# Spatially explicit criterion for invasive species control

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ABSTRACT

Because management funds available to control biological invasions are often limited, there is a need to rationalize efforts and identify priority locations where invasions are to be targeted first. This paper proposes a spatially explicit cost-benefit decision criterion for optimal resource allocation over space. We construct a cost-benefit optimization framework that incorporates spatially explicit costs and benefits of control as well as invasion spatial dynamics. This framework offers the theoretical foundations of a simple and operational method for the spatial management of invasive species under a limited budget. It takes the form of a decision criterion a landscape manager could use in order to choose how to allocate his annual budget for maximizing the benefit-cost ratio of management. We apply this criterion to the spatial management of primrose willow (*L.peploides*) in the Brière marshland in France and we offer management recommendations. A key contribution of the paper is to define and apply this decision support tool and to make heuristic recommendations on how to assist local decision makers in rationalizing their efforts.

## 1. Introduction

Biological invasions entail massive biodiversity losses and tremendous economic impacts (Pimentel et al., 2005; Vilà et al., 2011). Increasing trends and magnitudes of damages call for drastic management strategies (Olson, 2006; Hulme, 2009; Seebens et al., 2015) but financial resources are limited and while number of invasions constantly increases, dedicated budgets are stable and even sometimes decreases over time (McCarthy et al., 2012).

Prioritization to support cost-effective allocation of resources is crucial. Decisions need to be made on a sound basis to avoid wasting resources. McGeoch et al. (2016) distinguish three principal prioritization schemes : species prioritization, pathways prioritization, sites prioritization. Key of these schemes is to identify species, pathways or sites causing the highest risk to the environment and biodiversity as well as the greatest and cheapest opportunities for preventing this risk. In other words, these schemes aims at promoting management strategies with the highest benefit-cost ratios. Relative benefits and relative costs matters but these costs and benefits are to account for spatial externalities and species interdependences. Choice of controlling one biological invasion rather than another is to account for direct benefits from avoiding impacts caused by the two biological invasions but also indirect benefits related to the cascade of interdependences between species composing the ecosystem (Courchamp et al. (2011), Ballari et al. (2016)). Similarly, choice of controlling one pathway rather than another or one site rather than another is to account for species dynamics over space and therefore for direct impacts from controlling a site or a pathway but also for indirect impacts due to the spread of the biological invasion once this pathway is crossed or this site is invaded.

Numerous species prioritization schemes were proposed. They account principally with invasive species red lists (see Faulkner et al. (2014)) and scoring based methods (e.g. Nentwig et al. (2010), Blackburn et al. (2014), Kumschick et al. (2012, 2015)). These schemes rank priorities on the ground of species' impacts and invasiveness, putting small or no attention on relative costs of control. Cost-benefit optimization modeling approaches remedy this deficiency. However, to our knowledge, only two papers study species-prioritization using a cost-benefit framework: Carrasco et al. (2010) and Courtois et al. (2018). The former offers qualitative results on properties of priority species while the later provides a decision-support optimization tools for budget allocation.

Although pathways and site prioritization schemes are fewer, a number of standardized evidence-based approaches were proposed (e.g. Dawson et al. (2015), Essl et al. (2015)). Surprisingly and as noted by Dana et al. (2014), despite

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the important focus of optimization approaches on spatio-temporal allocation of efforts, few progress indicators and decision-support tools were produced on this ground. In her review on the economics of biological invasion management, Epanchin-Niell (2017) examines key research questions raised by the literature. Most papers reviewed are dedicated to the study of where, when, how much and how to prevent and control biological invasions.[1] Spatial prioritization is analyzed using optimization and cost-benefit approaches, with several papers formally studying effort allocation over space in order to efficiently respond to a biological invasion, e.g. Chades et al. (2011); Epanchin-Niell and Wilen (2012); Chalak et al. (2016); Baker and Bode (2016) and Baker (2017). However, if many qualitative results on management strategies are derived, almost no decision support tools are proposed in order to help landscape managers in their allocation choices. This echoes (Knight et al., 2008) point according to a key challenge for scientists is to fill the research-implementation gap and research be implemented in practice. As argued by (Baker, 2016) or (Courtois et al., 2018) management guidelines and friendly user software packages are needed for the support of decision making in managing biological invasions. To our knowledge, only one such tool exists at the moment, the SPADE decision-support tool (Beeton et al. (2015)) which has proved to be useful in managing eradication campaigns for invasive species. Complementing this work and in line with multi-species decision tools algorithms developed by (Courtois et al., 2018), this paper contributes to the building of a decision tool for invasive species management over space.

More precisely, in this study, we develop a site prioritization criterion enabling decision-makers to efficiently allocate their budget toward the spatial management of an invasive species that has arrived or established in a landscape. Echoing the increasing demand for simple tools that guide managers to optimize their investments based on objective and measurable criteria (Tilman, 2000; Roura-Pascual et al., 2009; Dana et al., 2014; Koch et al., 2016), we define the theoretical groundwork of a decision tool to derive management decisions. This criterion is grounded on optimization methods and solves for the optimal spatial allocation of resources to control an invasive species within a landscape. Applying this criterion to a case study of primrose willow (*Ludwigia peploides*) management in the Brière marshland in France, our achievement is twofold. First, we illustrate how such a criterion can be used by a landscape manager not aware of optimization technics. We raise recommendations on spatial management in our case study. Second, applying this criterion allows for raising technical recommendations on data collection and methods for standardized decision tools be used in the field.

The paper proceeds as follows. In section 2, we present the theory and the method. We develop a spatial cost-benefit model of invasive species management under limited budget constraint. First we present the ecological component of the model describing the law of motion of the species over space. Dynamics is described as a mobil public bad within a spatial network. Second, we present the economics components of the model and describe general benefit and cost functions applicable to a large set of biological invasion situations. Combining the two components, we obtain a maximization program under a set of constraints which solution can be approximated using gradient methods. Principal result is the definition of an algorithm to perform spatial allocation of a limited budget for the control of a biological invasion over space. In section 3, we apply this prioritization protocol to the management of a biological invasion threatening economic and ecological viability of a French regional park in France, primrose willow in the Brière marshland. We present the case study and explain how we made use of available data to apply the choice algorithm proposed. We illustrate with this case study how our theoretical algorithm can be applied given the limited and imperfect data collections on costs, benefits and invasion spatial dynamics. We deduce two kinds of recommendations : site prioritization recommendations and technical recommendations on how to improve data collections in order to apply such decision tools. Section 5 concludes and discusses relevant extensions of this work.

## 2. Theory and method

Consider a landscape composed of $N = 1; n$ distinct areas impacted or likely to be impacted by a biological invasion that is spreading. Assume that the manager of this landscape is endowed with a finite budget he allocates to control a biological invasion over space. We denote $\mathbf{X}$, the $n$-dimension column vector of effort whose $i$-th elements is $x_i$, the control effort of the invasion in area $i$. Because the manager does not know how much budget will be allocated to the control of the invasion over the next periods (i.e. years), because there are important uncertainties about intertemporal costs, intertemporal benefits and invasion dynamics over time, and because the manager often expects immediate returns from his efforts, we consider a myopic setting. This translates into considering that the manager chooses his spatial allocation of efforts abstracting from mid or long term dynamics and impacts. Choice is then made on the basis

[1] For other reviews on the economics bio-invasion management, see Olson (2006), Gren (2008) and Finnoff et al. (2010).

of the current state of the invasion, spatial dynamics within the period, how much it costs to control it over space and what will be the immediate returns from the allocation vector chosen.[2]

We present in this section a four layers cost-benefit optimization framework to assess optimal allocation of efforts over space. Considering a generic landscape (Fig1.A), we first describe the ecological problem and model spatial dynamics of a biological invasion over space (Fig1.B). We next describe the economic problem which consists in two major components: a cost function representing cost heterogeneity over space (Fig1.C) and a benefit function representing the impact of the invasion over space (Fig1.D). The model is a deterministic optimization approach aiming at studying best allocation of a limited budget to control a biological invasion over space (Fig1.F).

## 2.1. Population model

We define $\mathbf{Y^0}$ and $\mathbf{Y}$ as the $n$-dimension column vectors whose $i$-th elements respectively are $Y_i^0$ and $Y_i$, where $Y_i^0$ denotes the magnitude of the biological invasion in area $i$ at the beginning of the period and $Y_i$ this magnitude at the end of the period. Note that the magnitude of biological invasion may be measured as a number of species, an areal (in $m^2$) or a volume (in $m^3$).

Formally, we assume the magnitude of the invasion in area $i$ at the end of the period is a function of the prevalence of the invasion at the beginning of the period, its growth within area $i$ along the period, the spread of the biological invasion from area $j$ to area $i$, $\forall j \neq i$, and the control effort $x_i$ to reduce it in area $i$. The resulting law of motion of the magnitude of the species in area $i$ writes:

$$Y_i = \rho Y_i^0 + \gamma \sum_{j \neq i} w_{ji} \, Y_j - \beta x_i \, , \tag{1}$$

where $\rho$ is the invasive species intra-area growth parameter, $\gamma$ the inter-area dissemination parameter, $\beta$ the effort efficiency parameter, $w_{ji}$ the inter-area connectivity parameter (from $j$ to $i$), with $\rho$ and $\gamma \geq 0$, $w_{ji}$ and $\beta \in [0, 1]$.
We shall add the additional constraints that:

$$\forall i \;\; Y_i^0 \in [0, \overline{Y_i}], \;\; Y_i \in [0, \overline{Y_i}], \tag{2}$$

meaning that due to geographic constraints, the magnitude of the biological invasion is necessarily positive or nil and it is bounded. We assume $\overline{Y_i}$ is the maximum magnitude of the biological invasion in area $i$ and by definition this number is related to physical variables (*i.e.*). To ensure condition (2), we impose:

$$x_i \in \left[0, \overline{x}_i\right] \;\; \forall i, \tag{3}$$

with $\overline{x}_i$ a maximum effort level for $Y_i^0$ and $\overline{Y_i}$ to always be greater or equal to zero in any area $i$.[3]

We obtain a system describing the magnitude of the biological invasion in the $N$ areas over the period. In matrix form, this system reads as:

$$\mathbf{Y} = \rho \mathbf{Y^0} + \gamma \mathbf{W} * \mathbf{Y} - \beta \mathbf{X}, \tag{4}$$

and under the *weak* assumption that matrix $\mathbf{I}^n - \gamma \mathbf{W}$ is invertible, with $\mathbf{I}^n$ denoting the $(n \times n)$ identity matrix, the $N$-equations system (4) admits a solution that reads as:

$$\mathbf{Y} = \mathbf{\Lambda} * \left(\rho \mathbf{Y^0} - \beta \mathbf{X}\right), \tag{5}$$

where $\mathbf{\Lambda} \equiv [\mathbf{I}^n - \gamma \mathbf{W}]^{-1}$.

It follows that the magnitude of the biological invasion in each area decreases linearly both with efforts made in the area and in the areas that are connected. In the proceeding and for readability purpose, we denote $\mathcal{Y}(\mathbf{X})$ the affine mapping from management efforts to biological invasion magnitudes over space and we write:

$$\mathcal{Y}(\mathbf{X}) = \mathbf{\Lambda} * \left(\rho \mathbf{Y^0} - \beta \mathbf{X}\right). \tag{6}$$

[2]This myopic assumption may seem unrealistic to some readers but we claim that it is the most reasonable assumption given the objective of this project which is to design the theoretical ground of a practical and user-friendly decision tool to be used by landscape managers who have no sufficient information, nor even the will to perform an intertemporal optimization. We thus conform to this myopic assumption to make ou approach operational. An analysis of the approximation involved by this assumption is provided in appendix A.

[3]A method to assess maximum effort vector $\overline{\mathbf{X}}$ is provided in appendix B.

## 2.2. Economic model

We consider the manager is endowed with a finite budget we denote $G$. This budget is to be allocated toward the $n$-dimension column vector of effort $\mathbf{X}$ so as to maximize the benefit/cost ratio of control. Control cost is assumed to be heterogeneous over space meaning that due to geographic, environmental and accessibility constraints, marginal cost of control may differ from one area to the other.[4] We denote $c_i$ the marginal cost of controlling the biological invasion in area $i$ and $\mathbf{c}$ the marginal cost vector. It follows that the budget constraint of the manager writes:

$$\mathbf{c}^\top * \mathbf{X} \leq G\,. \tag{7}$$

We also assume benefits of control are heterogeneous over space. Impacts due to biological invasions often vary spatially because the magnitude of the invasion varies over space but also and foremost because biological invasions impact economics and ecosystems services that are spatially distributed. For our results to remain as general as possible, we do not specify impact functional forms and consider that for any area $i$, impact function $\mathcal{B}_i(Y_i)$ pertains to the class of $C^2$ functions, *i.e* those whose first and second order derivative both exist and are continuous. Although $\mathcal{B}_i$ often decreases with the magnitude of the invasion in the area and admits only negative values, one may conceive situations in which this function is positive and increasing, meaning that the biological invasion may produce positive benefits (see XXX). As we assume that the objective of the manager is to maximize the total benefits from his spatial allocation of efforts, we deduce that his objective function is $\sum_i \mathcal{B}_i(Y_i)$, that is $\mathcal{B}(\mathbf{Y})$ in matrix form. In order to express the manager's objective as a function of effort $\mathbf{X}$, we plug the solution of equations (6) into $\mathcal{B}(\mathbf{Y})$, and we obtain the composite function which is a mapping from the values taken by the vector of efforts $\mathbf{X}$ to the set of real numbers:

$$\mathcal{B}\circ\mathcal{Y}\,(\mathbf{X}) \equiv \mathcal{B}\,(\mathcal{Y}\,(\mathbf{X})) \equiv B\,(\mathbf{X})\,. \tag{8}$$

The spatial management problem is then the constrained maximization of a function of management efforts $\mathbf{X}$:

$$\max_{\mathbf{X}} B\,(\mathbf{X}) \tag{9}$$

subject to

$$\mathbf{c}^\top * \mathbf{X} \leq G \tag{10}$$

$$0 * \iota^{\mathbf{n}} \leq \mathbf{X} \leq \overline{\mathbf{X}}\,, \tag{11}$$

where the two remaining functions are the budget constraint and the admissible range of efforts. As the constraints form a compact and convex set and the objective pertains to the class of $C^2$ functions, the problem is well-behaved and there trivially exists a solution. As it is usual, we denote $\mathbf{X}^*$ this optimal solution.

## 2.3. Decision criterion

Optimization methods were developed in applied mathematics in order to approach the solution of this class of maximization problems. Due to generality of functional forms and mathematical complexity, iterative algorithms were designed in order to nearly approach and single out an optimal solution. Different competing algorithm aiming at solving this task exist and each involves a trade off between the accuracy and the tractability of the approach. Inspiring from this literature and following a method developed in a companion paper aiming at defining decision tools for multi-species eradication programs (Courtois et al. 2018), we approximate this solution and propose a decision criterion to assist managers for spatial control of a biological invasion under limited financial resources.[5]

[4] Cost heterogeneity across areas is likely as there usually exists several competing control technologies whose use and efficiency often rely on physical constraints.

[5] The criterion we define here is to be viewed as a refinement or an extension of the criterion presented in Courtois et al. (2018). The criterion of Courtois et al. (2018) aims at prioritizing allocation choice among a set of biological invasion abstracting from spatial consideration. Complementarily and technically related, the criterion we develop in the current paper aims at prioritizing allocation choices over space of a single biological invasion that is spreading in a landscape.

The method proceeds in two steps. First we estimate the gradient of the objective at a given vector $\hat{\mathbf{X}}^1$ by computing the first order Taylor approximation of the objective :

$$B(\mathbf{X}) \simeq B\left(\hat{\mathbf{X}}^1\right) + \nabla B\left(\hat{\mathbf{X}}^1\right) * \left(\mathbf{X} - \overline{\mathbf{X}}\right). \tag{12}$$

In $\hat{\mathbf{X}}^1 = 0 * \iota^n$, that is in the pre-policy situation in which control is nil all over the landscape, the magnitude of the biological invasion over space is described by $\hat{\mathbf{Y}}^1$ such that $\hat{\mathbf{Y}}^1 = \rho \mathbf{Y^0} + \gamma \mathbf{W} * \hat{\mathbf{Y}}^\mathbf{1}$. We deduce from (8), that the gradient of the objective function in $0 * \iota^\mathbf{n}$ is :

$$\nabla B\left(0 * \iota^\mathbf{n}\right) = \nabla \mathcal{B}(\hat{\mathbf{Y}}^1) * \nabla \mathcal{Y}\left(0 * \iota^\mathbf{n}\right). \tag{13}$$

As $\mathcal{Y}(\mathbf{X})$ describes a system of linear equations, we have $\nabla \mathcal{Y}(0 * \iota^\mathbf{n}) = \nabla \mathcal{Y}(\mathbf{X})$ which we can compute from (6) :

$$\nabla \mathcal{Y}(\mathbf{X}) = -\beta * \mathbf{\Lambda}. \tag{14}$$

In $\hat{\mathbf{Y}}^1$, the gradient $\nabla \mathcal{B}(\hat{\mathbf{Y}}^1)$ is a vector with typical element we denote $\nabla \mathcal{B}_i$ such that :

$$\nabla \mathcal{B}_i = \left.\frac{\partial \mathcal{B}(\hat{\mathbf{Y}}^1)}{\partial Y_i}\right|_{\mathbf{Y}=\hat{\mathbf{Y}}^1}. \tag{15}$$

We deduce that gradient (13) writes :

$$\nabla B\left(0 * \iota^\mathbf{n}\right) = -\nabla \mathcal{B}^\top(\hat{\mathbf{Y}}^1) * \beta * \mathbf{\Lambda}, \tag{16}$$

that is a n-dimensional line vector $[b_1^0, ..., b_n^0]$ where each typical element is :

$$b_i^0 = -\sum_{j=1}^{n} \left(\frac{\partial \mathcal{B}(\hat{\mathbf{Y}}^1)}{\partial Y_j} * \beta \Lambda_{ji}\right). \tag{17}$$

Observe that this gradient is made of two components that embed two effects. The first is the direct impact that results from a decrease in the magnitude of the invasion within an area, *i.e.* $\partial \mathcal{B}_i(\hat{Y}_i^1)/\partial Y_i \; \forall i$. As one can expect, control effort within an area impacts this area. The second effect is an indirect one and is related to mobile externalities between areas. Decrease in the magnitude of the invasion in an area is likely to impact other areas that are spatially connected to this area and this cascade of additional impacts between areas is conveyed in matrix $\mathbf{\Lambda} = [\mathbf{I}^n - \gamma \mathbf{W}]^{-1}$. This second effect is important and reflects the complexity of this allocation problem.

Second, we make use of this gradient in order to define a simple rule approximating the solution of our optimization problem. In line with Courtois et al. (2018), the point here is to engineer a simple algorithm that could be used by a decision maker whatever the shape of the benefit function under consideration. To produce this algorithm, we distinguish the case where this solution lies on the boundary of the efforts set (corner solution or segment between two corners) and where it does not.

When the solution is extreme, that is when it lies on the boundary of the efforts set, the optimization problem described in (9-11) can be approximated by a linearized problem in matrix form:

$$\begin{aligned} &\max_{\mathbf{X}} \; \nabla B\left(0 * \iota^\mathbf{n}\right) * \mathbf{X} + \text{ constant terms.} \\ &\text{subject to (7) and (11)} \end{aligned} \tag{18}$$

The solution of this linear programming problem is to put maximum efforts in the control of the biological invasion in the area where the benefit/cost ratio is the highest. Denoting $\phi_i^1$ the benefit-cost ratio of controlling the biological invasion in area $i$, we have:

$$\phi_i^1 \equiv \frac{b_i^1}{c_i}, \quad i = 1, ..., n, \tag{19}$$

and the manager should allocate his budget so as to put his effort in priority in the area where this ratio is the highest, to next control the invasion in the area with the second highest ratio and so on, up to the point where the budget is exhausted. This is important to note that the benefit-cost ratio of controlling the biological invasion in area $i$ does not depend only on the direct impacts of control in this area, but also on its indirect impacts on areas that are physically connected and where the invasion in $i$ will spread to, *i.e* through matrix $\mathbf{\Lambda}$. A consequence is that three variables drive allocation choice : relative marginal cost between areas, relative marginal direct impact between areas and relative marginal indirect impact through spatial connectivity.
We deduce that if $\mathcal{B}$ is such that the solution of the optimization problem is extreme (*e.g.* $\mathcal{B}$ is linear or convex in $\mathbf{Y}$), this solution[6] is found : (1) by computing for all area $i$ the benefit cost ratio $\phi_i^1$, (2) by allocating the budget in priority in the area where this ratio is the highest, (3) in case of remaining budget, by allocating efforts in the area with the second highest ratio and so on, up to the point budget $G$ is exhausted.

Let us now generalize this result by considering any arbitrary functional form $\mathcal{B}$ and therefore, approximating solutions that do not necessarily lie on the boundary of the efforts set, *i.e.* interior solutions. Following Simianer (2003) and Courtois et al. (2018), we define an iterative algorithm in order to approach the solution of this general problem as an incremental sum of corner solutions.
We define $g$ as a share of the total budget such that $g = G/s$, with $s$ the number of shares. We suppose $s$ is big enough so that $g$ is sufficiently small for being spent in one area only. Instead of considering the allocation problem of budget $G$, we focus on the allocations of the $s$ shares, starting from the first. By definition each budget share is small enough to be spent in a single area. To allocate the first budget share, we apply the method presented for extreme solution and invest in the area where the benefit-cost ratio is the highest, say in area $j$. The corresponding control effort is $x_j = g/c_j$. To allocate the second budget share, we denote $\hat{\mathbf{X}}^2$ the n-dimensional vector where each element is zero except element $j$ which is $g/c_j$. We then compute the new gradient $\nabla B(\hat{\mathbf{X}}^2)$ of the objective function in this point. Following the method described in (13-19), we can then assess for all areas, $i = 1, .., n$, the new benefit-cost ratio in this point. We call this ratio $\phi_i^2$ and we have for any share $h = 1, .., s$

$$\phi_i^h \equiv \frac{b_i^h}{c_i}, \quad i = 1, ..., n, \quad h = 1, .., s, \tag{20}$$

with,

$$b_i^h = -\sum_{j=1}^{n} \left( \frac{\partial \mathcal{B}(\hat{\mathbf{Y}}^h)}{\partial Y_j} * \beta \Lambda_{ji} \right) . \tag{21}$$

Each share $h$ is allocated to the control of the biological invasion in the area with the highest benefit cost ratio. The following algorithm[7] can then be used for approximating the solution of our spatial optimization program for any functional form:

**Algorithm 1.** *Spatial prioritization algorithm*

1. *Divide budget* $G$ *into* $s$ *shares, such that* $g = G/s$*, and consider the allocation problem of each of the* $s$ *shares starting from allocation of share 1*
2. *Assess for each area i the ratio*
$$\phi_i^h \equiv \frac{b_i^h}{c_i},$$
*where h is the share number,* $h = [1, .., s]$
3. *Allocate the share h to the control of the area with the highest positive score. In case of ties, an area is selected randomly among them.*
4. *Update* $\hat{\mathbf{Y}}^h$.

[6]Note that this solution is an approximation. Courtois et al. (2014) provide for this class of problems a measure of the approximation error as a function of the curvature of the objective function ; Courtois et al. (2018) prove that the solution of this approximation necessarily leads to an improvement compared to inaction.

[7]For interested readers, properties of this class of algorithm are examined in Courtois et al. (2018).

5. *Return to step 2, recalculate the ratios $\phi_i^{h+1}$ and allocate the next budget share, until all shares are allocated.*

In the next section we apply this algorithm to a management problem : the spatial control of primrose willow in the Brière marshland in France.

# 3. Empirical application

## 3.1. Case Study

The Brière marshland is a regional park located on the west coast of France. The wetland spans over 20000 hectares and includes several villages, pasture lands and navigable channels. The park is popular for its biodiversity and for its recreation activities among hiking, fishing, hunting and rowboat riding. This last activity in particular attracts tourists all along the year and constitutes an important financial resource for the inhabitants of the region. Pasture is also a key usage of the marshland as cattle breeding is the major agricultural activity within the zone.

First reported in 1991 on the west side of the marshland, primrose willow (*Ludwigia peploides*), an amphibious plant originating from south America, progressively spanned all over the marshland threatening both biodiversity and economic activities. Left uncontrolled, *L.peploides* admits an explosive rate of proliferation over space and propagates extremely fast through channels. Unattended, the plant disseminates over water but also on pasture lands nearby. Covering water ponds, it harms drastically fish populations and plant diversity. It also impedes access to channels and to grazing pastures. Both recreational and agricultural activities are then drastically threatened by the invasion making prevention and control a major challenge for the managers of the park.

Control of the biological invasion started at the end of the 1990's with an increasing pressure put on the invasion over the years. Budgets spent varied from one year to the other and tended to increase over time. In average over 110000 were spent yearly in grubbing-up campaigns in the 2007-2017 period. According to accessibility of the area but also to width and depth of the channels, manual and mechanical treatments were implemented. Despite efforts made, primrose willow spread extensively over the years and the plant can now be found in any cardinal point of the wetland. Control efforts were mainly aimed at avoiding extensive spatial propagation and maintaining navigability on major channels. However, since 2007, an increasing areal of grazing pastures of the marshland became impracticable. Problems of accessibility also harmed fishing and hunting activities. Groups of farmers principally located in the south of the marshland got particularly concerned due to the propagation of primrose willow in the vicinity of their land. Some of them decided unilaterally to spread salt in order to halt the invasion. This led the managers of the park to set a salt experiment in the south of the marshland with a progressive introduction of salty water from the ocean *via* the Loire estuary. It resulted in halting primrose willow but led also to a major disruption of local diversity with a tremendous decline of the fish population. Other areas of the park remain subject to annual grubbing-up campaigns and due to important financial constraints and recurrent criticisms from stakeholders impacted, an objective clearly stated by the park is to rationalize efforts so as to limit ecological and economics impacts from the biological invasion in a cost-effective manner accounting explicitly for spatial distribution of impacts. This objective translates in allocating efficiently annual financial resources in order to maximize benefits from the control strategy accounting for cost heterogeneity over space on one hand, for spatial dynamics of the invasion on the other.

## 3.2. Spatial allocation choice

Algorithm (1) can be used in order to analyze this spatial allocation choice problem. We proceeded in four steps: (1) We divided the marshland in five areas of interest of similar size (Fig.2). Frontiers were defined in order to account for benefit and cost heterogeneity as well as propagation pathways within and between areas ; (2) We made a choice experiment in order to value the relative benefits of controlling the biological invasion in the different areas defined. A focus was put on user preferences and willingness to pay (WTP) for spatial control policy ; (3) We calibrated control costs in the five areas defined. We built an invasibility map and assessed for each area the proportion of channels, ponds and pasture lands likely to be invaded. We then estimated the control cost of the biological invasion in each areas using available data from 19 years of previous control campaigns ; We calibrated on the basis of 17 years of yearly GIS report statements a functional form of the plant annual dynamics over space.

In order to apply Algorithm (1) we proceed in two steps. First, we estimate the objective function $\mathcal{B}(\mathbf{Y})$ as well as the dynamics of the magnitude of the biological invasion over space described theoretically by equation (4). We deduce from these two equations, the transformed objective function $B(\mathbf{X})$ from whose the gradient can be approximated at

any point so that any element $b_i^h, \forall i, \forall h$, of this gradient can be assessed. Second, we calibrate cost function $\boldsymbol{C}(\mathbf{X})$ so that any element $c_i, \forall i$ of the gradient of this function can be assessed as well.
In this following, we explain how we proceed to estimate these three functional forms and relegate in the appendix technical work as much as possible.

### *3.2.1. Spatial benefits*

Assessing benefits related to a decrease of the surface covered by primrose willow in the different areas of the marshland is a complex task. Some of those benefits are economics values and can easily be monetized. For instance, among the 317 agricultural holdings that possess or use lands in the marshland, permanent grasslands represent 14971 ha that is 40% of their utilized agricultural area. Among these lands, 1345 ha are surrounded by ponds and channels and are therefore directly threatened by primrose willow. We can easily estimate the fodder or grazing production loss related to any hectares invaded. Direct loss in terms of agri-environment-climate commitment payments from European CAP is easily assessed also. In 2015 for example, the subvention loss due to primrose willow in the marshland was estimated to be about 38588.[8] Direct impact on boat rides is also easily assessed as we know the number of companies practicing this activity (11), their income and the number of people they employ (39). However, beyond the direct loss from halting this activity, an important indirect impact is a likely collapse of tourism which constitutes a major source of employment and income in the marshland. Other activities such as hiking, fishing or hunting are also difficult to monetized beyond the direct price of hunting and fishing permits which constitute imperfect proxies to monetize the benefits derived from these activities. Finally and this is a major complexity with this assessment, many impacts are ecological ones. Several species among endangered and critically endangered ones (e.g. pikeperche, chub, lamprey) are threatened by primrose willow. Ecological chain is also impacted and due to species interdependences, many uncertainties remain on ecological impacts due to the invasion. These non-monetary bequest and existence values, make impossible a straightforward direct valuation for the loss of biodiversity due to primrose willow.
Several method may be used in order approximate these different values. Following the review by Breidert et al. (2006), two principal methods are available: *revealed preferences* methods and *stated preferences* methods. A key advantage of the later is that valuation is derived from surveys which allows for estimating the determinants of WTP for any type of goods (see Louviere et al. (2000)). Discrete choice experiment method is particularly appropriate for estimating non-markets good values and existence values such as biodiversity and a growing body of studies offer evaluation of spatially differentiated preferences (Brower et al. 2010, Interis and Petrolia, 2016). We therefore conducted interviews from July 2016 to July 2017 and ran a websurvey.[9] Overall we collected a sample of 302 face-to-face respondents and 238 web-respondents from whose we estimated spatially explicit WTP for the control of primrose willow in the five areas of the marshland.
Interviews proceeded in four steps. First we showed a video displaying general information on the invasion and its impacts in the marshland. Second we asked preliminary questions about how the respondents were acquainted with the marshland and with the problems caused by primrose willow. Third we submitted eight choice set in order to elicit WTP and spatial preferences. Fourth we ended the interview with a survey on the degree of comprehension and socio-economic characteristics of respondents.
Key of the introduction video is to explain the choice problem. We define the delimitation of the marshland, the five areas where control could be performed, the impact caused by the invasion in the different areas of the marshland and the current state of the invasion in each of these areas. Choice problem is then to ask whether control effort is to be implemented within the next five years and if need be, where in priority and how much respondents are willing to contribute for. Choice set are based on the map of the marshland. It comprises six attributes: the five areas and a cost attribute. Spatial attributes admit three infection levels : low (in green), intermediate (in yellow) and high (in red). Signification of colors is carefully explained in the introduction video and photos are provided. Each level is defined as a proportion of area invaded with two thresholds : 10 and 50% so that low levels $\in [0, 10]$, medium levels $\in [10, 50]$, high levels above 50% (i.e. see Appendix A.X for a detailed description of these colors). In order to avoid size effects preferences, areas were defined so as to be of identical size. We also assumed in the experiment that each area had the exact same invasibility surface. Payment attribute admits five levels : 0, 5, 15, 30 and 60  per year. Again, explanation was provided in the video and it was explained that this cost will be paid annually on local tax (which is the case right now). When responding a choice set, one option among three is selected, one of these options being the no policy

[8] Agri-environment-climate commitment payment received pertain to the class of incentives for grazing practices - sub-measure 10.1.4 Grassland GS1-17

[9] see Bougherrara et al. (2017) for a detailed analysis of this choice experiment and major results raised.

**Table 1**
Willingness to pay (per year)

| | Red to Yellow | Yellow to Green | Red to Green |
|---|---|---|---|
| Area 1 | - | $10.94^{***}$ | - |
| Area 2 | $15.93^{***}$ | $17.00^{***}$ | $32,93^{***}$ |
| Area 3 | $27.13^{***}$ | $15.60^{***}$ | $42.73^{***}$ |
| Area 4 | - | $10.58^{***}$ | - |
| Area 5 | - | $5.16^{***}$ | - |

option also called statu quo. An example of choice set is provided in Figure (**??**):

Note that spatial attributes levels represent states of the invasion in five years, current state being defined in the video displaying general informations. Note also that we ran an efficient protocol and excluded irrelevant alternatives (e.g. situation where invasion magnitude increases with the amount spent ; situations describing irrelevant invasion magnitudes). In particular, because magnitude of primrose willow cannot reach a critical level in areas 1, 4 and 5 within the 5 years period, we excluded these alternatives from the experiment.

All coefficients are highly significant and have consistent signs, meaning that all WTP increases with efforts. In average respondents are willing to pay 14 per year in order to avoid the statu quo. They express therefore strong support for control policy in all areas. They also exhibit spatial preferences, with areas where they are willing to pay twice as much in order primrose willow to be controlled. In particular, the study shows three categories of areas : high priority areas (areas 2 and 3), medium priority areas (areas 1 and 4) and low priority areas (area 5).[10] We use these WTP in order to calibrate benefit function $\mathcal{B}(\mathbf{Y})$. Because we have only three spatial attribute levels (two in few areas), we need to calibrate functional forms on the basis of a limited set of points. We assume piecewise linear calibration meaning that damages caused by the invasion is proportional to the number of hectares invaded.[11]. Using individual WTP described in Table (1), we depict in Figure (**??**) the individual benefits associated to a decrease in the proportion of primrose willow in each zone.

The left part of the figure depicts benefits associated to a decrease in the proportion of the primrose willow for any initial state of the invasion. We assume median points on the proportion of surface invaded and consider two intervals : [50;25[ (i.e. from red to yellow) and [25;0] (i.e. from yellow to green). Notice that we do not have information regarding WTP for reducing invasion magnitude from critical level (red) to medium level (yellow) in areas 1, 4 and 5. We then assume WTP increases homogeneously on the interval [0;50]. The right hand part of the figure depicts individual WTP to reduce the invasion when invasion state is medium or low ($\leq 25$) and it values payments to progressively switch from a medium level of invasion (yellow) to a low level of invasion.

Considering the average invasibility surface over the five areas and denoting it $\overline{\overline{Y}}$, with $\overline{\overline{Y}} \equiv 1135$ ha, we assess benefit function $\mathcal{B}_i(Y_i)$ for $i = [1, .., 5]$, with $Y_i$ the invaded surface in area $i$ measured in square meters and $\mathcal{B}$ the impact measured in . We obtain the following functional forms:

$$\mathcal{B}_i(Y_i) = \begin{cases} \tau_i \cdot Y_i & if\ Y_i^0/\overline{\overline{Y}} \leq 0.25, \\ \tau_i' \cdot Y_i & if\ Y_i^0/\overline{\overline{Y}} > 0.25, \end{cases} \tag{22}$$

with $Y_i^0$ the invaded surface in area $i$ at the beginning of the management period. At the individual level, gradient parameters $\tau$ and $\tau'$ are provided in Table(3).

In order to derive gradient parameters at the county level, we aggregate individual WTP considering the number of fiscal households in the county. Considering INSEE data 2015, we have 576113 fiscal households in the county and we deduce that aggregated functional form parameters for the entire cohort is:

[10] Interested readers may refer to our companion paper (Bougherara et al., 2017) in which we provide an extended analysis of the results with a focus on spatial preferences according to respondents locations as well as respondents activities in the marshland.

[11] This assumption is in our opinion reasonable as surfaces densely invaded are surfaces lost. Once critical level is reached, problem of accessibility threatens activities in the entire zone implying a change in the gradient of the function

**Table 2**
Functional form parameters (per capita)

| | Area 1 | Area 2 | Area 3 | Area 4 | Area 5 |
|---|---|---|---|---|---|
| $\tau$ | -3.856E-06 | -5.991E-06 | -5.498E-06 | -3.729E-06 | -1.819E-06 |
| $\tau'$ | -3.856E-06 | -2.807E-06 | -4.781E-06 | -3.729E-06 | -1.819E-06 |

**Table 3**
Functional form parameters (aggregated)

| | Area 1 | Area 2 | Area 3 | Area 4 | Area 5 |
|---|---|---|---|---|---|
| $\tau$ | -2.2212 | -3.4516 | -3.1674 | -2.1481 | -1.0477 |
| $\tau'$ | -2.2212 | -1.6172 | -2.7542 | -2.1481 | -1.0477 |

### *3.2.2. Spatial costs*

We propose in this section an empirical evaluation of primrose willow control costs, to calibrate the economic management model, and more specifically, to obtain marginal costs for each of the five ecological management areas. While accounting for cost heterogeneity across the latter is essential for helping environmental managers implement a genuine control strategy, the limited number of observations implies that homogeneity restrictions need to be made on parameters, as will be discussed below. Moreover, the nature of available data on control operations, obtained through management records, is such that only a simplified representation of control technology is possible.

#### *The data*

Yearly management of the biological invasion started in 1999. Data were collected on each grubbing-up operations, displaying information on location, total cost, manpower and surfaces treated. With an average of 15 operations per year, the dataset contains approximately 270 operations. Each operation information on the location of control, total cost, daily manpower and invaded surface treated. Because density of one square meter of primrose willow treated varies, **????**

Data were available on a sample of control sites, with information on the cost of extraction and disposal of primrose willow, and the volume of observed biomass (in kg net weight and in number of 25-kg bags), allowing to compute directly an indicator of average cost.

Observed average cost is not in general equal to marginal cost of control, as the former potentially includes a fixed cost, and the latter cannot in general be directly computed from the data. As a consequence, the objective here is to specify a parameterized cost function from which marginal cost can be derived in a parcimonious manner, as the first derivative of the cost function with respect to the activity level (treated area or volume of primrose willow extracted).

Because capital expenses (including amortization) and operation expenses other than labour are not available, we approximate the cost function $C$ as the number of man-hours of the site treated, multiplied by a unit, homogeneous wage rate (approximately 130 euros as reported by experts from local companies in biological invasion control).

The sample consists of 142 yearly observations on individual sites from 2007 to 2016, and some sites may be treated more than once during this period. A baseline of 7 hours of operation a day is assumed, but the average number of hours on a given site may exceed (or be less than) such figure, in which case the volume of work is adjusted accordingly. We consider that the daily number of hours worked for a particular site is equal to the total number of hours for extracting primrose willow and for transportation (disposal) of the biomass.

Treated areas are computed with the software Q-gis by merging botanical inventories and maps of operation sites, converting linear range measures into $m^2$ surface. For conversion, we consider an average 5-meter width of primrose willow on waterway banks, so that 200 meter of invaded waterway bank corresponds to an area of 1000 $m^2$. We distinguish sites according to two different factors: the type of the site (waterway) and its density of primrose willow biomass. We define Type 1 for major waterways, Type 2 for minor waterways, and Type 3 for land area. Density 1, 2 and 3 respectively denote density less than 10 percent, between 10 and 50 percent and more than 50 percent. **Such density thresholds are defined to represent continuous, discontinuous and dispersed aquatic grass beds**

**Table 4**
Descriptive statistics on costs and sites

| Variable | Mean | Standard deviation | Minimum | Maximum |
|---|---|---|---|---|
| Man-days (number) | 15.245 | 16.9021 | 1.52 | 107 |
| Operation hours | 14.1055 | 13.0669 | 2.5 | 78 |
| Disposal (transport) hours | 1.3522 | 2.2333 | 0 | 16 |
| 25-kg bags (number) | 144.7958 | 299.5148 | 1 | 3000 |
| Extracted biomass (kg) | 3619.894 | 7487.87 | 25 | 75000 |
| Area ($m^2$) Type 1, Density 1 | 3140.458 | 5353.003 | 0 | 27982 |
| Area ($m^2$) Type 1, Density 2 | 762.5915 | 2473.423 | 0 | 21456 |
| Area ($m^2$) Type 1, Density 3 | 339.4507 | 1316.982 | 0 | 10169 |
| Total area Type 1 | 16441.4 | 28087.17 | 0 | 201007 |
| Area ($m^2$) Type 2, Density 1 | 4132.063 | 7174.683 | 0 | 34985 |
| Area ($m^2$) Type 2, Density 2 | 1269.866 | 2759.268 | 0 | 21177 |
| Area ($m^2$) Type 2, Density 3 | 660.7746 | 1480.285 | 0 | 8587 |
| Total area Type 2 | 45885.57 | 63549.66 | 0 | 254454 |
| Operating cost | 1981.85 | 2197.275 | 197.6 | 1391 |
| Average cost for Type 1 | 0.2207 | 0.2852 | 0.0054 | 1.3423 |
| Average cost for Type 2 | 0.3469 | 0.6928 | 0.0032 | 4.1823 |
| Average cost, all types | 0.2461 | 0.5601 | 0.0030 | 4.1823 |

Notes. 142 observations. Period: 2007-2016. Type 1 corresponds to major waterways, type 2 is for minor waterways. Density 1, 2 and 3 respectively denote biomass density of primrose willow less than 10 percent, between 10 and 50 percent and more than 50 percent.

**Table 5**
Average number of man-days worked and density on extraction sites

| | Area | | | | |
|---|---|---|---|---|---|
| Site type | 1 | 2 | 3 | 4 | 5 |
| 1 | 2.21 | 2.28 | 1.52 | 2.19 | 7.50 |
| (% D1) | (0.4048) | (0.4121) | (0.5679) | (0.3295) | (0.1495) |
| (% D2) | (0.2109) | (0.1674) | (0.1903) | (0.2329) | (0.1264) |
| (% D3) | (0.3862) | (0.4204) | (0.2416) | (0.4374) | (0.0576) |
| 2 | 8.00 | 2.00 | 2.00 | 2.00 | 15.09 |
| (% D1) | (0.6850) | (0.5434) | (0.6239) | (0.5127) | (0.4316) |
| (% D2) | (0.1462) | (0.1761) | (0.2152) | (0.2565) | (0.0697) |
| (% D3) | (0.1686) | (0.4204) | (0.2416) | (0.4374) | (0.0542) |

Notes. Proportion of densities 1 2 and 3 (respectively, D1, D2 and D3) are in parentheses.

**respectively).**

In our analysis, we will not consider Type 3 sites, as extraction of primrose willow is not possible on land in our setting, given available technologies, therefore no extraction sites are observed for Type 3.

Table 3.2.2 presents descriptive statistics on the sample, from which area 1 appears to be characterized by a significantly lower average cost, compared with the four other areas.

To have a more detailed description of operating costs across sites (recalling that such costs are considered more or less proportional to number of man-days), we present in Table 3.2.2 the average number of days worked on a site, by area, site type and density, together with the proportion of each density.

To test for a significant difference between site types across areas, we compute the chi-square contingency test, distributed as a $\chi^2(5\text{x}2 = 19)$ variate under the null of no difference in the number of man-days. The test statistic equals 0.2616 ($p$-value of 1.00), so that we do not reject the null of no dependence between area and site type.

We then conduct a mean-equality test between the number of man-days in sites of type 1 and 2. Under the null hypothesis of equal sub-sample means, the test statistic is distributed as a Student and is equal to $t = -2.3850$, with a $p$-value of 0.0193, so that we reject the null. We conclude from these two tests than the cost function $\boldsymbol{C}$ can be considered homogeneous between areas, by that the cost structure will be different between Type 1 and Type 2 sites.

**Table 6**
Average total and average costs by are and type of extraction site

| | | Area | | | | |
|---|---|---|---|---|---|---|
| Site type | | 1 | 2 | 3 | 4 | 5 |
| 1 | TC | 197.60 | 287.30 | 296.40 | 284.30 | 975.41 |
| | mean AC | 0.0582 | 0.0546 | 0.0602 | 0.0422 | 0.1938 |
| | median AC | 0.0422 | 0.0546 | 0.0582 | 0.0582 | 0.1938 |
| 2 | TC | 1040.00 | 260.00 | 260.00 | 390.00 | 1957.76 |
| | mean AC | 0.0406 | 2.3214 | 1.6560 | 0.0149 | 0.5374 |
| | median AC | 0.0406 | 1.5568 | 1.5568 | 1.6560 | 0.5169 |
| 1 and 2 | TC | 618.80 | 273.65 | 278.20 | 338.65 | 1466.58 |
| | mean AC | 0.0476 | 1.1898 | 0.8581 | 0.0286 | 0.3656 |
| | median AC | 0.0414 | 0.8085 | 0.8075 | 0.8571 | 0.3554 |

Notes. In euros by extraction site. TC and AC respectively denote total and average operating cost.

Statistics on total operating and average operating costs by site are presented in Table 3.2.2. To limit the influence of extreme values on average operating cost, we also present the median of average operating cost in the Table. We note a very important difference between operating costs for major (Type 1) and minor waterways (Type 2), and, as expected, more concentrated values of average cost with the median than with the mean.

***Specifying the control cost function***

Several specifications can be considered for a parametric representation of the control operating cost (recall that we chose not to estimate the total cost function because of lack of data, but the average cost of control, as a function of man-days worked).

We consider for instance that parameters of such technology are constant for a given area, and do not not depend on the operation size (e.g., volume of biomass extracted from the waterway or the river bank). Nevertheless, cost parameters may depend upon each geographical area, to account for differences in biomass primrose willow density (of invasion). We first present data on costs and characteristics of associated areas, and then various cost specifications that we consider, namely

a) Total operation costs as the sum of cost components, conditioned by biomass density ;

b) A system of simultaneous equations expressing the control cost for a high-biomass density area alone, together with proportionality relationships between different density types ;

c) A homogeneous cost with respect to biomass density, where all densities are converted to high density.

As described above, invaded waterways in the five areas $Z_1$ to $Z_5$ are of two types (denoted $M_1$ and $M_2$) and can be characterized by three density levels, denoted $D_1$, $D_2$ and $D_3$. In Model a), the control cost function $C$ can be specified as

$$C = f\left[Y_{D_1},\, Y_{D_2},\, Y_{D_3} \mid Z_i,\, M_j,\, i = 1, \dots,\, 5\,; j = 1,2\right]$$

$$= f_{ij}\left[g_1\left(Y_{D_1}\right) + g_2\left(Y_{D_2}\right) + g_3\left(Y_{D_3}\right)\right],\; i = 1, \dots,\, 5\,; j = 1,2, \tag{23}$$

where $Y_{D_i}$, $i = 1, 2,\ 3$ represents an indicator of primrose willow invasion with density $i$, and we assume that biomass levels of density $D_1$, $D_2$ and $D_3$ have a specific, additive cost function $g_k\left(Y_{D_k}\right)$, $k = 1,\ 2,\ 3$. Recall that the biomass level, following the conversion described above, is measured in square meters of invaded area, implying that, in the following, primrose willow invasion can indifferently be described in terms of biomass volume or surface. **NB : pas clair ici, car on n'a pas introduit avant de notion de masse, de poids ?**

Marginal cost, denoted $c_i$, is obtained as the first derivative of $C$ with respect to the primrose willow volume or area:

$$c_i = \frac{\partial C}{\partial Y_{D_i}},\ 1, 2,\ 3. \tag{24}$$

A first estimation can be performed by regressing cost on all density levels with a quadratic specification and a constant term, as in Jardine and Sanchirico (2018).

$$C = f_j\left[g_1\left(Y_{D_1}\right) + g_2\left(Y_{D_2}\right) + g_3\left(Y_{D_3}\right)\right] = \alpha_0 + \sum_{k=1}^{3}\left(\beta_{1k}Y_{D_k} + 1/2\,\beta_{2k}\,Y_{D_k}^2\right). \tag{25}$$

In the case of Model b), we consider a system of simultaneous equations that takes into account proportionality relationships between biomass density levels, and which considers cost as a function of density $D_3$ only (provided that corresponding parameters are significantly different from 0). The system of equations is the following:

$$C = \beta_1 Y_{D3} + \varepsilon_1, \tag{26}$$
$$Y_{D_3} = \gamma_2 Y_{D_2} + \varepsilon_2, \tag{27}$$
$$Y_{D_2} = \gamma_1 Y_{D_1} + \varepsilon_3, \tag{28}$$

where $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$ are random terms, and parameters $\beta_1$, $\gamma_2$ and $\gamma_1$ respectively denote average cost per unit of biomass of density $D_3$, and proportionality factors between $D_2$ and $D_3$, and between $D_2$ and $D_1$. Such model corresponds to strategy of controlling primrose willow invasion that ''targets"biomass of type $D_3$ (continuous **aquatic grass beds**) as a first in a series of concentric circles. Cost associated with areas of density $D_2$ and $D_1$ can then be retrieved from estimated proportionality relationships between $D_3$ and $D_2$ on the one hand, and $D_2$ and $D_1$ on the other. A strong assumption is that control cost is directly proportional to the volume of primrose willow biomass, with a proportionality factor (representing average cost per unit of biomass extracted ) that decreases as one moves away from area of density $D_3$. Finally, in the third specification of Model c), we convert densities $D_1$ and $D_2$ in terms of density $D_3$, to express cost as a function of the latter only. In a sequential way, densities $D_1$ and $D_2$ are first regressed on $D_3$ to obtain predictions similar to proportionality rules of Model b). In a second step, the ''predicted" density $\hat{D}_3(D_1, D_2)$ is used as an explanatory variable of cost for each area ($Z_1$ to $Z_5$). In the third and last step, the mean of marginal costs is computed for each area.

### *Cost estimation results*

Estimation results of Model a) are presented in Table **??**. In terms of parameter precision and also of expected sign of parameters, this model specification performs poorly, especially regarding the condition that convexity of the cost function is required to have a positive and possibly increasing marginal cost with respect to biomass extracted. As can be seen from Table 3.2.2, except for density $D_3$, quadratic terms are all negative (implying a concave cost function), and the majority of parameters of order 1 (linear terms) are not significant. Similar results obtain with the constrained model without intercept term (with $\beta_0 = 0$).

A possible explanation for non expected signs (or the non significance) of parameter estimates is that, for a given area, the density levels are not independent. If a significant proportionality relationship exists between $D_1$ and $D_2$ on the one hand, and between $D_2$ and $D_3$ on the other, then parameters in the cost function, associated with biomass of different densities, are likely to be biased.

Table 3.2.2 presents estimation results of Model b) (constrained model of simultaneous equations).

One confirms from Table 3.2.2 that the proportionality factor decreases along with the distance with respect to density $D_3$. For example, for all areas in the last column of the table, the ratio between $D_3$ and $D_2$ equals 0.47 and decreases to $0.47\times 0.719 = 0.338$ between $D_3$ and $D_1$. Note that areas of Type 2 experience a ratio between densities $D_2$ and $D_1$very close to 1 (0.9967), implying that the treatment cost should also be close for these two densities. The system of equations can be solved to provide an estimate of average costs for different densities:

**Table 7**
Quadratic cost function parameter estimates

| Parameter | Site Type 1 | Site Type 2 | All site types |
|---|---|---|---|
| $\beta_0$ | 0.1963* | 0.4959*** | 0.5393*** |
| | (0.1147) | (0.1106) | (0.1352) |
| $\beta_{11}$ | 0.1819 | 0.1480 | 0.6625*** |
| | (0.1163) | (0.2025) | (0.1657) |
| $\beta_{21}$ | -0.0193 | -0.0177 | -0.1147*** |
| | (0.0241) | (0.0423) | (0.0340) |
| $\beta_{12}$ | 0.0725 | 0.1293 | 0.0495 |
| | (0.0122) | (0.0554) | (0.0882) |
| $\beta_{22}$ | -0.0133*** | -0.0188 | -0.0125* |
| | (0.0033) | (0.1353) | (0.0067) |
| $\beta_{13}$ | -0.0564 | 0.1535 | 0.1540* |
| | (0.0518) | (0.1239) | (0.0794) |
| $\beta_{23}$ | 0.0134*** | 0.0884*** | 0.0017 |
| | (0.0029) | (0.0172) | (0.0054) |
| $R^2$ | 0.7365 | 0.7901 | 0.1579 |
| Observations | 43 | 43 | 142 |

Notes. Estimated standard errors are in parentheses. *, ** and *** respectively indicate a parameter significant at the 10, 5 and 1 percent level.

**Table 8**
Constrained simultaneous-equation parameter estimates

| Parameter | Site Type 1 | Site Type 2 | All site types |
|---|---|---|---|
| $\beta_1$ | 0.0782*** | 0.3443*** | 0.2456*** |
| | (0.0117) | (0.0306) | (0.0392) |
| $\gamma_2$ | 0.7338*** | 0.5333*** | 0.4700*** |
| | (0.1811) | (0.1429) | (0.0818) |
| $\gamma_1$ | 0.9428*** | 0.9967*** | 0.7190*** |
| | (0.2652) | (0.1925) | (0.1181) |
| Observations | 43 | 43 | 142 |

Notes. Estimated standard errors are in parentheses. *, ** and *** respectively indicate a parameter significant at the 10, 5 and 1 percent level.

$$\frac{C}{Y_{D3}} = \beta_1, \tag{29}$$

$$\frac{C}{Y_{D2}} = \frac{C}{Y_{D3}} \times \frac{Y_{D3}}{Y_{D2}} = \beta_1\, \gamma_2\,, \tag{30}$$

$$\frac{C}{Y_{D1}} = \frac{C}{Y_{D3}} \times \frac{Y_{D3}}{Y_{D2}} \times \frac{Y_{D2}}{Y_{D1}} = \beta_1\, \gamma_2\, \gamma_1. \tag{31}$$

$$\tag{32}$$

The econometric estimation above provides us with marginal costs of control by density of primrose willow (1, 2, 3) and by type of site (1, 2). Every area being heterogeneous with respect to both of these variables, we compute a weighted average of marginal costs over the five areas of our study, using as weights the proportions of land corresponding to each density and type of site in each area (see Table **??**).

Table 3.2.2 presents estimates of marginal costs by site type and density, and Table 3.2.2 reports final marginal cost estimates by area.

In the model specification without intercept term, marginal cost estimates are constant (but they are different across site types), and the differences between values in different areas are therefore due to the uneven distribution of site type

**Table 9**
Marginal cost estimates, by site type and density

| | With intercept | No intercept |
|---|---|---|
| Site Type 1 | | |
| Density 1 | 0.0152 | 0.0198 |
| Density 2 | 0.0767 | 0.1001 |
| Density 3 | 0.3177 | 0.4146 |
| Site Type 2 | | |
| Density 1 | 0.0398 | 0.0481 |
| Density 2 | 0.2009 | 0.2426 |
| Density 3 | 0.8319 | 1.0049 |

Notes. Marginal costs are in euro / $m^2$.

**Table 10**
Marginal cost estimates by area, Model b)

| Area | With intercept | No intercept |
|---|---|---|
| 1 | 0.2689 | 0.3274 |
| 2 | 0.3132 | 0.3811 |
| 3 | 0.4524 | 0.5629 |
| 4 | 0.2790 | 0.3398 |
| 5 | 0.1085 | 0.1338 |

Notes. Marginal costs are in euro / $m^2$.

by area.

Let us turn to the last specification, Model c), in which all densities on a given site are converted to $D_3$ density. This specification allows us to calibrate the economic model of invasion control in a simplified way, as the latter considers a single state variable for the biomass of primrose willow. Moreover, specification c) avoids the rather strong assumption that cost proportionality would be maintained (see Model b) above) while obtaining marginal cost estimates for densities $D_1$ and $D_2$ from marginal cost of density $D_3$. A limitation of Model c) however, is that the distribution of invaded waterways among the three densities is estimated as an average, with parameters specific to each area, which can be different from the actual distribution on a given site. Note however that the same drawback also applies to Model b). Estimation results for Model b) are presented in Table 3.2.2 for areas of Type 1 and Type 2, and with and without intercept.

Marginal costs estimates, in terms of density $D_3$, are the following: for site type 1, 0.1498 with intercept and 0.1955 without intercept ; for site 2, 0.2636 with intercept and 0.3185 with out intercept. Marginal costs by area are finally estimated by area, using as weights the proportions of land corresponding to each density and type of site in each area. Results are presented in Table 3.2.2.

Estimated marginal costs with specification of Model c) are very similar across the five areas, much more in fact than ith specification b) (in which density $D_3$ is the ''target"and proportionality rules apply with densities $D_1$ and $D_2$. Note also that estimates with or without intercept terms (proportional or linear model of cost) are not very different in magnitude, for specification c).

In order to match as closely as possible the structure of the economic model of control, in a simple yet realistic representation of the heterogeneity of control sites, we consider Model c) without intercept as our specification of choice. In this specification, cost parameters are specific to the site type and not the density of primrose willow, and marginal costs by area are computed as the weighted average of marginal cost estimates over the two types of sites (Type 1 and Type 2).

### *3.2.3. Spatial dynamics*

The purpose of this section is to estimate the propagation equation of primrose willow, accounting for both its dynamics and spatial diffusion along the geographical areas of our study. In our representation of biological invasion control, we consider a myopic model that does not integrate long or medium term dynamics. Decisions are made today for results that will be observed tomorrow, implying that all that is required is a representation of the spatial dynamics

**Table 11**
Cost function estimates with densities converted to $D_3$, by site type

| Parameter | Site Type 1 | Site Type 2 | All site types | |
|---|---|---|---|---|
| | | Site type 1 | | |
| $m^2_{D_2}$ = | 0.8712*** $m^2_{D_3}$, | $R^2 = 0.29$ | | |
| | (0.2006) | | | |
| $m^2_{D_1}$ = | 0.24911 $m^2_{D_3}$, | $R^2 = 0.18$ | | |
| , | (0.5106) | | | |
| Cost = | 584.1154*** | + 0.1498*** $m^2_{D_3}$, | $R^2 = 0.31$ | (With intercept) |
| | (172.7326) | (0.0336) | | |
| Cost = | 0.1995*** $m^2_{D_3}$, | $R^2 = 0.42$ | | (No intercept) |
| | (0.0344) | | | |
| Observations | 43 | | | |
| | | Site type 2 | | |
| $m^2_{D_2}$ = | 0.8285*** $m^2_{D_3}$, | $R^2 = 0.10$ | | |
| , | (0.2502) | | | |
| $m^2_{D_1}$ = | 1.3265** $m^2_{D_3}$, | $R^2 = 0.08$ | | |
| | (0.5319) | | | |
| Cost = | 818.5917*** | + 0.2636*** $m^2_{D_3}$, | $R^2 = 0.62$ | (With intercept) |
| | (233.4627) | (0.0312) | | |
| Cost = | 0.3185*** $m^2_{D_3}$, | $R^2$= 0.71 | | (No intercept) |
| | (0.0305) | | | |
| Observations | 43 | | | |

Notes. Estimated standard errors are in parentheses. *, ** and *** respectively indicate a parameter significant at the 10, 5 and 1 percent level.

**Table 12**
Marginal cost estimates by area, densities converted to $D_3$

| Area | With intercept | No intercept |
|---|---|---|
| 1 | 0.2383 | 0.2911 |
| 2 | 0.2397 | 0.2927 |
| 3 | 0.2437 | 0.2969 |
| 4 | 0.2373 | 0.2901 |
| 5 | 0.2080 | 0.2584 |

Notes. Marginal costs are in euro / $m^2$.

of the biological invasion within this period of time. Say that we focus on the 2018 choice problem. We know the magnitude of primrose willow in the five areas of the marshland at the beginning of the period (April 2018) and we aim at evaluating the magnitude of the biological invasion at the end of the period (April 2019) in any area $i$ and given any control strategy implemented in all areas. Such evaluation requires the estimation of a dynamic equation of primrose willow that includes spatial spillover effects among areas.

### *The data*

For estimation purposes, we have at our disposal a large data set collected yearly by the regional park, starting in 1999 with the beginning of the control strategy implemented in the marshland, and ending in 2017 (19 years). Two pieces of panel data compiled in a GIS database were collected: (1) a spatially explicit inventory of the prevalence of the biological invasion ; (2) a spatial inventory of control implemented. Inventories of species prevalence were performed at the middle of the growing season, that is in April-May. For technical reasons, these inventories were made accounting for the areal covered by the biological invasion (in $m^2$).[12]

[12] Note that three densities were distinguished in those inventories (from low - scattered and discontinuous - to high - dense and continuous). We elicited statistically a proportional relationship between these three densities - i.e. any square meter covered by primrose willow proves to be composed of $x$% of primrose willow in high density, $y$% in medium density and $1 - x - y$% in low density.

**REMARQUE : Besoin de passer ces infos plus haut, dans la section spatial costs of control, car REDONDANCE**

We then end up working with a single multi-density unit which simplifies slightly our model (see Appendix A.2). As primrose willow cannot be controlled when established in grasslands, we distinguish surfaces that can be managed (ponds, channels) and that cannot (grasslands).[13] We then aggregate GIS data in each area and for the 18 years, so as to obtain surfaces covered by the biological invasion that can be managed, and those that cannot. For obtaining more precise estimates of the dynamic spatial equation, we isolate as much as possible intra- and inter-areas species growth. For this purpose, we consider sub-areas and split each of our five areas in five areas in order to best account for invasion pathways (see Figure (**??**)). We then obtain 25 sub-areas for which we qualify the magnitude of the biological invasion (in $m^2$) at each year period, so that the initial number of observations is $25 \times 19 = 475$. Spatially explicit control data were collected along each year. Inventories were also made accounting for area treated (in $m^2$), with each square meter treated cleared of any visible trace of primrose willow.[14] These two pieces of data are sufficient to estimate spatio-temporal equation (**??**) by setting matrix **W** according to experts estimates (see Appendix A.4 Figure(**??**)).

Each observation unit ''sub-area - year" is associated with three density levels ($D_1$, $D_2$ and $D_3$, as in Section ???, and for each density level, the sample contains records on the biomass invasion level, area not controllable (land, in particular, in $m^2$), potentially controllable area, and area controlled (all in $m^2$).

As in section ??? for spatial costs of control (Model c), and to better correspond to the economic model of control, a single dynamic model needs to be estimated. We therefore convert all invaded areas in terms of density $D_3$, namely $Y_{D_3}$ (with notation of Section ???), where conversion parameters are those of proportionality relationships estimated in Section ??? The restriction implied by such simplified specification is that the distribution of the three densities in each area in constant through time (homotheticity in the diffusion process among densities $D_1$, $D_2$ and $D_3$.

Inspection of the initial dataset reveals that 3 sub-areas (in area 5) contain only 4 positive observations on areas (out of 19×3=57), and only 2 of them have been controlled (in 2016 and 2017). For this reason, we drop these 3 sub-areas and end up with a final sample that contains $22 \times 19 = 418$ observations. The issue of dealing with non-positive areas is discussed further below in this section.

Table **??** presents descriptive statistics on the sample, by density level ($D_1$, $D_2$ and $D_3$) as well as for the new variables corresponding to normalized area with respect to density $D_3$.

There are various available specifications for the spatial econometric model with panel data (repeated cross sections). As discussed in Baltagi (2015), econometric models for panel data allow for a dedicated treatment of unobserved heterogeneity (in the form of time-invariant individual effects). This is particularly interesting when unobserved individual characteristics may drive part of the dynamics of the dependent variable, while being correlated with some explanatory variables (see for example Boumahdi and Thomas, 2008). However, such correlation (a source of potential bias in parameter estimates) is also possible with random error terms that vary both across individuals (in our case, sub-areas) and time (years). In such case, methods to control for unobserved heterogeneity such as fixed effects or some instrumental-variable methods (Hausman-Taylor, 1981 ; Cornell and Rupert, 19???, for example) are ineffective, and a different model specification may be considered.

We consider the following spatial dynamic model:

$$Y_{it} - X_{it} = \rho(Y_{i,t-1} - X_{i,t-1}) + \delta Y_{i,t-1} + \gamma W Y_{it} + \beta Z_{it} + \alpha_i + \lambda_t + \varepsilon_{it}, \tag{33}$$

$$i = 1, \ldots, N; t = 1, 2, \ldots, T,$$

where $W$ is a spatial weight matrix (or contiguity matrix) with dimension $N \times N$, $Z$ is a vector of explanatory variables, $\alpha_i$ and $\lambda_t$ are unobserved individual and time effects respectively , and $\varepsilon_{it}$ is an error term, identically and independently distributed. The model integrates both the dynamics of the dependent variable and the (contemporaneous) spillover effects from other sub-areas. Spatial correlation is specified only for the contemporaneous dependent variable through matrix $W$, and we do not consider spatial effects from other explanatory variables (explanatory variables $Z$), nor on the lagged dependent variable. Baltagi, Fingleton and Pirotte (2014a, 2014b) discuss the properties of the most frequently used spatial econometric models, which include SAR (Spatial AutoRegressive Model) with lagged dependent variable

[13]Note that surfaces that cannot be managed constitute residual reservoirs of biological invasion. However, as grasslands are distributed all over the marshland, prioritizing one area versus the other in order to avoid the formation of residual reservoirs is not a choice alternative.

[14]Note that cleared of visible traces does not mean cleaned as residues always remain.

**Table 13**
Descriptive statistics on primrose willow invasion

| Variable | Mean | Standard deviation | Minimum | Maximum |
|---|---|---|---|---|
| | | Density $D_1$ (area in ha) | | |
| Invaded$_1$ | 85.6795 | 16.1720 | 0 | 113.6824 |
| Not controllable$_1$ | 51.1947 | 12.7845 | 0 | 96.0957 |
| Controllable$_1$ | 34.4847 | 47.5733 | 0 | 24.4133 |
| Controlled$_1$ | 17.3124 | 30.9731 | 0 | 17.7023 |
| | | Density $D_2$ (area in ha) | | |
| Invaded$_2$ | 4.7552 | 10.5894 | 0 | 88.2981 |
| Not controllable$_2$ | 2.9763 | 7.8410 | 0 | 77.7493 |
| Controllable$_2$ | 1.7788 | 4.1209 | 0 | 38.5489 |
| Controlled$_2$ | 0.9642 | 3.5570 | 0 | 38.5489 |
| | | Density $D_3$ (area in ha) | | |
| Invaded$_3$ | 8.4603 | 28.6518 | 0 | 28.0302 |
| Not controllable$_3$ | 6.2499 | 2.27092 | 0 | 22.2549 |
| Controllable$_3$ | 2.2103 | 6.7322 | 0 | 57.7532 |
| Controlled$_3$ | 0.4525 | 3.1776 | 0 | 55.2901 |
| | | Total area, converted in density $D_3$ (area in ha) | | |
| Invaded | 35.1068 | 75.1394 | 0 | 585.1269 |
| Not controllable | 22.4417 | 59.2550 | 0 | 461.4947 |
| Controllable | 12.6651 | 19.9126 | 0 | 132.3721 |
| Controlled | 5.8435 | 12.5333 | 0 | 132.3721 |

Notes. 418 observations over sub-areas 1 to 22 and 19 years (1999-2017). Surface areas (invaded, non controllable, controlled) are in ha.

in time or both in time and space, in its dynamic or static version ; SDM (Spatial Durbin Model) in a static or dynamic form ; SAC (Spatial Autocorrelation Model) ; SEM (Spatial Error Model). In our application, we consider the dynamic SDM (dynamic Spatial Durbin Model)[15] without time effects. The use of a corrected dependent variable (and its lagged value on the right-hand side), by substracting from the observed invasion level the abstracted level, is similar as in Jardine and Sanchirico (2018). Unobserved individual effects are controlled for by a Fixed Effects procedure, consistent with the dynamic spatial representation in panel data.

The dynamic SDM specification has been compared with others presented above, in particular the SAR and SAC models, and estimation results are fairly similar as far as parameter magnitude is concerned.

### *Estimation results*

Before estimating the above model, we need to cope with the issue of zero observations for a proportion of the sample, i.e., when controlled area is 0 for some time periods, following, e.g., a particularly effective operation campaign. Instead of specifying a structural model combining the dynamics of diffusion with the decisions of control or not (leading to positive or zero observations for $X_{it}$ and $X_{i,t-1}$, see for example Lacroix and Thomas, 2011), we consider a simple procedure as follows. We estimate the probability that the controlled area is positive with a Fixed-Effects Logit model, using as explanatory variables the two following ratios:

$$r_{1it} = \text{ controllable area}_{i,t-1}/(\text{controllable area}_{i,t-1} + \text{non controllable area}_{i,t-1}),$$

$$r_{2it} = \text{controlled area}_{i,t-1}/\text{controllable area}_{i,t-1},$$
$$i = 1, 2, \dots, 22 \quad \text{and} \quad t = 2000, \dots, 2017.$$

The estimated probability that $X_{it} > 0$, denoted $\hat{P}_{X,it}$, is then used as a control variable in the dynamic spatial model. Parameter estimates of the probability $\hat{P}_{X,it}$ are reported in Table 3.2.3.

A second step consists in computing the contingency matrix $W$ with five areas, which is used to evaluate the spillover effects $\gamma W^* Y_{it}$. The matrix is obtained from maps described above in Section ???, using the distance between

[15] See for example Bouayad-Agha and Védrine, 2010 for a discussion on estimation strategies with this type of models.

**Table 14**
Estimation of the probability of a positive controlled area

| Variable | Estimate | Standard error | $t$ Student | $p$-value |
|---|---|---|---|---|
| $r_{1it}$ | -6.0233 | 1.1103 | -5.42 | 0.000 |
| $r_{2it}$ | 0.9208 | 1.2983 | 0.71 | 0.478 |

Notes. 361 observations. Estimation method: Fixed-Effects binary Logit. Log likeihood= -103.087. Global significance test LR $\chi^2(2) = 122.22$ ($p$-value=0.00). $r_{1it}$ and $r_{2it}$ respectively denote the ratio of lagged controlled rapport over the lagged total area, and the ratio of the lagged controlled area over the lagged controllable area.

**Table 15**
Estimation of the dynamic spatial model of primrose willow diffusion

| | Model 33 | | Model 33′ | |
|---|---|---|---|---|
| Variable | Estimate | Standard error | $t$ Estimate | Standard error |
| $\rho$ | 0.8409*** | 0.1868 | 1.0084*** | 0.0185 |
| $\delta$ | 0.0523 | 0.2240 | - | - |
| $\gamma$ | 0.1192*** | 0.0288 | 0.0851*** | 0.0315 |
| $\beta$ | -7.7077 | 6.2755 | -2.8062 | 2.7159 |
| Constant | 0.7777* | 0.4491 | 0.6826 | 2.0193 |
| Test $\rho = 1$ | $\chi^2(1) = 0.72$ | ($p$-value= 0.39) | $\chi^2(1) = 0.21$ | ($p$-value= 0.65) |

Notes. 361 observations (22 sub-areas, 17 years). Estimation method: Generalized Method of Moments (Arellano-Bond). Robust standard errors (clustering on sub-areas) are in parentheses. *, ** and *** respectively indicate a parameter significant at the 10, 5 and 1 percent level. Fixed effects are normalized to have a zero mean, so that an intercept term can be estimated.

areas and the contiguity of the areas (length of frontiers between areas). With restrictions imposed that the matrix columns and rows sum to 1 and that elements in the main diagonal are all zero, we have:

$$W^* = \begin{bmatrix} 0 & 0.375 & 0.375 & 0 & 0 \\ 0.5666 & 0 & 0.375 & 0 & 0 \\ 0.5666 & 0.46875 & 0 & 0.4375 & 0 \\ 0 & 0 & 0.1041 & 0 & 0.375 \\ 0 & 0 & 0 & 0.125 & 0 \end{bmatrix}$$

Based on estimates and computations above, parameter estimates of Model 33 are provided in Table 3.2.3, where vector $Z_{it}$ contains the explanatory variable $\hat{P}_{X,it}$. Robust standard errors are used, by clustering on sub-areas and are bias-corrected (method of Windmeijer, 2005). To confirm the relevance of such simplification, we run a second estimation (Model 33′ ) omitting $\delta Y_{i,t-1}$, and we find similar results (see last two columns of Table 3.2.3).

With robust standard errors, the Sargan specification test cannot be computed its distribution is unknown when the homoskedasticity assumption is relaxed). However, the specification test based on first- and second-order serial correlation of differenced error terms (see Arellano and Bond, 1991) indicates no specification error.

The autoregressive parameter $\rho$ is relatively close to 1, and a unit-root test (Breitung and Das, 2005) concludes that it can be considered equal to 1. Accounting for positive vs. zero observations in the model does not modify significantly model performance, as parameter $\beta$ associated with $\hat{P}_{X,it}$ is not significantly different from 0. Moreover, the effect of the lagged dependent variable $Y_{i,t-1}$, on top of the lagged difference $(Y_{i,t-1} - X_{i,t-1})$, is not significant either. Direct and indirect effects of controlling primrose willow are directly computed, from $\beta$ estimate for the former and from $\gamma W$ for the latter. One can conclude from this estimation that, for calibrating the economic model of invasion control, it may be enough to consider the spatial spillover effects (produced by $\hat{\gamma} W$, as the growth rate of the primrose willow is almost zero (because of the unit root). Such estimated growth rate for the ''net" biomass $(Y_{it} - X_{it})$ is lower than the one found by Jardine and Sanchirico (2018).

In Table 3.2.3, estimates of the intercept term represent the average yearly growth (in level) of primrose willow (fixed effects are normalized to have a zero mean), once contribution from other areas (and invasion control) is accounted for. The latter, i.e., the term $\gamma W Y_{it}$, has a mean of 1.4506 (standard deviation 1.86) and can be directly compared with $Y_{it}$, which has a mean of 35.1068 (standard deviation of 75.14).

**Table 16**
State of the invasion

| | | Area 1 | Area 2 | Area 3 | Area 4 | Area 5 |
|---|---|---|---|---|---|---|
| Surface at risk | (in ha) | 1502.92 | 1655.5 | 1145.86 | 947.21 | 441.43 |
| Surface invaded (2018) | (in ha) | 110.60 | 660.54 | 287.33 | 192.27 | 24.42 |
| Invaded surface at risk (2017) | (in %) | 7.39 | 39.93 | 25.05 | 20.27 | 5.44 |
| Invaded area surface (2017) | (in %) | 3.87 | 23.03 | 10 | 6.69 | 0.84 |

**Table 17**
Data and estimates used for simulation

| Area | Area under invasion risk (ha) | Area invaded (in 2018) (ha) | Average cost (euro/m$^2$) | Marginal cost (euro/m$^2$) | Marginal benefit (euro/m$^2$) |
|---|---|---|---|---|---|
| 1 | 1502.92 | 110.60 | 0.0414 | 0.2911 | 0.1406 |
| 2 | 1655.50 | 660.54 | 0.8085 | 0.2927 | 0.2185 |
| 3 | 1145.86 | 287.33 | 0.8075 | 0.2969 | 0.2005 |
| 4 | 947.21 | 192.27 | 0.8571 | 0.2901 | 0.1360 |
| 5 | 441.43 | 24.42 | 0.3554 | 0.2584 | 0.0663 |

| Area | $b_i/c_i$ (euro/m$^2$) | Controllable area (ha) | Average site area (ha) | Controllable area per 5000 euro (ha) | Marginal benefit (enlarged population) (euro/m$^2$) | $b_i/c_i$ (enlarged population) (euro/m$^2$) |
|---|---|---|---|---|---|---|
| 1 | 0.4829 | 24.1545 | 15.250 | 12.0770 | 2.2212 | 7.6304 |
| 2 | 0.7465 | 1.2368 | 0.2654 | 6.1840 | 3.4516 | 11.7923 |
| 3 | 0.6753 | 1.2384 | 0.1872 | 6.1920 | 3.1674 | 10.6682 |
| 4 | 0.4688 | 1.1667 | 16.4530 | 5.8335 | 2.1481 | 7.4047 |
| 5 | 0.2565 | 2.8137 | 33.5081 | 14.0685 | 1.0477 | 4.0546 |

# 4. Results

We apply in this section our strategy for controlling primrose willow invasion, based on our estimates of marginal benefits and costs of control, and the estimated dynamic and spatial diffusion process of invasion in the case of the Brière marshland. The algorithm of Section 2 is calibrated and implemented over a range of possible control budgets, and social benefits are computed over the five invaded areas in the marshland. For such calculation, we consider two populations as targets of the control policy: local or enlarged population (i.e., households in the Loire-Atlantique *département*).

Focusing on current state of the invasion, we observe that no area reached a critical state of invasion (>50%). Gradients of benefit functions are then depicted in the right hand part of figure (**??**).

## 4.1. Calibration

Data used for the simulation concern waterways under invasion risk from primrose willow. We convert all information on costs and benefits in terms of surface of controllable areas (ha). Table 17 presents data and parameters used in the simulation experiment. Areas in m$^2$ from previous sections are converted in hectares (division by 10,000) but for visibility reasons, controllable area for 1 euro is presented in the table in m$^2$.

Cost estimates from section ??? inform the ratio benefit over cost, which is a major factor in the control policy of section 2. Average cost is interesting to use for computing the area that can be fully controlled for a given budget. In Table 17, we see that, although marginal costs are fairly similar across areas, average costs of control are different, particularly between area 1 and the four others. This leads to very different controllable areas (presented in m$^2$ for visibility purposes), from Area 1 (24 m2) and area 4 (1.16 m2). Areas considered as the benchmark for the simulation are the total areas invaded by primrose willow for year 2018.

It is also interesting to compare marginal benefits by unit of surface (in euro / ha) with the controllable area for 1 euro. In this respect, Area 1 is clearly the most interesting because it is both characterized by an important area under risk (1503 ha, second out of five), and a modest area already invaded (110.60 ha), with a very low average cost and a large ratio benefit over cost. At the other end, Area 4 is characterized by a low benefit - cost ratio (0.1360) and a high average cost (0.85).

The ratio benefit-cost is essential for the simulation, as this parameter will determine the sequence of controlled areas, according to the algorithm described in Section 2. In the case of constant marginal benefits computed from the local population, this ratio varies from 0.25 for Area 5 to 0.74 for Area 2, which would therefore be controlled first. With the enlarged population and constant marginal benefits, the sequence of controlled areas would be, based on the ranking of $b_i/c_i$, 2, 3,1,4,5. Because marginal costs are less variable than marginal benefits, it is the latter that influence the most the ranking of areas regarding priority of invasion control. The diffusion model of primrose willow is calibrated from estimates in section 3.2.3:

$$Y_1 = Y_0 + (X_1 - X_0) + 0.6826 + 0.0851W, \tag{34}$$

where the intercept term represents the annual variation (in level) of the invaded area. Il is important to note that, during simulation, it is essential to confront such variation with the frequency of removal decisions (i.e., budget ''shares", or operations). We divide the constant term by the number of operations (10, here) so that the invasion growth by primrose willow between the beginning ($Y_0$) and the end of the period ($Y_1$) is equal to that parameter. The matrix of spatial, $W$, is available from Section 3.2.3.

Seven values for the total budget are considered: 100 k, 250 k, 500 k, 750 k, 1 M, 2 Mand 3 M. Total budget is divided in 10 shares, corresponding to a early planning of the invasion control policy. To compute the value of benefits from the control policy over the 10 budget shares, we compute the gross benefits (number of hectares controlled by area, between initial and final periods, multiplied by the marginal benefit associated with the area) and the bet benefits (by substracting marginal costs from the former).

Implementing the algorithm of Section 2 proceeds as follows.

1 Divide budget $B$ in $s$ shares;

2 Initialise with first share ($= B/s$), $h = 1$;

3 Rank the $n$ (=5) areas by decreasing order of the ratio $\phi_i^1 = \frac{b_i}{c_i},\ i = 1,\ 2,\ \dots,\ n$;

4 Allocate budget shares $hB$ to areas, from area corresponding to $\phi_1^1$ to area corresponding to $\phi_n^1$;

5 Compute the invaded area before control, $\bar{Y}_{i1} = d_0 + \ \rho Y_{i0}\ +\ \gamma W Y_i,\ \ i = 1,\ 2,\ \dots,\ n$;

6 Evaluate the controlled area $X_{i1}, i\ = 1,\ 2,\ \dots,\ n$ associated with budget share $hB$ for area $i$;

7 If $Y_{i1} - \beta X_{i1} < 0,\ \ i = 1,\ 2,\ \dots,\ n$, set $X_{i1} = 0$;

8 Compute invaded area $i$ after control, $Y_{i1} = Y_{i1} - X_{i1},\ \ i = 1,\ 2,\ \dots,\ n$;

9 Compute primrowe willow propagation over all areas, $\gamma W Y_1$;

10 Update the invaded area, $Y_{i1} = Y_{i1} + Y_{i0}\ +\ \gamma W Y_1,\ \ i = 1,\ 2,\ \dots,\ n$;

11 Compute gross and net benefits: $b_i\ \times \left(\ \bar{Y}_{i1} - Y_{i1}\right)$ and $(b_i -\ c_i) \times \left(\ \bar{Y}_{i1} - Y_{i1}\right),\ i = 1,\ 2,\ \dots,\ n$;

12 Increment $h = h + 1$ and start from step [4] above ;

13 Stop if $h = s$.

This straightforward ranking algorithm implies a uniform budget share ($B/s$) across all areas, with a similar ranking over the subsequent years, from the largest to the smallest benefit-cost ratio. Gross and net benefits can be compared for different total budget values, such benefits being calculated from the comparison between controlled surface area and not controlled ones.

**Table 18**
Invasion control effort across areas (in ha)

| Area | Budget 100 k | 250 k | 500 k | 750 k | 1 M | 2 M | 3 M |
|---|---|---|---|---|---|---|---|
| 1 | 48.3092 | 120.773 | 241.5458 | 362.3188 | 483.0918 | 966.1836 | 1449.2754 |
| 2 | 2.4738 | 6.1842 | 12.3686 | 18.5528 | 24.7372 | 48.744 | 74.2116 |
| 3 | 2.4768 | 6.1842 | 12.384 | 17.008 | 24.7678 | 49.5356 | 74.3034 |
| 4 | 2.3334 | 5.8336 | 11.6672 | 17.1408 | 23.324 | 46.669 | 70.0034 |
| 5 | 2.8137 | 7.0343 | 14.0687 | 21.03 | 28.1373 | 56.2746 | 84.4119 |

**Table 19**
Ratio benefit / budget by budget level (in )

| Budget | 100 k | 250 k | 500 k | 750 k | 1 M | 2 M | 3 M |
|---|---|---|---|---|---|---|---|
| Ratio benefit / budget | 13.59 | 13.13 | 8.34 | 6.74 | 5.91 | 4.58 | 4.13 |

Graphs **??** to **??** present invaded areas by primrose willow for the 10 budget shares of the control policy, according to each budget level and area, with the ‘’$Y0$ control“ as benchmark case. Graph **??** presents total control effort over the 10 budget shares, as a function of total budget, on the five areas. These graphs clearly illustrate that significant budget levels are necessary to yield a visible control of primrose willow invasion over the five areas of our study. A sharp drop with respect to the benchmark situation (0 invasion control) is obtained only with a budget greater than or equal to 2 M(except for Area 5 that has a smaller surface). Area 1 is the only case for which full invasion control is achieved with this budget level, which can be explained by a particularly low average control cost in Area 1, leading to a greater potentially controllable area. Graph **??** presents the ratio between total benefits from control policy and budget, for the five areas. Total benefits are obtained by summing up surface areas where control policy avoids invasion, multiplied by marginal benefit associated with the area. The ratio between total benefits from a control policy and budget is, in all cases, greater than 1, and this ratio decreases with respect to budget level.

Because average control cost (or surface) is lower in Areas 1 and 5, their invasion is entirely controlled (equal to 0) with significant budget levels. Last, total benefits from control policy are greater than removal (control) budgets, with benefit-budget decreasing with respect to budget (about 13 for a 100k, to more than 3 for a budget of 3M).

## 5. Discussion and management recommendations

The next step and obvious continuation of this work is to build an easy to use computer interface enabling managers to allocate their budget efficiently.

## Appendix

### A.1 Description of datas

### A.2 Density target assumption

### A.3. Invasiveness map

Three levels of invasiveness risk were distinguished:

- Level 0 : No or very low risk of herbarium establishment and no risk of proliferation.
- Level 1: Implantation possible but without generalized proliferation, risk of proliferation in localized wet depressions.
- Level 2: highly probable proliferation.

Invasiveness map was then constructed in two steps. As invasiveness risk is usually habitat dependent, we first classified habitats. Reed beds, mixed helophyte groups and aquatic environments under afforestation and/or regularly salted were assigned a risk of level 1. Permanent or temporary aquatic environments (bare vessels), sunny and of freshwater, more or less colonized by sedges, were assigned a risk of level 2. Second, because invasiveness risk in wet meadows habitat varies according to geographic variables (in particular altitude), we used existing datas of past invasions in order to assess altitude threshold between level 0 and level 1 on the one hand, between level 1 and level 2 in the other hand.

Overall the obtained invasiveness map is the following with red pixels denoting invasiveness risk of level 2, green pixels denoting invasiveness risk of level 1.

### A.4. Connectivity matrix

### A.5. Density colors - Choice experiment

**High invasion level (Red)** : Primrose willow forms continuous banks on most water bodies and channels. At least 50% of banks are densely colonized. The biological invasion is present in any water environments. At least 50% of grasslands are densely colonized and we find the species almost anywhere but in dense reed beds. Activities whatsoever are strongly impacted by the invasion and are often impossible.

**Medium invasion level (Yellow)** : Primrose willow can be found in any water environment. The species forms dispersed or discontinuous grass beds on most of the water bodies, canals and meadows. It also forms continuous grass beds with at least 10% of banks and grasslands densely colonized. Potentially it can colonize locally the entire hydraulic section of a canal or small areas of water. Activities whatsoever are impacted but remain possible.

**Low invasion level (Yellow)** : Primrose willow is absent or almost absent from the area. When present, the species forms dispersed or discontinuous grass beds. Occasionally it can form continuous grass beds but in very small proportions (<10%). The central part of canals and large bodies of water is free of primrose. On very wet grasslands, it can be locally massive, but its presence is nil on most sites. Activities are not impacted.

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