{ and } S < Q, \tag{74}$$

$$\frac{\partial q_S}{\partial Q} = -\frac{\gamma}{1-\gamma}\left(\frac{C_t}{S_t}\right)^{\frac{1}{\sigma}} \left(\frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta - 2\right)\frac{mS}{Q^2}/(\rho_S)^2 < 0 \text{ if } \frac{1}{\sigma}(1-\gamma^*) + \gamma^*\eta > 2. \tag{75}$$

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