Private management of epidemics

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Abstract

Optimal control of epidemics is a major challenge as control is costly and damages are substantial. Complementing the raising literature on the topic, we focus in this paper on coordination and cooperation issues related to control strategies. Modeling an epidemics affecting perennial crops over space and time, we consider a dynamic game where several land owners choose whether to control an epidemics within their property. Analyzing the game both in a cooperative and non-cooperative fashion, we draw insights on initial conditions likely to produce inefficiencies and coordination issues due to private management. We characterize game situations according to spread intensity and infection levels and focus on landowners strategic behaviors generating inefficiencies within a network.

Keywords: spatio-temporal model, plant epidemics, networks, coordination, decentralized game, Nash equilibrium, inefficiencies
JEL: Q28, Q57, Q58

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1 Introduction

Pathogens and diseases cause severe losses and constitute major threats to health and agriculture. Optimal control is a key challenge in epidemiological, phytopathological and economics modelling. In particular where, when and how to address these threats are major concerns as control costs are usually high and management budgets are limited (Dybiec et al.,2004,2005). Mathematical modelling is useful to discuss these questions as it allows to simultaneously account for economics and epidemiological aspects and draw recommendations for optimal decision-making.

Most of the literature on the topic considers management strategies as if a single decision maker had the ability to carry out actions to prevent and control the spread over space (Horan et al. (2005), Aadland et al. (2015), Atallah et al. (2017), Parnell et al. (2010)) . This framework is appropriate for deriving insights about first best optimal management that a single landowner or health agency in charge of an epidemics could target. However, agricultural land is usually owned by multiple landowners that choose individually how to deal with their property. Human epidemics often spread toward national borders and individuals have freedom of choice regarding their health. This makes optimal management both a cooperation and a coordination problem as viruses and pathogens spread but management is performed by agents that do not necessarily cooperate nor even coordinate between each others. There is therefore space for management inefficiencies as well as coordination issues.

Relatively little work has been carried out to investigate decentralized (uncoordinated) management. Epidemics pertains to a larger class of mobile public bad comprising pest, invasive species and mobile pollutions (Smith et al. 2009). Principal feature of this class of problems is the spatially explicite dispersion of the bad and the requirement to implement a spatially oriented strategy. In the area of pollution management, some important contributions have been made. Brock and Xepapadeas (2008) and Brock et al. (2014) consider a dynamic game with an incomplete internalization of the spatial externality assuming optimizing agents as myopic, i.e. considering external effects as outside their control. Contrasting with these papers, de Frutos and Martín-Herrán (2017) study a dynamic optimization control model in a spatial setting accounting for the transportation of pollution across space. The key of their paper is to both add an explicit treatment of the spatial pollution externality and an analysis of decentralized management. Considering a two-player non-cooperative continuous game in space and time with a linear quadratic objective, they characterize the feedback Nash equilibrium of a space-discretized model. Two principal insights are made. First accounting for spatial aspects drastically modifies the outcome of the game and therefore the emission policies. Second, decentralized management is inefficient but inefficiencies can be solved by implementing a tax scheme. Although this paper is relevant for our analysis, a major difference between transboundary pollution and epidemics, pest or invasive species management is that pollution travels but does not reproduce endogenously. No outbreak problems are in place and the key to the management strategy is to efficiently address the negative spatial externality between states or landowners.

Several papers on epidemics, pest and invasion management examine the cooperation issue comparing centralized and decentralized control. Bhat and Huffaker (2007) consider a spatially dynamic game between two landowners impacted by a negative spatial externality and show that the potential economic gains from cooperation can be substantial. Complementing this result, Atallah et al. (2017) show that benefits from cooperation increase with the level of heterogeneity between patches and that players moving first impacts other players strategy either forwarding further efforts or laissez-faire. Benefits from cooperation in addressing a pest with spatial spread are more fully examined by Epanchin-Niell and Wilen (2014). They show that benefits from cooperation can be realized even if cooperation in efforts is partial, with a degree of cooperation inversely correlated with control costs. In line with this result, Fenichel et al.

(2014) show numerically that the more valuable the land, the more ambitious the control plan implemented unilaterally by landowners. Grimsrud et al. (2008) show that the lower the magnitude of an invasion, the higher is the benefit from cooperation.
A common feature of these papers is that they rest upon numerical analysis and simulations of specific case studies. A recent exception that provides a theoretical treatment of the non-cooperative game across several heterogeneous landowners controlling a mobile, renewable public bad is Costello et al. (2017). Considering an infinite-discrete-time linear-quadratic differential game, the authors analyse decentralized owners incentives and contrast decentralized decisions with socially optimal control pattern across space and time. Two principal insights are made. The first is on conditions for decentralized management to be inefficient. They show that when marginal dynamic cost of the bad is high, eradication is optimal in both cooperative and non-cooperative management settings. Similarly, if the marginal dynamic cost is low then inefficiencies due to decentralized management is low. However, if this cost is moderate, inefficiencies can be significant making coordination schemes economically relevant. The second insight is on the importance of initial conditions on public bad stocks, on spread parameters and on heterogeneity between parcels. They show in particular that control increases with the stock of public bad, that the benefit from cooperation increases with the rate of spread, that heterogeneity of infection across space incentivizes low efforts in the non-cooperative setting.

Complementing Costello et al. (2017) and de Frutos and Martín-Herrán (2017) , this paper analyse non cooperative behavior across a set of landowners controlling a mobile public bad over a landscape. Without loss of generality we take as a motivating example for our model the management of the epidemics of Plum Pox Virus (PPV) affecting fruit trees. Initial conditions are described by the number of infected and uninfected trees and we consider at each period, a sequence involving control, growth and spread. Our model depart from Costello et al. (2017) or de Frutos and Martín-Herrán (2017) in three principal respects. First and foremost, we assume a finite horizon setting. Strategies to control PPV as well as many plant epidemics and invasive species are usually set over a finite time period. Similarly, many human epidemics are cyclical involving management strategies over finite time span. Second, we assume linear damage function. Although the epidemics damage may be convex as assumed by Costello et al. (2017), many epidemics affecting perennial crops admit linear damages as the economic impact is proportional to the volume of perennial crops infected. For example, in the case of PPV, the valence of the disease is measured by the number of trees infected and each tree infected translates in a fixed economic loss. Third, we specify management costs using a fixed control rate whereas any control level along a convex abatement cost can be chosen in Costello et al. (2017). In our model, as this is often the case in epidemics management, control is not continuous and the regulator can either detect and control the epidemics within a patch or not.

These three modeling features are crucial in terms of policy recommendations for at least three reasons. First and foremost, our model sheds light on the fact that multiplicity of equilibria may be an issue in this class of problems. In addition to being useful in understanding inefficiencies due to lack of cooperation, our model is also useful in characterizing coordination issues by delineating the parameter space for multiplicity of equilibria to arise. While the problem of multiplicity has been broadly eluded from this literature, we show that it constitutes a salient issue. Second and contrasting with Costello et al. (2017) or Fenichel et al. (2014), strategic interactions in our model are not pure strategic complements. The nature of strategic interactions has been highlighted as a key concept within this literature and it has been showed that strategic complementarity between landowners incite control in neighboring patches and vice-versa. This led Fenichel et al. (2014) to argue that there is no free riding in their model and Costello et al. (2017) to argue that their mobile public bad game is a spatial analog of the "weaker link" public good described by Cornes (1993). We show that according to parameter space, our model may exhibits inter-temporal strategic substituability, with potential freeriding behaviors, inter-temporal and contemporaneous complementarity. This has important consequences on general recommendations as, in the class of problems we describe, the incentive to control the public bad does not necessarily increase with the control effort of other landowners. Third, as we model a finite horizon game, our model allows us to discuss strategic transitions. In line with Fenichel et al. (2014), our model allows us to discuss thresholds on infection levels at the origin of strategic transitions. This constitutes a major difference with Costello et al. (2017) as in their model, optimal control for a given time period does not depend on the infectious state at the beginning of the period.

The paper proceeds as follows. Section 2 introduces our model. We first present the motivating example at the origin of the paper, that is the management of the epidemics of PPV. Building on this example, we define the eco-epidemiological model. We end the section with a presentation of the epidemic game. Section 3 presents our results. We analyze in details the simple case of a two patches two time periods model with a specific focus on subgame perfect Nash equilibrium solution. We first analyze strategic interactions. Second, we explore the parameters space and describe some specific outcomes of the epidemic game. This includes an analytical characterization of initial infectious levels leading to multiplicity of equilibria in a fully symmetric case. Third we deal with social inefficiencies that emerge from private management. Section 4 concludes and discusses relevant extensions of this work.

2 The model

2.1 Epidemiological model

Spatio-temporal structure of the problem Our model consists in a discretized T steps N patches framework that we simplify into a two steps two patches model in the analysis part. Consider a spatially explicit landscape composed of N patches or orchards we denote i, with $i = 1, .., N$. For each patch, we follow two state variables, the quantities of infected (I_i) and susceptible (S_i) trees. At the beginning of each period, t, $t = 0, .., T-1$, patch i contains ${I_i}^t \in \mathbb{R}^+$ infected trees and ${S_i}^t \in \mathbb{R}^+$ suitable trees that are susceptible of being infected over the next periods.

In our settings T is finite. All orchards have a finite lifespan and we assume that at the end of the T periods, all remaining trees do not have any residual value. This important restriction corresponds to a case in which production cycles are synchronized among patches. We assume that all trees are from the same cohort, and that replanting does not occur during the T periods [1]. It follows that in patch i, the total number of trees $I_i + S_i$ may not be constant but decreasing over the T periods, due to the removal of infected trees. Our problem is thus framed using a spatialized susceptible infected removed (SIR) model.
In our decentralized framework, patch i is owned by landowner i and at each period t, landowner i freely chooses whether he will implement active management. In this model we consider management by removal of infected trees which has been identified as the main lever against Sharka (Rimbaud et al. (2015)).

If a patch is managed, a fraction ρ_{max} of the infected trees is removed. The parameter ρ_{max} indicates how management is efficient. In the cases we study, $0 < \rho_{max} < 1$ meaning that the outcome of management is imperfect in the sense that infected trees cannot be completely removed. In the Sharka example, this inefficiency is explained by the presence of asymptomatic trees (Dallot et al. 2004). At each period, a landowner faces a binary choice: to manage its

[1] This is the case for Prunus crops where all trees have the same age within an orchard

orchard (which consists in removing a fraction ρ_{max} of infected trees) or to let the virus spreads within and outside the orchard. The binary variable $\rho_i^t \in \{0; \rho_{max}\}$ captures the management decision for patch i at time t.

We also assume that, at the beginning of each period, landowners take their decision with a perfect knowledge of the system state. They are able to observe $\rho_{max} \times I$ infectious trees, and deduce the total number of infected trees using the knowledge of ρ_{max}.

Diffusion model

From time t to time $t+1$ the system evolves according to difference equation that we will call the evolutionary law. It is constructed as a sequence of events. Management occurs at the beginning of the period, here time t. After management, it remains $I^t(1-\rho_i^t)$ infected trees in patch i. Following the "management phase", "growth and spread" of the infection takes place from the remaining infected trees. In order to model heterogeneous dispersal of the pathogen over space we introduce the parameters r_{ji} $(i,j) \in [1:N]^2$. r_{ji} represents the average number of new infections in patch i per infected tree in patch j during a time period. It is not obvious that these parameters are time independent as the number of new infections might depend on the number of susceptible trees within an orchard. However when the level of infection is small comparing to the size of orchards ($I << S$), they can be considered as constant and approached using measurable data. Experts can show how to estimate those parameters for the Sharka case using available information. The hypotheses ($I << S$) also makes us avoid the upper bound problems corresponding to cases in which all trees are infected within a patch. We only focus on orchards with small to intermediate prevalence. The evolutionary law links the states of the system in times t and $t+1$. Associating diffusion and management, we write:

$$I_i^{t+1} = I_i^t(1-\rho_i^t) + \sum_{j=1}^{N} I_j^t(1-\rho_j^t) r_{ji}, \tag{1}$$

$$S_i^{t+1} = S_i^t - \sum_{j=1}^{N} I_j^t(1-\rho_j^t) r_{ji}, \tag{2}$$

with, by assumption, ${S_i}^t > 0$.
I_i^{t+1} is the sum of remaining infected trees after management plus newly infected trees in period t. S_i^{t+1} is simply constructed as the quantity of susceptible trees at time t minus the number of new infections in period t. We define f, as the function describing the evolutionary dynamics for all patches such that $(I^{t+1}, S^{t+1}) = f(I^t, S^t, \rho^t)$.

2.2 Economic model

Costs and benefits The profit generated by the exploitation of a patch is modeled as the discounted sum of benefits and costs over the T time periods. For a period, the gross profit generated by an orchard is adapted from Rimbaud (2015), with some differences. We consider that production by infected trees keeps a value, which is an important feature of our model. We will note u_i the value of the yearly production per infected tree in patch i and v_i the value of the production by a sane tree. Because the disease generates damages, we obviously have $u < v$. Moreover, the omission of the production costs constant with respect to the optimization problem (e.g. cost independent from the outbreak management) is another difference between our model and Rimbaud (2015).

Before giving expressions for benefits and costs, we need to clarify the temporal framework. we consider that period t links times t and $t+1$, so the first period is period 0.

Benefits for period t are computed according to the state of the system after management and diffusion (thus using I_{t+1} and S_{t+1}) because the harvest occurs at the end of period t.

$$B_i{}^t = f(S_i^t, I^t, \rho^t)'.(v_i, u_i) \tag{3}$$
$$= S_i^{t+1} v_i + I_i^{t+1} u_i \tag{4}$$

Management costs take into account a monitoring cost that depends on the size of the patch, and a cost depending on the number of infected trees removed. The total management cost writes:

$$C_i{}^t = \mathbb{1}_{\rho_i=\rho_{max}}(c_a + \frac{\rho_i{}^t}{\rho_{max}} c_h A_i) + c_r \rho_i^t I_i^t. \tag{5}$$

where c_a represents an access cost to the patch, A_i is the area of patch i, c_h is an inspection cost per unit of surface, $\frac{\rho_i{}^t}{\rho_{max}}$ is the fraction of the patch managed, and c_r an additional cost per infected tree removed. Considering, $\rho_i^t \in \{0; \rho_{max}\}$ total cost for landowner i at period t is either $C_i{}^t = 0$ if $\rho_i^t = 0$, or $C_i{}^t = c_a + c_h A_i + c_r \rho_i I_i^t$ if $\rho_i^t = \rho_{max}$. Writing $c_f = c_a + c_h A_i$ for the fixed cost, equation 5 becomes:

$$C_i{}^t = \frac{\rho_i{}^t}{\rho_{max}}(c_a + c_h A_i) + c_r \rho_i^t I_i^t. \tag{6}$$

Profit function Taking into account benefits and costs, we express the profit function π_i^t corresponding to period t:

$$\pi_i^t = B_i^t - C_i^t \tag{7}$$
$$\pi_i^t(\rho^t, I^t, S^t) = \left(S_i^{t+1} v_i + I_i^{t+1} u_i - \frac{\rho_i^t}{\rho_{max}}(c_f) - c_r \rho_i^t I_i^t \right) \tag{8}$$

subject to:

$$(I^{t+1}, S^{t+1}) = f(I^t, S^t, \rho^t).$$

We assume that bills are paid at same time as the harvest occurs. It is therefore not necessary to discount costs differently from benefits. Denoting the discount factor δ^t, with $0 \leq \delta^t \leq 1$, the discounted profit of landowner i over the T periods writes:

$$V_i^T(I^0, S^0, \rho^0, ..., \rho^{T-1}) = \sum_{t=0}^{T-1} \delta^t \left(S_i^{t+1} v_i + I_i^{t+1} u_i - c_r \rho_i^t I_i^t - \frac{\rho_i^t}{\rho_{max}}(c_f) \right) \tag{9}$$

subject to :

$$I^{t+1}, S^{t+1} = f(I^t, S^t, \rho^t).$$

The figure 1 illustrates the structure of this model in the two-times steps two-patches case.

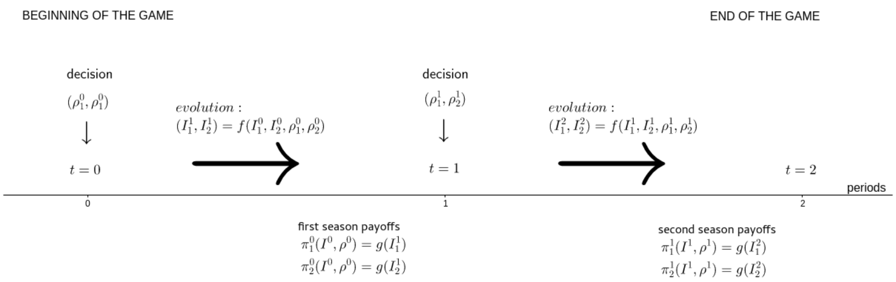

Figure 1: Temporal structure of the game

2.3 Decision making framework, solution concept

We model the decision making process using a game theoretic framework with a feedback information structure. In these settings, landowners decide whether they are willing to manage their patch at the beginning of each period, with a perfect knowledge of system state. Each agent maximizes its own profit and behaves rationally, taking into account that other agents do the same. This kind of discrete time dynamic game is associated with the Subgame Perfect Nash Equilibrium (SPNE) solution concept and we will only focus on pure strategy equilibria. This paper focuses on the 2 players / 2 time periods / 2 patches case (2/2/2 model) whose structure is illustrated in figure 1.

Feedback strategies and management paths

We solve the game using the feedback information structure with perfect knowledge of the state variables. In this modeling framework, at each step, agents build their strategies as functions of the state variables. In the case of the two steps epidemic game, a strategy is a function $\sigma(t, I^t)$, with $t \in \{0, 1\}$ and we still assume $I << S$.

Given initial conditions and a pair of strategies, the outcome of the 2/2/2 game can be predicted. We thus associate a management patch to each set of strategies and initial condition. A management path in the 2/2/2 game will be denoted as a vector $\rho = (\rho^0, \rho^1) = (\rho_1^0, \rho_2^0, \rho_1^1, \rho_2^1)$, where ρ_i^t is player's i action at time t.

We re-define as well the payoff function, with feedback strategies as arguments:

$$J_j^\tau(I^\tau, S^\tau, \sigma_1, \sigma_2) = \sum_{t=\tau}^{T-1} \delta^t \pi_j^t(I^t, S^t, \sigma_1(t, I^t), \sigma_2(t, I^t)) \tag{10}$$

subject to :

$$(I^{t+1}, S^{t+1}) = f(I^t, S^t, \sigma_1(t, I^t), \sigma_2(t, I^t)).$$

We introduce as well the value function W_j as player j's discounted payoff emerging from an equilibrium path starting at $t = \tau$:

$$W_j^\tau = \sum_{t=\tau}^{T-1} \pi_j^t(I^t, \sigma_j^*(t, I^t), \sigma_{-j}^*(t, I^t)) \tag{11}$$

$$\text{with } I^{t+1} = f(I^t, \sigma_j^*(t, I^t), \sigma_{-j}^*(t, I^t)) \tag{12}$$

The optimization problem

The problem we need to solve for finding feedback equilibrium in pure strategies is summarized by the system of equations (for $j, -j$, and $t \in \{0, 1\}$):

$$\sigma_j^*(t, I^t) = \arg\max_{\rho_j^t} \{\pi_j((I^t), \rho_j^t, \sigma_{-j}^*(t, I^t)) + \delta W^{t+1}(I^{t+1})\} \tag{13}$$

$$(I^{t+1}, S^{t+1}) = f(I^t, S^t, \sigma_1(t, I^t), \sigma_2(t, I^t)).$$

which can be solved using backwards induction. This justifies that, for a particular initial condition, our multistage game can be solved using a game tree such that the one we present now.

Extensive form representation

Considering a particular initial condition, the game might then be represented in extensive form as a game tree 2 where each branch corresponds to a potential management path and each leaf is associated with the players payoffs. This representation allows to get insight on the backwards reasoning. We then get insight on the global solution of the two step model, applying a backwards induction algorithm to many different initial states. In the case the computation of the complete resolution of the game is heavy, the solution can be reconstructed point by point using extensive form formulation.

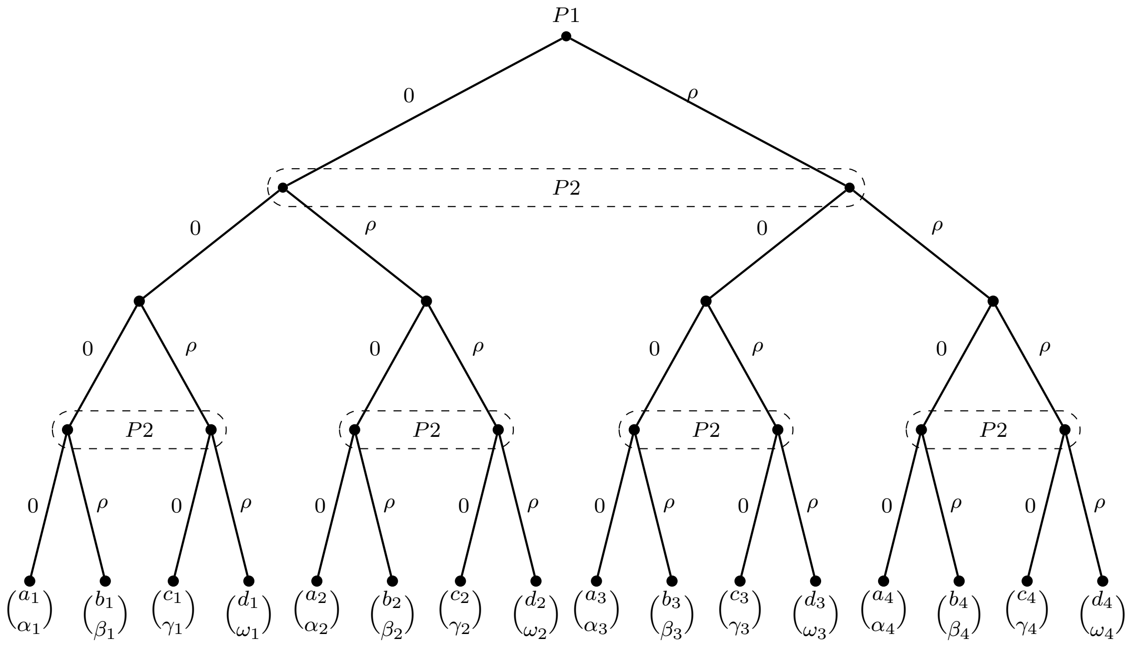

Figure 2: Illustration of the different potential outcomes for the two patches two time periods game using a game tree. Letters at the tree's leaves correspond to players payoffs. Those are computed according to the value functions (equation 9) along the different branches: $a_1 = V_1^2(0,0,0,0,I^0,S^0)$, $\alpha_1 = V_2^2(0,0,0,0,I^0,S^0)$ and so on...

2.4 Methods and results presentation

Example presented along this document are produced using the algorithm presented in section 5.1, using parameters introduced in section 5.2. We still are in the case of a two-patches two-time periods model. Parameters have been chosen in order to generate rich behaviors and explore the model features. We remain in a symmetric case, excepting with respect to the initial state. We present here two numerical experiments, that differ only with respect to the detection rate (ρ_{max}).

3 Results

In this results section we first analyze strategic interactions within our simple two players / two patches / two stages game. Second, we explore the feedback equilibrium structure and show situations with multiple pure strategy equilibria, situations without pure strategy equilibria, situations with free-ridding, and explain how the strategic effect contributes to the different equilibria. Third we discuss social inefficiencies emerging from private management.

3.1 Strategic effects in the epidemic game

The 2 players / 2 patches / 2 periods epidemic game seems to be quite a simple game. However, it stills presents many parameters and players' decisions depend on a bunch of different effects. Players action have direct effects on the outbreak propagation. With the feedback information structure actions might also have indirect effects through the influence on other players actions (see for example Fudenberg and Tirole (1984)). Our first results make clear how those effects are shaped in the epidemic game, and specifically how inter-period substitutability might be present according to the initial condition. Second this section clarifies how direct effects and strategic effects are linked (and might be opposed) in a player best response formula.

There are not immediate strategic interaction in the one stage game
The one stage game has a solution in dominant strategies, meaning that the best action of each player does not depend on other players actions but only depends on a threshold on the quantity of infected trees. In this model actions by other player do not not have an immediate effect on the efficiency of players i action. This result is summarized in proposition 1.

Proposition 1 : Assume that $r_{ii}(v_i - u_i) - u_i - c_r > 0$; the one stage game from $t = 1$ admits at least one equilibrium in dominant strategies; player's i action is governed by a threshold α_i for the infection level in his patch:

$$\rho_i^{1*}(I_i^1) = \begin{cases} \rho_{max} \text{ if } I_i^1 > \alpha_i \\ 0 \text{ if } I_i^1 < \alpha_i \\ \rho_{max} \text{ or } 0 \text{ if } I_i^1 = \alpha_i \end{cases} \quad \text{with } \alpha_i = \frac{1}{r_{ii}(v_i - u_i) - u_i - c_r} \frac{c_f}{\rho_{max}}$$

□

The threshold α_i appears because detection effort is associated with a cost independent of the number of infected trees. When there are fewer infected trees the same effort allows to avoid less infections (because detections are less frequent) but still has the same fixed cost. Management becomes profitable from a certain number of infected trees. The monetary balance associated with the removal of one infected tree is, for one period: $F_i \equiv (r_{ii}(v_i - u_i) - u_i - c_r)$ €. Each time an infected tree is removed, r_{ii} infections are avoided in patch in the next growth period saving $(v_i - u_i)$, whereas production value and the removal cost are lost. If $F_i > 0$, action is profitable for player i, even for a single step time horizon, in the case where the product of F_i and the number of avoided infections exceeds the fixed cost c_f. If F_i is not positive, it means that the amount of money that would be spent for management is more than what is saved from infections avoided in case of removal. In this case there is no management in the one stage game, whatever the level of infection is. However, it might become profitable if agents would consider longer time horizon as there is propagation of the infection at each step. Importantly, such a threshold structures implicates that eradication is not achievable (because management is only partially efficient). The economic question addressed here is therefore related to the relevant level of control.

Second step optimal action depends on the first step action: emergence of strategic effects The fact that the one stage game displays dominant strategies simplifies the understanding of the two stages game. Using the knowledge of the evolutionary law, the optimal decisions in the second step might be precisely predicted as functions of the initial state I^0 and the knowledge of players actions at time 0, $\rho^0 = (\rho_1^0, \rho_2^0)$. Therefore, player i's second step action in the two stages game is function of (I^0 and ρ^0) but not of ρ_j^1, player's j action at the beginning of the second step.

Proposition 2 : In the game, second step actions for a rational player are determined by I^0 and ρ^0 (recall that the other player's second action does not intervene): for $i \in \{1; 2\}$,

- if $\rho^0 = (0, 0)$ then: $\rho_i^1 = \begin{cases} \rho_{max} \text{ if } I_j^0 > \frac{\alpha_i - I_1^0(1+r_{ii})}{r_{ji}} \\ 0 \text{ if } I_j^0 < \frac{\alpha_i - I_i^0(i+r_{ii})}{r_{ji}} \end{cases}$

- if $\rho^0 = (0, \rho_{max})$ then: $\rho_i^1 = \begin{cases} \rho_{max} \text{ if } I_j^0 > \frac{\alpha_i - I_i^0(1+r_{ii})}{r_{ji}(1-\rho_{max})} \\ 0 \text{ if } I_j^0 < \frac{\alpha_i - I_i^0(1+r_{ii})}{r_{ji}(1-\rho_{max})} \end{cases}$

- if $\rho^0 = (\rho_{max}, 0)$ then: $\rho_i^1 = \begin{cases} \rho_{max} \text{ if } I_j^0 > \frac{\alpha_i - I_i^0(1+r_{ii})(1-\rho_{max})}{r_{ji}} \\ 0 \text{ if } I_j^0 < \frac{\alpha_i - I_i^0(1+r_{ii})(1-\rho_{max})}{r_{ji}} \end{cases}$

- if $\rho^0 = (\rho_{max}, \rho_{max})$ then: $\rho_i^1 = \begin{cases} \rho_{max} \text{ if } I_j^0 > \frac{\alpha_i - I_i^0(1+r_{ii})(1-\rho_{max})}{r_{ji}(1-\rho_{max})} \\ 0 \text{ if } I_j^0 < \frac{\alpha_i - I_i^0(1+r_{ii})(1-\rho_{max})}{r_{ji}(1-\rho_{max})} \end{cases}$

□

Proposition 2 has a nice geometric interpretation. Each inequality is drawn from a potential first step action and defines a line in the plan (I_1^0, I_2^0). Four lines are related with action of player 1 and four other lines are related with action of player 2. If a point I is above, say the line corresponding to action ρ_1^1 in the case where $\rho_0 = (\rho_{max}, 0)$, it means that in the case this first step indeed occurs, player 1 is going to do $\rho_1^1 = \rho_{max}$ in the second step. Figure 3 illustrates the 8 lines in a symmetric example. This graphical representation makes clearly appear a strategic component in the 2/2/2 game and shows how it depends on the initial infectious condition, each polygon representing a particular combination of inter-temporal effects. Indeed, there are plenty of situations in which, a variation in player i first step action generates a change in the second step outcome. Let's see how this might be read.

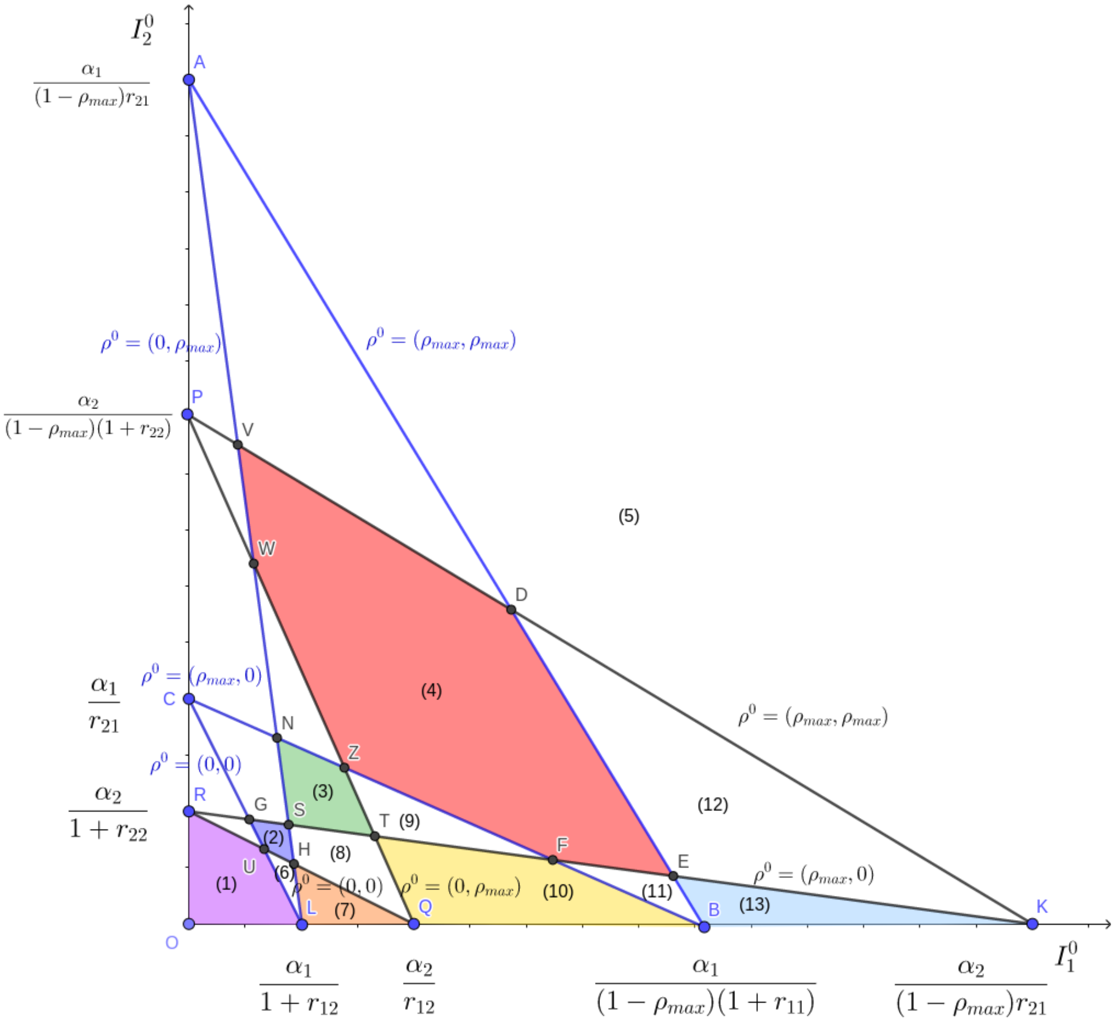

Figure 3: Illustration of the 8 lines defined in proposition 2 for parameters corresponding to a symmetric example. For each line, the associated first step is indicated on the graph. Each numbered zone corresponds to a particular strategic situation. Representations of these situations in term of games paths are provided in supplementary material (table 19).

In order to highlight inter-temporal strategic interactions in this epidemic game, we introduce the first step equivalent game. It is a representation that indicates, for each potential couple of first step actions the following optimal second step.

		player 2	
		0	ρ
player 1	0	$0, 0, \sigma_1^{1*}(f(0,0)), \sigma_2^{1*}(f(0,0))$	$0, \rho, \sigma_1^{1*}(f(0,\rho)), \sigma_2^{1*}(f(0,\rho))$
	ρ	$\rho, 0, \sigma_1^{1*}(f(\rho,0)), \sigma_2^{1*}(f((\rho,0))$	$\rho, \rho, \sigma_1^{1*}(f(\rho,\rho)), \sigma_2^{1*}(f(\rho,\rho))$

Figure 4: Representation of the equivalent game in terms of management paths according to first step choices. Each box of the table is filled as follows: (ρ_1^0, ρ_2^0, $\sigma_1^{1*}(f(\rho_1^0, \rho_2^0, I^0))$, $\sigma_2^{1*}(f(\rho_1^0, \rho_2^0, I^0))$) where ρ_1^0, and ρ_2^0 are management options at $t = 0$ corresponding to a box, $f(\rho_1^0, \rho_2^0, I^0) = I^1$ is the resulting state at $t = 1$ computing using the evolutionary law, $\sigma_1^{1*}(f(0,0))$, and $\sigma_2^{1*}(f(0,0))$ correspond to the Nash outcome for the second step subgames (see proposition 1).

Such a representation allows to qualitatively analyze inter-temporal strategic interactions by looking at the outcome of a change in player i's first step action on second step optimal actions. Let's have a look at an example (that corresponds to what happens in zone 8 in figure 3):

$(0, 0, \rho, \rho)$	$(0, \rho, \rho, 0)$
$(\rho, 0, 0, 0)$	$(\rho, \rho, 0, 0)$

Figure 5: Example of a first step equivalent game representation. Each box represents a management path (ρ_1^0, ρ_2^0, $\sigma_1^{1*}(f(\rho_1^0, \rho_2^0, I^0))$, $\sigma_2^{1*}(f(\rho_1^0, \rho_2^0, I^0))$) where ρ^0 is fixed, and ρ^1 is the following optimal couple of decisions. This representation can be constructed by a careful interpretation of the position of zone 8 with respect to the 8 lines in figure 3

An analysis of the first step equivalent game gives informations about four strategic situations. The effect of a variation in player 1's (resp. player's 2) action when player 2's (resp player 1) action is 0, the effect of a change in player 1's (resp. player 2) action when player 2 is ρ_{max}. The notation $\Delta_{\rho_i^0:0\to\rho_{max}}(X) = X(\rho_i^0 = \rho_{max}) - X(\rho_i^0 = 0)$ indicates the variation of the function X when ρ_i^0 goes from 0 to ρ_{max}. Unless it is precised we will consider variations from 0 to ρ_{max} and use the abbreviation $\Delta_{\rho_1^0}$ for $\Delta_{\rho_i^0:0\to\rho_{max}}$. Here an interpretation of table 5 leads to:

inter-temporal effect (player 1's point of view)	
$\rho_2^0 = 0$	$\rho_2^0 = \rho_{max}$
$\Delta_{\rho_1^0}(\rho_1^1) = -\rho_{max}; \Delta_{\rho_1^0}(\rho_2^1) = -\rho_{max}$	$\Delta_{\rho_1^0}(\rho_1^1) = -\rho_{max}; \Delta_{\rho_1^0}(\rho_2^1) = 0$
inter-temporal effect (player 2's point of view)	
$\rho_1^0 = 0$	$\rho_1^0 = \rho_{max}$
$\Delta_{\rho_2^0}(\rho_1^1) = 0; \Delta_{\rho_2^0}(\rho_2^1) = -\rho_{max}$	$\Delta_{\rho_2^0}(\rho_1^1) = 0; \Delta_{\rho_2^0}(\rho_2^1) = 0$

Figure 6: Inter-temporal strategic effects in the game displayed in figure 5

This example (5) illustrates a rich asymmetrical strategic situation: the effect of a change in player one first step action depends on what is player 2's first step action. If $\rho_2^0 = 0$ a increase in ρ_1^0 makes both stop managing at the second step. In the case where $\rho_2^0 = \rho_{max}$, the strategic influence on player's 2 second step action is not present anymore. In this zone, player 2 first step decision has no impact on player 1's second step action. We also remark that player 2 has analyze a trade-off between action in the first stage and action in the second stage when $\rho_1^0 = 0$.

Proposition 3 : According to the initial condition, the inter-period strategic effect associated with an increase of the first step action is either neutral or negative (this a substitutability interaction).

$$\Delta_{\rho_i^0}(\rho_j^1) = -\rho_{max} \text{ if } I_i^0 r_{ij} + I_j^0(1+r_{jj})(1-\rho_j^0) > \alpha_j \quad (14)$$

$$I_i^0 r_{ij}(1-\rho_{max}) + I_j^0(1+r_{jj})(1-\rho_j^0) < \alpha_j \quad (15)$$

$$\begin{aligned}\Delta_{\rho_i^0}(\rho_j^1) = 0 \text{ if } & I_i^0 r_{ij} + I_j^0(1+r_{jj})(1-\rho_j^0) > \alpha_j \\ & I_i^0 r_{ij}(1-\rho_{max}) + I_j^0(1+r_{jj})(1-\rho_j^0) > \alpha_j \\ \text{or if} & \\ & I_i^0 r_{ij} + I_j^0(1+r_{jj})(1-\rho_j^0) < \alpha_j \\ & I_i^0 r_{ij}(1-\rho_{max}) + I_j^0(1+r_{jj})(1-\rho_j^0) < \alpha_j\end{aligned}$$

□

Note that the size of the zone where player i's first step action is associated with a strategic effect is determined by 2 lines (equations 14 and 15). Its size depends on the product $\rho_{max} r_{ij}$: if management efficiency and inter-patch transmission are high, a strategic inter-temporal effect is present for a larger set of initial condition. The understanding of the strategic behavior within the game is valuable in itself. However it is worth comparing with other effect in order to understand the formation of equilibria in the game.

3.1.1 Best response in the first step equivalent game (feedback best response)

Because the second stage presents solutions in dominant strategies, the 2/2/2 game might be studied thanks to the analysis of the first step equivalent game. Payoffs in the first stage equivalent game are given by the function Z:

$$Z_i(I^0, S^0, \rho_i^0, \rho_{-i}^0) = \pi_i^0(I^0, S^0, \rho_i^0, \rho_{-i}^0) + \delta W_i^1(f(I^0, S^0, \rho^0)). \quad (16)$$

The best response to ρ_{-i} in the first stage equivalent game can be expressed by computing $\Delta_{\rho_i^0} Z_i(I^0, S^0, \rho_i^0, \rho_{-i}^0) = Z_i(I^0, S^0, \rho_i^0, \rho_{-i}^0) - Z_i(I^0, S^0, \rho_i^0, \rho_{-i}^0)$. As already mentioned, a change in player i's first step action has various effects. Proposition 4 details the best response formula effect by effect and, table 7 gives the interpretation of the different effects.

Effects formulation is made very general and some effects depend on the inter-temporal relationship between actions. The variations in combinations of actions should therefore be specified in order to look at a particular case. In this model,

$$\begin{aligned}\Delta_{\rho_i^0}(\rho_k^0) &= 0 \\ &\text{or} = -\rho_{max}\end{aligned}$$

according to I^0 and ρ_j^0.
An increase in the first step action can also create an overlap between actions in the first step and action in the second step. (The idea is that some infections avoided due to management in the first step would have been avoided due to management in the second step). The variation $\Delta_{\rho_i^0}(\rho_i^0 \rho_k^1)$, $k \in \{i, j\}$ allows to capture this overlap that occurs when there is management at both steps:

$$\Delta_{\rho_i^0}(\rho_i^0 \rho_k^1) = \rho_{max}^2 \text{ if } \text{ and } \rho_k^1(\rho_i^0 = \rho_{max}) = \rho_{max} \quad (17)$$

$$= 0 \text{ if } \rho_k^1(\rho_i^0 = \rho_{max}) = 0 \quad (18)$$

Proposition 4 :
Player i's best response to player j's action (ρ_j^0) is given by the sign of the following expression:

$$
\begin{aligned}
\Delta_{\rho_i^0, 0\to\rho_{max}}(Z_i(I^0, S^0, \rho_j^0)) &= (\Delta_{\rho_i^0}\Pi_i^1 + \delta\Delta_{\rho_i^0}W_i^2) \\
&= \rho_{max} I_i^0((v_i - u_i)r_{ii} - u_i - c_r) - c_f \\
&+ \delta(\\
(v_i - u_i)(& \\
& I_i^0 \rho_{max} r_{ii} \\
& + I_i^0 \rho_{max} r_{ii}(1 + r_{ii}) \\
& + I_i^0 \rho_{max} r_{ij} r_{ji} \\
& - I_i^0 \Delta_{\rho_i^0}(\rho_i^0 \rho_j^1) r_{ij} r_{ji} \\
& - I_i^0 \Delta_{\rho_i^0}(\rho_i^0 \rho_i^1)(1 + r_{ii}) r_{ii} \\
& + \Delta_{\rho_i^0}(\rho_i^1)(I_i^0(1 + r_{ii}) r_{ii} + I_j^0(1 - \rho_j^0) r_{ji} r_{ii}) \\
& + \Delta_{\rho_i^0}(\rho_j^1)(I_j^0(1 + r_{jj})(1 - \rho_j^0) r_{ji} + I_i^0 r_{ij} r_{ji}) \\
&) \\
& - u_i I_i^0 \rho_{max} \\
-(u_i + c_r)(& \\
& \Delta_{\rho_i^0}(\rho_i^1)(I_i^0(1 + r_{ii}) + I_j^0(1 - \rho_j^0) r_{ji})) \\
& - I_i^0 \Delta_{\rho_i^0}(\rho_i^0 \rho_i^1)(1 + r_{ii}) \\
&) \\
-\frac{1}{\rho_{max}} & \Delta_{\rho_i^0}(\rho_i^1) c_f \\
&).
\end{aligned}
$$

If it is positive, player i prefers to do ρ_{max} at the first step. $\square$

direct effects in the first step (discounted by δ)	
infections avoided in the first step	$\rho_{max} I_i^0 (v_i - u_i) r_{ii}$
additional management cost in the first step	$\rho_{max} I_i^0 (-u_i - c_r) - c_f$
direct effects in the second step (discounted by δ)	
first step infections would have generated damages in the second step	$(v_i - u_i) I_i^0 \rho_{max} r_{ii}$
infections avoided in the second step due to infections avoided in the first step	$(v_i - u_i) I_i^0 \rho_{max} (r_{ii}(1 + r_{ii}) + r_{ij} r_{ji})$
overlapping between ρ_i^0 and ρ_j^1 in the case where $\rho_j^1 = \rho_{max}$	$-I_i^0 \ (v_i - u_i) \ \Delta_{\rho_i^0}(\rho_i^0 \rho_j^1) \ r_{ij} r_{ji}$
overlapping between ρ_i^0 and ρ_i^1 in the case where $\rho_i^1 = \rho_{max}$	$-I_i^0 \ (v_i - u_i) \ \Delta_{\rho_i^0}(\rho_i^0 \rho_i^1) \ (1 + r_{ii}) r_{ii}$
overlapping between ρ_i^0 and ρ_i^1 with respect to management costs	$-I_i^0 \Delta_{\rho_i^0}(\rho_i^0 \rho_i^1)(1 + r_{ii})$
trees removed in the first step would have produced in the second step	$-u_i I_i^0 \rho_{max}$
effects due to changes in second step action (discounted by δ)	
strategic effect through a change in player j's second step action	$(v_i - u_i) \ \Delta_{\rho_i^0} \ (\rho_j^1) \ (I_j^0 (1 + r_{jj}) \ (1 - \rho_j^0) r_{ji} + I_i^0 r_{ij} r_{ji})$
variation in infections due to a change in ρ_i^1	$(v_i - u_i) \ + \Delta_{\rho_i^0}(\rho_i^1) \ (\ I_i^0 (1 + r_{ii}) r_{ii} + I_j^0 (1 - \rho_j^0) r_{ji} r_{ii})$
variation in variable cost and production lost due to a change in ρ_i^1	$-(u_i + c_r) \ (\Delta_{\rho_i^0}(\rho_i^1) \ (I_i^0 (1 + r_{ii}) + I_j^0 (1 - \rho_j^0) r_{ji}))$
variation in the fixed cost due to a change in ρ_i^1	$-\frac{1}{\rho_{max}} \Delta_{\rho_i^0}(\rho_i^1) c_f$

Figure 7: Analysis of the different effects in the best response formula

Interestingly, in case the strategic effect is present, $\Delta_{\rho_i^0}(\rho_k^0) = -\rho_{max}$ (see proposition 3 for a characterization), and it plays against management in the first step. The inter-period strategic effect is an incentive for differing management. The next result section will show what important consequences this might have on the equilibrium structure. Without strategic influence, a potential high overlapping is another incentive for differing action. We also note how complicated the trade-off between action in the first step and action in the second step might be. In the following section, we focus different equilibrium structures emerging from the game, explaining which economic forces are key stones in the construction of those effects. It is also clear from the best response formula, that a strategic immediate interaction appears in the first step of the game, through the mediation of the temporal dimension.

Strategic interactions in the first stage equivalent game
The orientations of those strategic interactions are this time more complicated. They can be retrieved by the sign of $\Delta_{\rho_j^0}\Delta_{\rho_i^0}Z_i(I^0, S^0, \rho_i^0, \rho_j^0)$. Here according to parameters and initial condition this interaction might be either complementarity or substitutability or even be neutral. The interpretation of this new quantity is not straightforward given that $\Delta_{\rho_j^0}\Delta_{\rho_i^0}(\rho_k^0)$ might be either $-\rho_{max}$, 0, or ρ_{max}, meaning that inter-temporal interactions might appear or disappear according to the other player first action.

3.2 Analysis of the feedback equilibrium structures

In this section we focus on the equilibria structures in the 2/2/2 symmetric game. Given that the best response formula is constituted by different competing effect, rich equilibria structure are expected. In this part using both analysis and numerical example, we examine a bunch of interesting features of the epidemic game. We characterize zones where the equilibrium is associated with maximal and minimal effort. We then analyze examples in which the inter-period strategic effect is pivotal (meaning that it determines the behavior of one player). An example then illustrates the construction of a case without any equilibrium in pure strategy. This list is ended by the characterization of two kinds of multiplicity of equilibria, in fully symmetric situation.

3.2.1 Maximal and minimal efforts

This analysis have been restricted to situations in which $I^t << S^t$. This implicates that the cases in which almost all trees are infected (which can generate strategic interactions) are not considered here. Given $I^t << S^t$, there is of initial infection level that is high enough for maximal action being a dominant strategy for both players. Similarly there is a zone for which no intervention threshold can be reached. All this is summarized in proposition 5. Proposition 5 is useful in order to understand how our model behaves in terms of thresholds: players do not manage when the infectious level is too low. Player manage for sure when the infection level is sufficiently high (benefits are higher than costs even only considering single patches and without looking at inter-patches fluxes). Between those two zones a decision might impact the other player behavior (see figure 3) and strategic interactions arise.

Proposition 5 : Within the initial condition state space, there is a zone where:

1. initial infection is sufficiently high so that both players do maximal effort (without inter-player strategic considerations):
$(\rho_{max}, \rho_{max}, \rho_{max}, \rho_{max})$ is the unique Nash equilibrium if and only if $(I_1^0, I_2^0) \in \Lambda_{max}$, where Λ_{max} is defined by the set of inequalities:

$$\begin{cases} I_2^0 > \frac{\alpha_1 - I_1^0(1-\rho_{\rho_{max}})(1+r_{11})}{(1-\rho_{max})r_{21}} \\ I_2^0 > \frac{\alpha_2 - I_1^0(1-\rho_{max})r_{12}}{(1-\rho_{max})(1+r_{22})} \\ I_2^0 > k_2 \\ I_1^0 > k_1 \end{cases}$$

where α_i is defined in proposition 1 and k_1 and k_2 are some constants:

$$k_i = \frac{1}{\rho_{max}}(c_f) \times \frac{1}{Y}$$

where,

$$\begin{aligned} Y = &(r_{ii}v_i - (1 + r_{ii})u_i + c_r) + \delta \times (\\ &(v_i - u_i)(r_{ii}(1 + (1 + r_{ii})(1 - \rho_{max})) + r_{i,-i}r_{-i,i}(1 - \rho_{max})) \\ &+ \rho_{max}(1 + r_{ii})c_r - u_i(1 - \rho_{max})) \end{aligned}$$

2. initial infection is small enough so that both players do not react: $(0, 0, 0, 0)$ is the unique Nash equilibrium of the game if and only if $(I_1^0, I_2^0) \in \Lambda_{min}$; where Λ_{min} is defined by the set of inequalities:

$$\begin{cases} I_2^0 < \epsilon_2 \\ I_1^0 < \epsilon_1 \\ I_1^0, I_2^0 \geq 0 \end{cases}$$

where ϵ_1 and ϵ_2 are some constants.
□

We notice that k_i does not depend on I_{-i} meaning that it defines a vertical or an horizontal line in the plan (I_1^0, I_2^0). We therefore find a new threshold, independent of I_{-i} once we know that I belongs to Λ_{max}.

3.2.2 Transect analysis and pivotal strategic effect

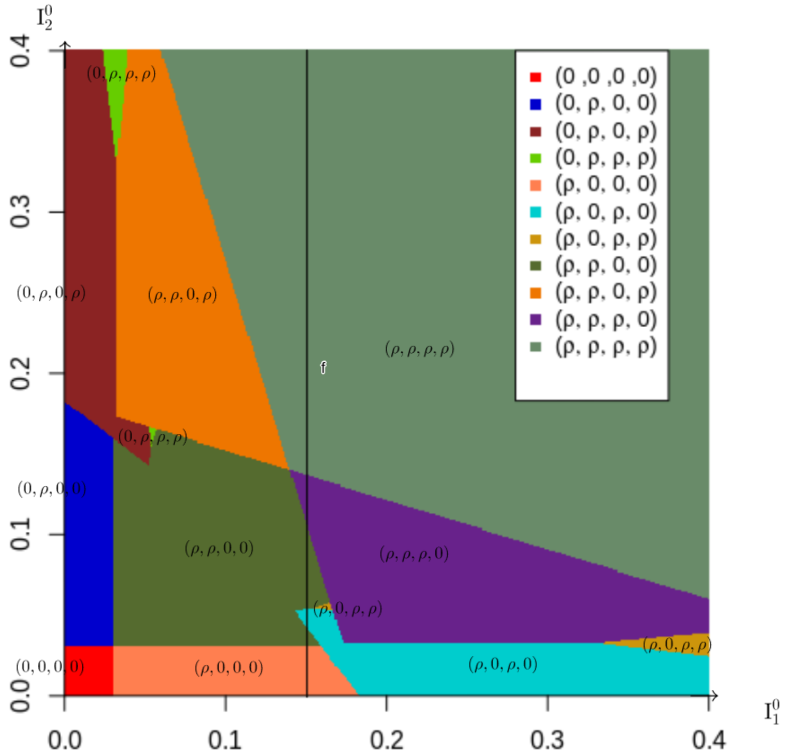

Figure 8: Management paths according to the initial condition, in the feedback Nash equilibrium in an example where intra patch diffusion is twice higher than inter patch diffusion (parameter are given in section 5.2). We will be interested in the transect $I_1^0 = 0, 15$.

Figure 8 illustrates the feedback equilibrium paths according to initial situations. Line f in the graph allows to follow the evolution of the equilibrium policies when I_2^0 increases, for a fixed I_0^1. If we follow the line f from the bottom to the top, and recalling that a path is written as a vector $(\rho_1^0, \rho_2^0, \rho_1^1, \rho_2^1)$, we observe the following successions of equilibria paths: $(\rho, 0, 0, 0)$; $(\rho, \rho, 0, 0)$; $(\rho, 0, \rho, 0)$; $(\rho, \rho, 0, 0)$; $(\rho, \rho, \rho, 0)$; (ρ, ρ, ρ, ρ). Along this transect, it is surprising that as I_2^0 increases the equilibrium path goes from $(\rho, \rho, 0, 0)$ to $(\rho, 0, \rho, 0)$ and then comes back to $(\rho, \rho, 0, 0)$. The structure of first step equivalent games in the corresponding zones are helpful in order to understand how such a sequence of equilibria might be generated.

The analysis of first step equivalent games in table 9 unveils that a strategic effect appears when $(\rho, 0, \rho, 0)$ becomes the equilibrium path. Indeed the comparison between games 3 and 4 (see table 9) shows that in game 4, contrary to game 3 $\Delta_{\rho_2^0}(\rho_1^1 | rho_1^0 = \rho_{max}) = -\rho_{max}$. I_0^2 is now high enough such that, if $\rho_2^0 = 0$, the threshold α_1 is reached due to an inter-patch flux. On the zone corresponding to game 4, the strategic incentive makes player 2 prefer to avoid management at the first step. However as I_2^0 keeps increasing, player 2's strategic incentive for not managing becomes dominated by the need to control the propagation. Here the change in the equilibrium path is not explicated by a modification in the strategic structure. The initial

I_2^0 range	equivalent game		SPNE	game number
$0.053 - 0.055$	$(0, 0, \rho, \rho)$	$(0, \rho, \rho, 0)$	$(\rho, \rho, 0, 0)$	5
	$(\rho, 0, \rho, 0)$	$(\rho, \rho, 0, 0)$		
$0.043 - 0.053$	$(0, 0, \rho, \rho)$	$(0, \rho, \rho, 0)$	$(\rho, 0, \rho, 0)$	4
	$(\rho, 0, \rho, 0)$	$(\rho, \rho, 0, 0)$		
$0.031 - 0.043$	$(0, 0, \rho, \rho)$	$(0, \rho, \rho, 0)$	$(\rho, \rho, 0, 0)$	3
	$(\rho, 0, 0, 0)$	$(\rho, \rho, 0, 0)$		

Figure 9: Equivalent games along the transect f, I_2^0 increases from the bottom to the top, here games for $I_2^0 \in [0.031; 0.055]$ are presented (the complete sequence is given in the appendix). Parameters correspond to the example illustrated in figure 8. I_1^0 is fixed and $I_0^1 = 0.15$

infectious level simply impacts the relative magnitudes of the different effects. As we have seen, there are opposed effects in the best response function, and when I_2^0 increases, player's 2 incentive for a first step management increases faster than the indirect benefit from a second step extra management by player 1.

This example is illustrative of how the strategic effect can be pivotal in the building of the equilibria. In game 4, we are in a situation in which player 2 is willing to differ its management only if it considers that action by player 1 in the first step is going to be triggered. We can define a pivotal strategic effect as:

In the example we have developed here, the strategic effect is pivotal for the patch where there is the less initial infection. In this case, the strategic effect explains why player 2's policy is $(0, 0)$ instead of $(\rho_{max}, 0)$.

We say that the strategic effect is pivotal for the stabilization of $\rho_i^{0*} = 0$ if:

$\Delta_{\rho_i^0}(\rho_j^1) = -\rho_{max}($ presence of a strategic effect $)$

$\Delta_{\rho_i^0}(Z_i(I^0, \rho_i^0, \rho_j^{0*} = .)) < 0$

(given the strategic effect it is preferable to not manage at the first step for player 1)

$\Delta_{\rho_i^0}(Z_i(I^0, \rho_i^0, \rho_j^{0*} = .)) - strategic\ effect > 0$

(without the strategic effect, player 1 prefers to operate at the first step)

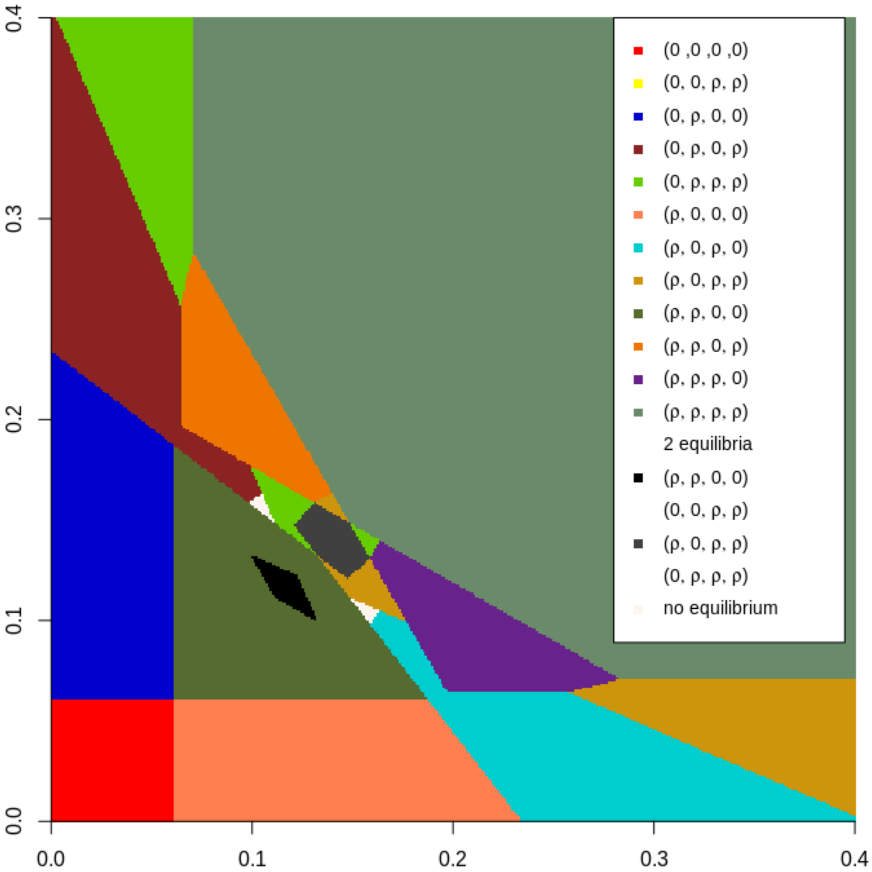

Figure 10: Subgame perfect equilibrium path according to the initial condition in an example with low management efficiency rate and high inter-patch connectivity. Note the presence of zones with multiple equilibria, and other zones with absence of equilibrium path (in white).

Paradoxical free-ridding We then focus on the equilibria paths in another symmetric situation which are illustrated in figure 10. This new example corresponds to a case where the detection rate is low, and intra and inter-patch diffusion coefficients are close to each-other (see section 5.2 for details).

In figure 10 we remark a zone where $(\rho_{max}, 0, \rho_{max}, \rho_{max})$ is the unique pure strategy SPNE path ; whereas $(I_1^0 < I_2^0)$. However in this equilibrium path, player 1 manages two times and player 2 manages once. This seems to be counter-intuitive as we might think that the highest the initial number of infections is, the highest the incentive to manage will be. It happens that in this case, player 1 has a dominant strategy in its first decision: he is interested in managing whatever player 2 does.

Conversely, player 2 benefits from a strategic effect when $\rho_1^0 = \rho_{max}$: when he does not manage in the first step, he triggers action by player 1 at the second step (we remark the patch that benefits from the strategic effect is the one with the higher level of infection). So if player 1 follows its dominant strategy, player 2 has the power to direct player's 1 second step action. And for those parameters, this is enough for player 2 to be interested only in managing at the second step. Whit this reasoning based on pure strategies, the predictability of player's one action makes the pure strategies equilibrium being detrimental for him, whereas we could expect the contrary at first glance. When I_2^0 increases, the strategic effect is not able anymore to drive player 2 behavior towards a manipulation strategy. It simply more profitable for player 2 to control as soon as possible.

		player 2	
		0	ρ
player 1	0	$(0,0,\rho,\rho)$ a_1, a_2	$(0,\rho,\rho,0)$ b_1, b_2
	ρ	$(\rho,0,\rho,\rho)$ c_1, c_2	$(\rho,\rho,0,0)$ d_1, d_2

has $\rho^* = (\rho, 0, \rho, \rho)$ as single equilibrium path. Those parameters verify:
$a_1 < c_1$
$d_2 < c_2$
$b_1 < d_1$
$a_2 < b_2$.

3.2.3 Absence of equilibrium

In figure 10, two symmetric zones without pure strategy SPNE appear in white. This example lead to proposition 6 which concerns one of those two cases.

Proposition 6 : We are able to find parameters (see the example developed in figure 10 where it corresponds to the white zone where $I_1^0 < I_2^0$) for which the game:

		player 2	
		0	ρ
player 1	0	$(0,0,\rho,\rho)$ a_1, a_2	$(0,\rho,\rho,\rho)$ b_1, b_2
	ρ	$(\rho,0,\rho,\rho)$ c_1, c_2	$(\rho,\rho,0,0)$ d_1, d_2

has no pure strategy Nash equilibrium. Those parameters verify:
$a_1 < c_1$
$d_1 < b_1$
$b_2 < a_2$
$c_2 < d_2$.
□

The structure of the first step equivalent game clarifies the impact of strategic effects in this case. First let's have look at player 1 behavior. There is a strategic effect only when $\rho_2^0 = \rho_{max}$ which explains why:
$\Delta^0_{\rho_1^0}(Z_1(I^0, S^0, \rho_2^0 = \rho_{max})) < 0.$
whereas,
$\Delta_{\rho_1^0}(Z_1(I^0, S^0, \rho_2^0 = 0)) > 0.$

When inspecting player's 2 incentives, we remark that there is also a strategic effect when

$\rho_1^0 = \rho_{max}$. However, we are in a zone where $I_1^0 < I_2^0$ and the incentive for controlling early dominates the strategic effect in this case. We have therefore.
$\Delta^0_{\rho_1^0}(Z_1(I^0, S^0, \rho_2^0 = 0)) > 0$
When $\rho_1^0 = 0$, management by player 2 (i.e. $\rho_2^0 = \rho_{max}$) does not allow to avoid another action in the second step (ρ_2^1 remains ρ_{max}). Here, this contributes to explain why player 2 prefers to differ its action:
$\Delta^0_{\rho_1^0}(Z_1(I^0, S^0, \rho_2^0 = \rho_{max})) < 0$.
In this first stage equivalent game, there is complementarity with respect to player 2 action and substitutability with respect to player one action.

3.2.4 Multiplicity of equilibria: characterization in the case $I_1^0 = I_2^0$

We see in the numerical example illustrated in figure 10 that there are parameters for which multiplicity of equilibria arise. In this case, two zones with multiple equilibrium appear. In the first $(0, 0, \rho, \rho)$ and $(\rho, \rho, 0, 0)$ are the two equilibrium paths. In the second, $(\rho, 0, \rho, \rho)$ and $(0, \rho, \rho, \rho)$ are the equilibrium paths. If we only look at the first actions (e.g. the choice in the first step equivalent game), we can say that the first situation corresponds to a coordination game, and that the second situation corresponds to an anti-coordination game. Different first stage equivalent game might lead to the same type of multiplicity of equilibria. In the following section, we focus on a theoretical analysis of two types of games in the particular case where initial condition are identical ($I_1^0 = I_2^0$). We are interested in characterizing the parameters for which those games appear, and, within those games the combinations of parameters compatible with situations leading to multiplicity of equilibria.

Structure of the first step game according to the initial condition in the symmetric game ($I_1^0 = I_2^0$) New notations are introduced in order to simplify the model taking into account the symmetry. We note $r = r_{11} = r_{22}$ the intra-patch propagation, $r' = r_{12} = r_{21}$ the inter-patch propagation , v the production value by uninfected trees and u the production value by infected trees. We note also $\alpha = \alpha_1 = \alpha_2$ which corresponds to the threshold defined in proposition 1. Due to the symmetry, these parameters are common to both patches.

The first step consists in the characterization of the initial condition leading to first step equivalent games that contain the equilibria paths we study. Following the methodology developed in section 1, we construct the first step equivalent games along the bisector axis (figure 12) and delineate the boundaries between the subgames (figure 11).

Lemma 1 : If $r > r'$, and $\rho_{max} \in]0, 1[$, we observe, as I_i^0 increases, the succession of first step equivalent games described in figure (12), with the boundaries A,B,C,D,E on the initial condition presented in figure 11.

□

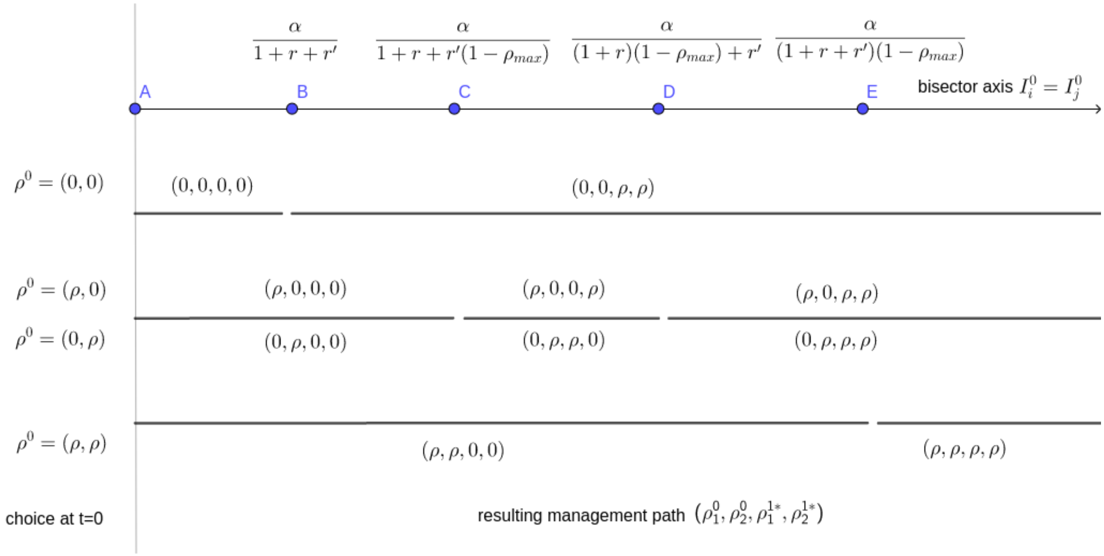

Figure 11: Representation of potential strategic paths according to the first step choice and the initial condition. It corresponds to what happens in the bisector of the graph in figure 3.

I_i^0 range	equivalent game		game number
$> E$	$(0,0,\rho,\rho)$	$(0,\rho,\rho,\rho)$	5
	$(\rho,0,\rho,\rho)$	(ρ,ρ,ρ,ρ)	
$D-E$	$(0,0,\rho,\rho)$	$(0,\rho,\rho,\rho)$	4
	$(\rho,0,\rho,\rho)$	$(\rho,\rho,0,0)$	
$C-D$	$(0,0,\rho,\rho)$	$(0,\rho,\rho,0)$	3
	$(\rho,0,0,\rho)$	$(\rho,\rho,0,0)$	
$B-C$	$(0,0,\rho,\rho)$	$(0,\rho,0,0)$	2
	$(\rho,0,0,0)$	$(\rho,\rho,0,0)$	
$A-B$	$(0,0,0,0)$	$(0,\rho,0,0)$	1
	$(\rho,0,0,0)$	$(\rho,\rho,0,0)$	

Figure 12: Structure of first step game along the bisector where $I_1^0 = I_2^0$. I increases from the bottom to the top and games are constructed using figure (11).

Multiplicity of equilibria: coordination and anti-coordination problems might arise
We notice that the paths involved in zones with multiplicity are present in game 2 for the first case and game 4 for the second case see figure 12. Conditions on payoffs within those games finish the characterization.

Lemma 2 : We observe a coordination first stage game of the form:

		player 2	
		0	ρ
player 1	0	$(0,0,\rho,\rho)$ a,a	$(0,\rho,0,0)$ γ,ω
	ρ	$(\rho,0,0,0)$ ω,γ	$(\rho,\rho,0,0)$ d,d

where $d > \gamma$, $a > \omega$;

if and only if there exist $I_i^0 > 0$ such that:

$$I_i^0 \in [\frac{\alpha}{1+r+r'}, \frac{\alpha}{1+r+r'(1-\rho_{max})}] \quad (19)$$

$$\Delta_{\rho_i^0, 0\to\rho_{max}}(Z_i(I^0, S^0, \rho_j^0 = 0)) < 0 \quad (20)$$

$$\Delta_{\rho_i^0, 0\to\rho_{max}}(Z_i(I^0, S^0, \rho_j^0 = \rho_{max})) > 0. \quad (21)$$

□

Formulas 35, 36 are expanded in the supplementary material (5.5). A brief glance at those formulas allows to see that an high r' favors the emergence of complementarity in such a game. A small value of the parameter u often has a similar effect.

Note that the first condition 34 implicates that we set ourselves in a zone leading to the particular first step equivalent game we want to study. Conditions 35 and 36 ensure that this game is a coordination game. In this case it happens that there are parameters such that those conditions define a non-empty set.

Why is that possible ? To understand why such a complementarity emerges in the first step equivalent game, we can come back to the best response formula, and simplify it taking into the structure of the game in this zone. When the first step action goes from 0 to ρ_{max}, the variation of second step actions depends on players j first step. When $\rho_j^0 = 0$, $\Delta\rho_i^1 = -\rho_{max}$ and $\Delta\rho_j^1 = -\rho_{max}$; and when $\rho_j^0 = \rho_{max}$, $\Delta\rho_i^1 = 0$ and $\Delta\rho_j^1 = 0$. When $\rho_j = 0$ an increase in player i action leads to a variation in action at $t = 1$. In particular, when player i do not manage at $t = 0$ it triggers action by player j which generates a strategic effect. Player i then benefits of player j's action in the second step without paying its cost. This explains why player i best response depends on player j action, a mechanism able to generate multiplicity in this symmetric case.

Anti-coordination game

In our example, we also observe a zone of multiplicity characterized by an anti-coordination game.

Lemma 3 : We observe an anti-coordination game of the form:

		player 2	
		0	ρ
player 1	0	$(0,0,\rho,\rho)$ a,a	$(0,\rho,\rho,\rho)$ γ,ω
	ρ	$(\rho,0,\rho,\rho)$ ω,γ	$(\rho,\rho,0,0)$ d,d

where $\gamma > d$, $\omega > a$;

if and only if there exist I_i^0 such that :

$$I_i^0 \in [\frac{\alpha}{(1+r)(1-\rho_{max})+r'}, \frac{\alpha}{(1+r+r')(1-\rho_{max})}] \tag{22}$$

$$\Delta_{\rho_i^0}(Z_i(I^0, S^0, \rho_j^0 = 0)) > 0 \tag{23}$$

$$\Delta_{\rho_i^0}(Z_i(I^0, S^0, \rho_j^0 = \rho_{max})) < 0, \tag{24}$$

$$\tag{25}$$

where formulas 23, 24, are expanded in the supplementary material (see section 5.5) □

Here again the strategic effect contributes to explain why we observe a situation with multiplicity of equilibria. This time a strategic effect is present when $\rho_j = 0$, and plays against the path $(\rho, \rho, 0, 0)$, whereas no strategic effect give an incentive for the path $(0, 0, \rho, \rho)$. Interestingly the equilibria path in pure strategies are asymmetrical. Even in presence of a symmetric initial condition, one of the players contributes more to the effort in the equilibrium.

Proposition 7 : To summarize those observations, we can say that multiplicity of equilibria might emerge from this game. It can even occur in the perfectly symmetric case ($I_1^0 = I_2^0$). For some parameters, both coordination problems and anti-coordination problems can arise according to the initial condition.

□ Note that we provided only some sufficient conditions for observing multiplicity in previous lemmas. Other situations (first step equivalent games) might lead to multiplicity of equilibria.

3.3 Social inefficiencies arising from private management

In the epidemic game, inefficiencies might analytically be localized as both centralized and decentralized models are solvable. However, this involves heavy operations (and long solutions formulations) in the most general case, because many sub-cases need to be distinguished. We choose therefore to focus on examples (with given parameters values). This section aims at illustrating the different phenomenons that might contribute to social inefficiency in such a model.

Pareto solution
For each initial condition, we call Pareto solution the management path that leads to the highest joined utility. It corresponds to the policy that maximizes the total profit over the whole landscape:

$$\arg\max_{\rho\in P} J(I^0, S^0, \rho) = \sum_{t=0}^{T-1}\sum_{i=1}^{N} \delta^t \left(S_i^{t+1} v_i + I_i^{t+1} u_i - c_r \rho_i I_i^t - \frac{\rho_i^t}{\rho_{max}}(c_a + c_h A_i)\right) \tag{26}$$

$$\text{subject to :} \tag{27}$$

$$I^{t+1} = f(I^t, \rho^t) \tag{28}$$

where P represents the management paths space. The action space is finite and discrete so for a given initial condition, we are able to compute payoffs for all possible management paths and simply select the highest. We also introduce $J^*(I^0, S^0, \rho) = \max_\rho V(I^0, S^0, \rho)$

Efficiency Obviously (by definition) the joined utility criterion leads to a better or equal result as the decentralized criterion. This is due to the fact that agents do not take into account damages generated in neighbors patches due to fluxes from their patch. The inefficiency associated with private management is simply measured as the difference between the global welfare produced under central planer and private managements. In this section, our aim is to better understand inefficiencies in the two players / two periods / two patches model.

An important objective of this paper is to clarify which initial conditions lead to inefficiency in our simple two-steps two-time periods model.

3.3.1 Overview of inefficiency in a numerical example, with symmetric agents:

Figure 13 shows a complex structure for the inefficiency (difference between the joined utility maximization and the sum of utilities from the Nash feedback solution). This example displays some local gradients but we observe discontinuities where inefficiency goes from high to low values. There are also zones where the inefficiency gradient is clearly higher then average. This let's assume some specific phenomenon occurring there. In the next section we give some explanations by comparing feature of the feedback solution of the epidemic game with feature of the joined utility solution.

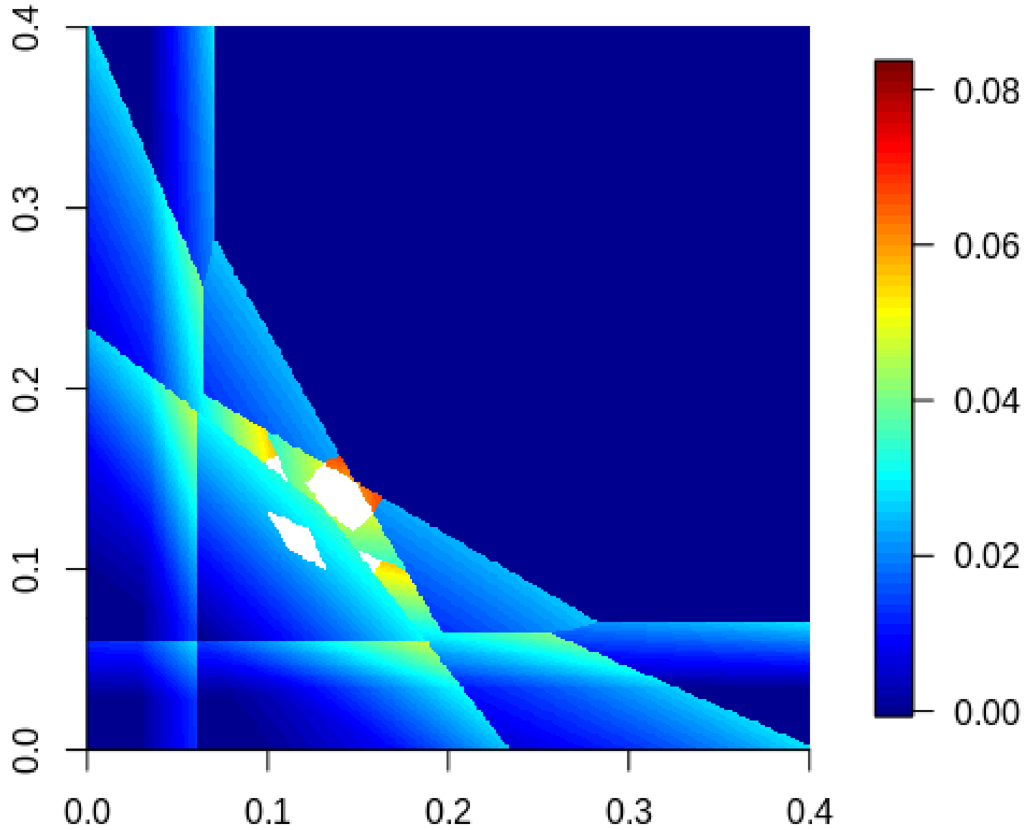

Figure 13: Map of inefficiencies (in percentage of the social optimum value) according to the initial conditions in a symmetric example with a small detection rate. In white zones there are either multiple equilibria or no equilibrium.

3.3.2 Myopic inefficiency (case of minimal and maximal policies)

The first reason explaining why centralized and decentralized solutions might not coincide is simply that there are basic differences in what is taken into account in the optimization. The social planer integrates the idea that a local decision might increase as well the utility of neighbors. Contrary to the selfish sole owner he takes into account the positive externality associated with the game. We are going to illustrate this using cases of maximal and minimal policies illustrate well the problem. Using backwards induction, it is easy to find conditions on the initial infection level for which maximal or minimal effort are equilibrium paths in the epidemic game .

As explained earlier efficiency is defined by comparing the central planer solution with the outcome of the game.

Proposition 8 : It is possible to characterize zones in the plan (I_1^0, I_2^0) for which the optimal policy under the central planer point of view is:

1. full action: $(\rho^1, \rho^2) = (\rho_{max}, \rho_{max}, \rho_{max}, \rho_{max})$ is the optimal policy if and only if $(I_1^0, I_2^0) \in \Omega_{max}$
2. no action at all: $(\rho^1, \rho^2) = (0, 0, 0, 0)$ is the optimal policy if and only if $(I_1^0, I_2^0) \in \Omega_{min}$.

where Ω_{max} is the set of (I_1^0, I_2^0) such that:

$$\begin{cases} I_2^0 > \frac{\beta_1 - I_1^0(1-\rho_{max})(1+r_{11})}{(1-\rho_{max})r_{21}} \\ I_2^0 > \frac{\beta_2 - I_1^0(1-\rho_{max})r_{12}}{(1-\rho_{max})(1+r_{22})} \\ I_2^0 > q_2 \\ I_1^0 > q_1 \end{cases}$$

and Ω_{min} is the set of (I_1^0, I_2^0) such that:

$$\begin{cases} I_2^0 < m_2 \\ I_1^0 < m_1 \\ I_2^0, I_2^0 > 0 \end{cases}$$

where $\beta_i \equiv \frac{1}{D_i}(c_a + c_h \frac{1}{\rho_{max}} A_i)$ where $D_i \equiv (v_i - u_i)r_{ii} + (v_j - u_j)r_{ij} - u_i - c_r$ and m_i are some constant that will be defined later. □

Those last constants can be compared to those introduced in the remarks accompanying proposition 1. In particular D_i a similar interpretation than F_i, excepting that it takes into account the externalities associated with inter-patch diffusion. $D_i \geq F_i$: action is more profitable under the central planer point of view.

The myopic effect allows to position zones where inefficiency occurs for sure. However between the two zones defined in proposition 6, strategic effects might have an impact on the game equilibria. Therefore the myopic effect does not alone explain inefficiencies positions and levels and the strategic aspect should again be taken into account.

Proposition 9 : Comparing results from propositions 5 and 8, we characterize zones in the plan (I_1^0, I_2^0) where:

1. A) full action is both an optimal solution of the central planer problem and the Nash equilibrium of the feedback game.
   B) full action is a Pareto solution but it is not a Nash equilibrium of the game.
2. A) No action is both an optimal solution of the central planer problem and a Nash equilibrium of the feedback game.
   B) No action is the Nash equilibrium of the game, but it is not the Pareto solution.

□

The figures (14) illustrates the proposition 5 part 1. This propositions characterizes analytically a large area of inefficiency, where intervention is necessary in order to reach the Pareto solution.

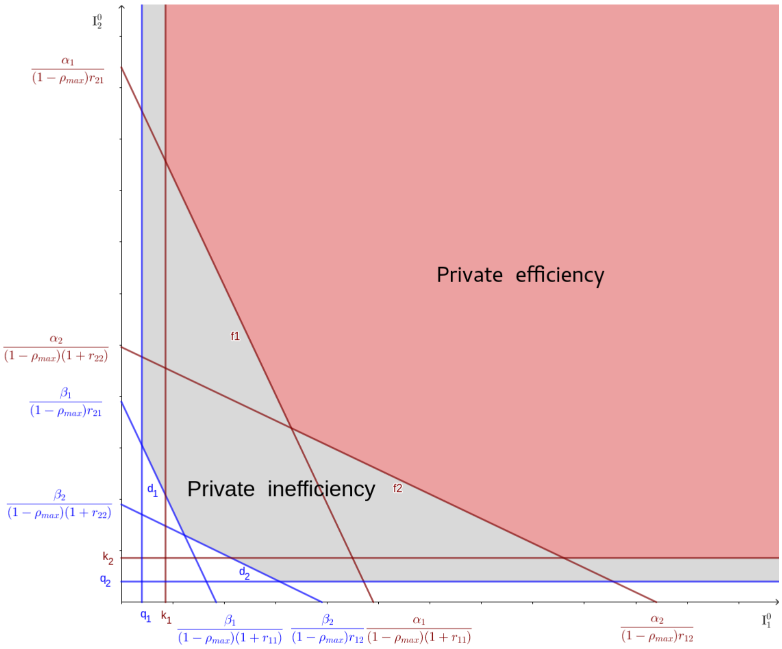

Figure 14: Illustration for proposition 4, part 1, the red area corresponds to values of (I_1^0, I_2^0) such that $(\rho^0, \rho^1) = (\rho_{max}, \rho_{max}, \rho_{max}, \rho_{max})$ is both the Nash equilibrium of the game and an optimal solution of the central planner problem. Within the gray area, extreme eradication is still a Pareto solution, but it is not a Nash equilibrium of the game. In this case, private management is inefficient.

3.3.3 Multiplicity and potential equilibrium selection problem

Multiplicity of equilibria introduces an uncertainty in the outcome of the game. In case two equilibria are present, we do not know a priori which equilibrium will be selected or even whether one of the equilibria will be played. In the example developed in figure 15, all the different equilibrium paths described lead to inefficiency. However, we notice, for example in the comparison of $(0, 0, \rho, \rho)$ with $(\rho, \rho, 0, 0)$, that the second equilibrium path is socially better than the first. In the other situation, we meet the same problem: above the bisector ($x = y$ line) $(0, \rho, \rho, \rho)$ is socially better $(\rho, 0, \rho, \rho)$. However in this case the equilibrium selection problem is more tricky because the socially better equilibrium is not preferred by player 2.

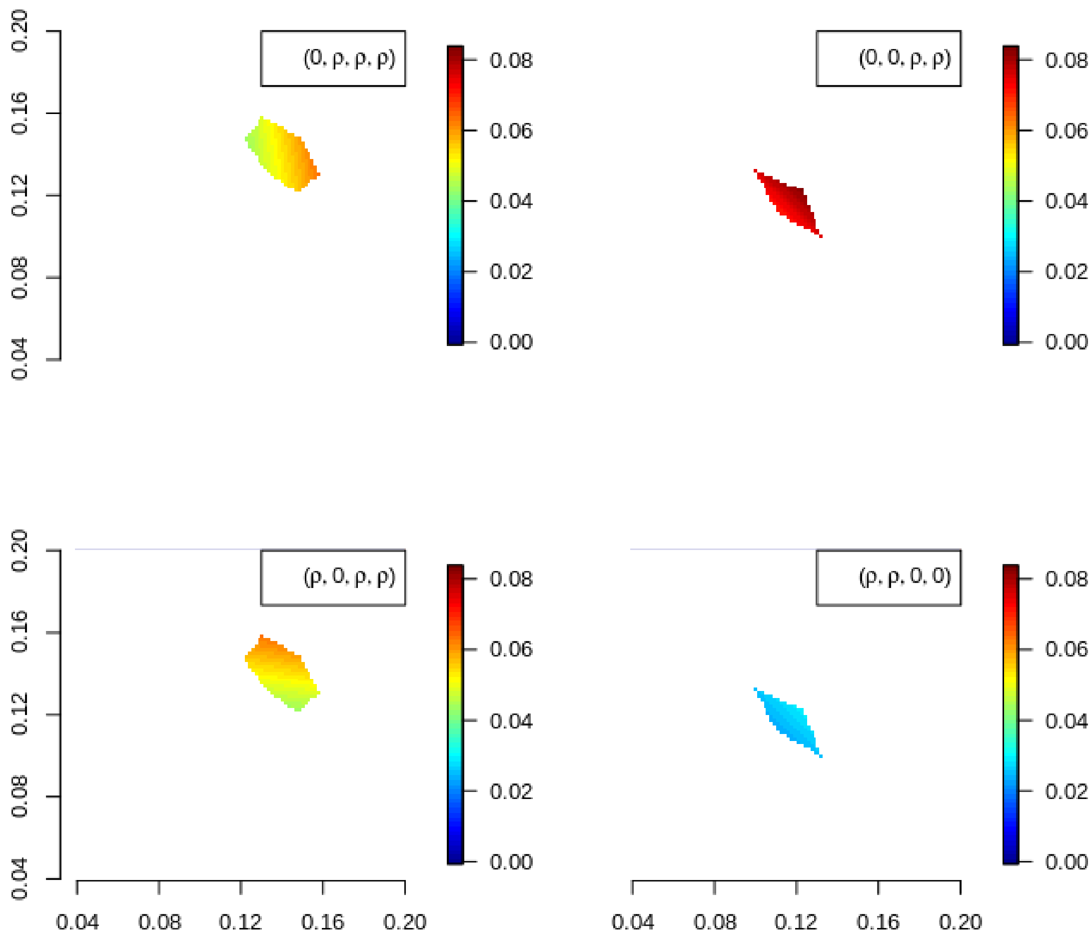

Figure 15: Map of inefficiencies (in percentage of the social optimum value) according to the initial conditions in the zones with multiple SPNE in the same example as in figure 13. Each subplot indicates the inefficiency for a particular pure strategy equilibrium path.

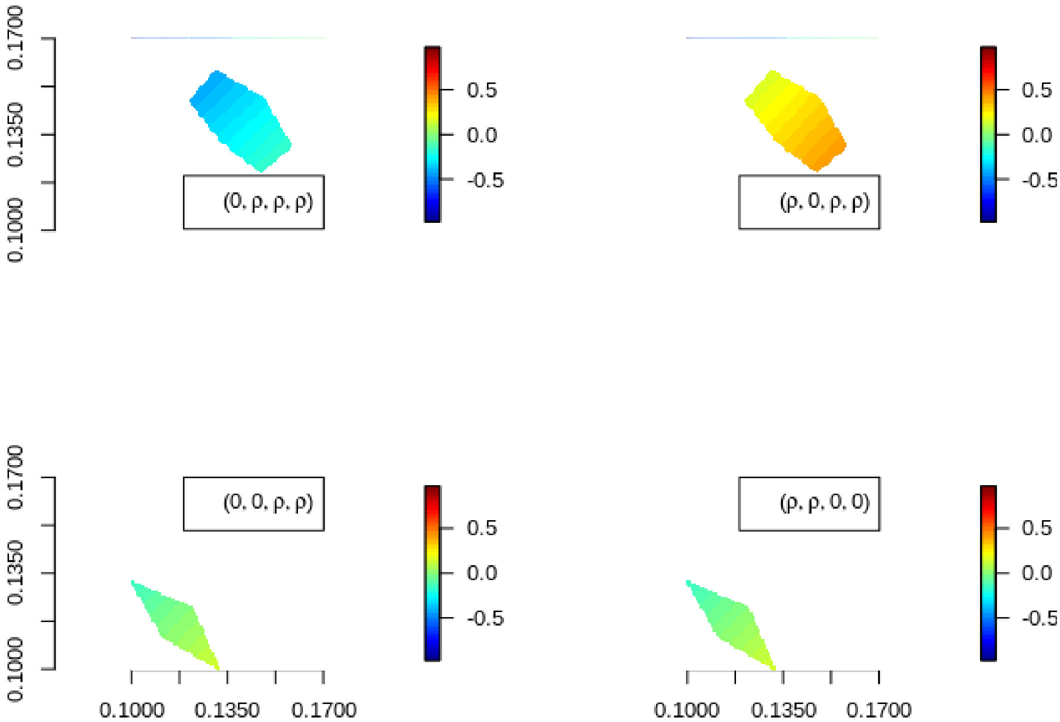

Figure 16: Map of the difference in player's payoffs (player 2 payoff - player 1 payoff) for pure strategies equilibrium paths in the zones with multiplicity. The parameters values are similar to those used for figure 13

4 Discussion and conclusion

Contributions

The bio-economic model developed in this paper frames an original decentralized finite resource management problem. The analysis of the associated game theoretic problem allows to formulate agents' best response function and identify different effects that impact agents' profit maximization. From this we understand how our two players two patches two time periods model can lead to various peculiar results, due to the opposition between a strategic feedback effect and other direct effects of a first step management action. Those results include situations without equilibria, games with multiple equilibria (synchronization games, hawk dove games), games where the feedback strategic effect leads to a free-ridding situation. It is worth noticing that those observations are not commonly produced by models investigated in the field of public bad management and are not reported neither in Costello et al. (2017), nor in Fenichel et al. (2014). Surprisingly, our model joins a famous strand of literature in industrial economics in which the strategic effect in two periods model is analyzed (Fudenberg and Tirole (1984), Bulow et al. (1985)). As in de Frutos and Martín-Herrán (2017), our model bridges the gap between this field of game theory and environmental economics. The second part of our results consists in analyzing inefficiencies emerging from decentralized management. It is first shown how inefficiency emerge from the simple fact that private owners do not consider the impact of their actions on neighbors profit. Then numerical examples allow to illustrate how additional inefficiencies might be generated and amplified by strategic interactions and multiplicity of equilibria.

Management implications, limitations, and perspectives

The exploration of behaviors generated by this model helps to understand the reasons why social inefficiencies might emerge. Particular free-ridding behaviors and inefficiencies levels depend on the parameters and in particular on patches inter-connection level, as well as on the initial infection level. One of the lessons we learn from our stylized model is that such behaviors should be considered at least in some cases. For the analysis of a particular case, a careful parameter analysis is necessary in order to determine whether:

- it will be worth managing for private owners
- feedback effect can be neglected in the analysis (due to small interconnections for instance)
- initial conditions and parameters are such that strategic interactions will impact agents' behavior (and generate inefficiencies).

Our model remains stylized but allows to illustrate strategic phenomenons and ask questions that could be included in more sophisticated (but often analytically intractable) approaches. The modeler, when considering a particular problem, should also consider the nature of the simultaneous strategic interaction: the immediate efficiency of a treatment might depend on whether neighbors treat or do not treat; whereas we have only focused on the case without such an immediate interaction.

To put our findings in the whole complexity of the epidemiological dynamics, many other phenomenons could be incorporated in a global model: the age of the crops, (that could be different across patches), the time horizon length, the release of the hypotheses of a small infectious level. With regard to strategic interactions the hypothesis of rational behaviors seems to be contestable, and one might assume that agents reason according to simpler heuristics as soon as the situation becomes a bit complex. As an example Atallah et al. (2017) works on a predefined set of management strategies.

Many extensions would be worth developing at this point but are left for further work. Among other things, we could mention the analysis of a multi-periods multi-players, multi-players model. Another obvious continuation would be to analyze strategic interactions in the field through a careful parameters estimation. This could be interesting to compare different public bad management problems. Last but not least, this exploration of inefficiencies opens the question of the mechanisms available in order to make the decentralized solution converge towards the social optimum.

5 Appendix

5.1 Backwards induction

Resolution of the game using the game tree structure Discrete time dynamic games are often represented in extensive form using game trees. Those trees represent all management paths from an initial condition as well as their associated payoffs. Figure 2 represents the 2 patches 2 time steps game in such a way. In what follows, we will note a management path as $(\rho_1^0, \rho_2^0, \rho_1^1, \rho_2^1)$, with both first elements being decisions at $t = 0$, or, more synthetically (ρ^0, ρ^1) with the first element being a vector gathering decisions at $t = 0$.

From a given initial condition, SPNE trajectories can be looked for using backward induction. This process can easily be translated into an algorithm:

1. Build the game tree as in figure (2)
2. From the game tree build the subgame matrices as illustrated in figure (17)
3. Find the Nach equilibrium of each subgame (17) [2]
4. Keep the players' payoffs corresponding to those Nash equilibria and construct a new game matrix (as in figure 18) using the paths identified in figure (4).
5. The resolution of this new static game gives the Subgame Perfect Nash Equilibria of the game.

We use this algorithm in our R implementation.

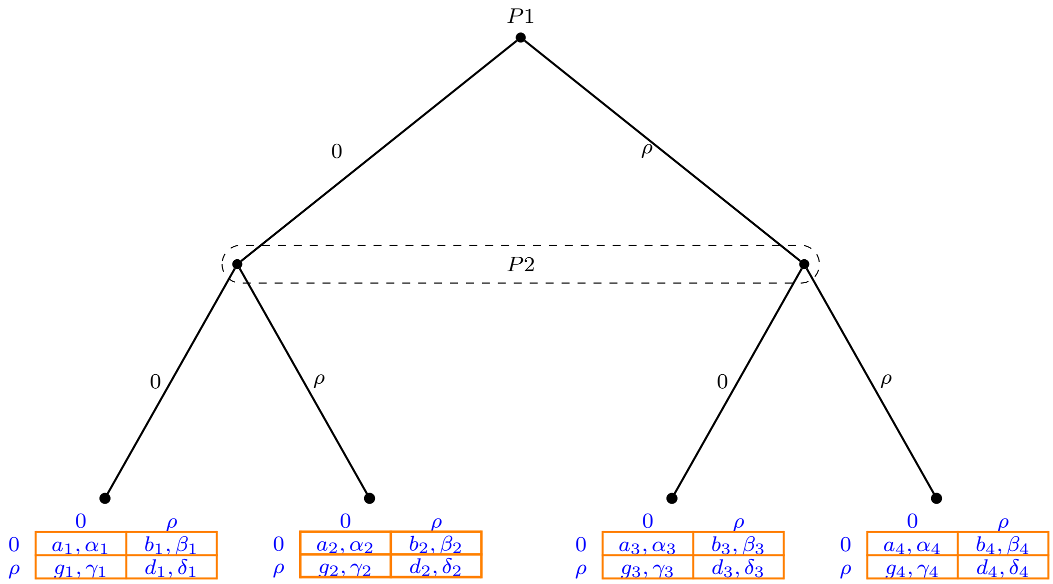

Figure 17: Simplification of the dynamic game: each couple of decisions at $t = 0$ leads to a second step subgame. This second step subgame is directly built from the game tree 17.

[2]When, rarely, a subgame has multiple equilibrium, as explained in proposition 1, each of them should be considered when constructing the list of Nash equilibria of the global game. These distinctions do not lead to differences in players payoffs.

5.2 Parameters examples

5.2.1 First symmetric example

Our model has many parameters. We often remain as general as possible in the writing. However for building numerical examples, we need to focus on some specific cases with precise values. Parameters for the first example are $r_{11} = r_{22} = 1.6$, $r_{12} = r_{21} = 0.8$, $U = (1, 1)$, $V = (3, 3)$, $c_a = (0.05, 0.05)$, $c_h = 2$, $c_r = 0.01$, $A = (0.1, 0.1)$, $\rho_{max} = 0.6$, $\delta = 0.96$. With those parameters, the propagation between orchard is twice smaller as propagation intra orchard. Detection rate is realistic with respect to Sharka disease.

5.2.2 Second symmetric example

Parameters for the second example are $r_{11} = r_{22} = 1.6$, $r_{12} = r_{21} = 1.5$, $U = (1, 1)$, $V = (3, 3)$, $c_a = (0.05, 0.05)$, $c_h = 2$, $c_r = 0.01$, $A = (0.1, 0.1)$, $\rho_{max} = 0.25$, $\delta = 0.96$. With those parameters, the detection rate is small and inter-patch propagation is close to inter-patch propagation.

5.3 Extreme behavior analysis

5.4 First step equivalent game (additional details)

From this, a new game matrix is constructed taking into account the different options available at $t = 0$ as well as the consequences of potential choices at $t = 0$ on the $t = 1$ decisions. Each first step couple of choices is associated with a second couple of decisions as developed in proposition 2 (excepting when initial conditions are such that players are indifferent at the second step). We propose a representation of the game using two matrices. The first matrix simply indicates the complete management paths arising from each couple of decisions at $t = 0$, when $t = 1$ subgames are solved using the Nash equilibrium concept. This is illustrated in figure 4, where ρ_1^{1*} and ρ_2^{1*} are determined using the result of proposition 1 and I^1 is computed using I^0 and the ρ^0 corresponding to the appropriate box in the table.

		player 2	
		0	ρ_{max}
player 1	0	a_1, a_2	b_1, b_2
	ρ_{max}	c_1, c_2	d_1, d_2

Figure 18: Equivalent game matrix for decision options at the first step. This representation will be referred as the first step equivalent game matrix.

where

$$\begin{aligned} a_1 &= V_1^2((0, 0, \sigma_1^{1*}(f(0, 0)), \sigma_2^{1*}(f(0, 0))), I^0, S^0) \qquad (29) \\ &= \pi_1((0, 0), I^0, S^0) + \delta W_1^1(f((0, 0), I^0, S^0)) \\ a_2 &= V_2^2((0, 0, \sigma_1^{1*}(f(0, 0)), \sigma_2^{1*}(f(0, 0))), I^0, S^0) \qquad (30) \\ &= \pi_2((0, 0), I^0, S^0) + \delta W_2^1(f((0, 0), I^0, S^0)) \end{aligned}$$

where W_i^1 is the value function for a one step game constructed using the solution of subgame 1 (see proposition 1 for more explanations):

$$W_i^1(I^1, S^1) = \begin{cases} \pi_i^1(I^1, S^1, \rho_{max}, \rho_{max}) & \text{if } I_1^1 > \alpha_1 \text{ and } I_2^1 > \alpha_2 \\ \pi_i^1(I^1, S^1, \rho_{max}, 0) & \text{if } I_1^1 > \alpha_1 \text{ and } I_2^1 < \alpha_2 \\ \pi_i^1(I^1, S^1, 0, \rho_{max}) & \text{if } I_1^1 < \alpha_1 \text{ and } I_2^1 > \alpha_2 \\ \pi_i^1(I^1, S^1, 0, 0) & \text{if } I_1^1 < \alpha_1 \text{ and } I_2^1 < \alpha_2. \end{cases}$$

Other payoffs are computed using a similar reasoning.

The equations introduced in proposition 2 are illustrated in figure 3. Each line is drown assuming a particular ρ^0. It represents a boundary between initial conditions leading to $\rho_1^i = \rho_{max}$ (above the line) or $\rho_i^1 = 0$ (below the line). There is obviously symmetric results regarding the other player second step action which leads us to a global mapping of the plan (I_1^0, I_2^0) according to players second step actions. Details of the different first step equivalent game are given in table 19 with the interpretation in terms of inter-temporal strategic interactions.

game number	equivalent game		inter-temporal effect		
			$\Delta_{\rho_i^1}$	$\rho_j = 0$	$\rho_j = \rho_{max}$
1	$(0,0,0,0)$	$(0,\rho,0,0)$	(p1)	no effect	no effect
	$(\rho,0,0,0)$	$(\rho,\rho,0,0)$	(p2)	no effect	no effect
2	$(0,0,\rho,\rho)$	$(0,\rho,0,0)$	(p1)	substitution(1,2)	no effect
	$(\rho,0,0,0)$	$(\rho,\rho,0,0)$	(p2)	substitution(1,2)	no effect
3	$(0,0,\rho,\rho)$	$(0,\rho,\rho,0)$	(p1)	substitution(1)	substitution(1)
	$(\rho,0,0,\rho)$	$(\rho,\rho,0,0)$	(p2)	substitution(2)	substitution(2)
4	$(0,0,\rho,\rho)$	$(0,\rho,\rho,\rho)$	(p1)	no effect	substitution(1,2)
	$(\rho,0,\rho,\rho)$	$(\rho,\rho,0,0)$	(p2)	no effect	substitution(1,2)
5	$(0,0,\rho,\rho)$	$(0,\rho,\rho,\rho)$	(p1)	no effect	no effect
	$(\rho,0,\rho,\rho)$	(ρ,ρ,ρ,ρ)	(p2)	no effect	no effect
6	$(0,0,\rho,0)$	$(0,\rho,0,0)$	(p1)	substitution(1)	no effect
	$(\rho,0,0,0)$	$(\rho,\rho,0,0)$	(p2)	substitution(1)	no effect
7	$(0,0,\rho,0)$	$(0,\rho,\rho,0)$	(p1)	substitution(1)	substitution(1)
	$(\rho,0,0,0)$	$(\rho,\rho,0,0)$	(p2)	no effect	no effect
8	$(0,0,\rho,\rho)$	$(0,\rho,\rho,0)$	(p1)	substitution(1,2)	substitution(1)
	$(\rho,0,0,0)$	$(\rho,\rho,0,0)$	(p2)	substitution(2)	no effect
9	$(0,0,\rho,\rho)$	$(0,\rho,\rho,\rho)$	(p1)	substitution(1)	substitution(1,2)
	$(\rho,0,0,\rho)$	$(\rho,\rho,0,0)$	(p2)	no effect	substitution(2)
10	$(0,0,\rho,\rho)$	$(0,\rho,\rho,\rho)$	(p1)	substitution(1,2)	substitution(1,2)
	$(\rho,0,0,0)$	$(\rho,\rho,0,0)$	(p2)	no effect	no effect
11	$(0,0,\rho,\rho)$	$(0,\rho,\rho,\rho)$	(p1)	substitution(2)	substitution(1,2)
	$(\rho,0,\rho,0)$	$(\rho,\rho,0,0)$	(p2)	no effect	substitution(1)
12	$(0,0,\rho,\rho)$	$(0,\rho,\rho,\rho)$	(p1)	no effect	substitution(2)
	$(\rho,0,\rho,\rho)$	$(\rho,\rho,\rho,0)$	(p2)	no effect	substitution(2)
13	$(0,0,\rho,\rho)$	$(0,\rho,\rho,\rho)$	(p1)	substitution(2)	substitution(2)
	$(\rho,0,\rho,0)$	$(\rho,\rho,\rho,0)$	(p2)	no effect	no effect

Figure 19: Structure of the equivalent games in the plan (I_1^0, I_2^0), corresponding to the representation in figure 3. Columns 4 and 5 indicate the effect of a change in ρ_i^0, on second step actions (ρ_i^1 and ρ_j^1) (when $\rho_j^0 = 0$ and when $\rho_j^0 = \rho_{max}$). Here, substitution(k) in column $\rho_j = \rho_{max}$ and line p_i means that when ρ_i^0 changes, ρ_k^1 changes as well (in the opposite direction). Substitution(1,2) means that both second step action change after an unilateral deviation by player i in the first step.

5.5 Multiplicity analysis

Conditions for observing multiplicity in the coordination game 2 (see figure ??)

$$I_i^0 \in [\frac{\alpha}{1+r+r'}, \frac{\alpha}{1+r+r'(1-\rho_{max})}] \tag{31}$$

$$\Delta_{\rho_i^0,0\to\rho_{max}}(Z_i(I^0,S^0,\rho_j^0=0)) < 0 \tag{32}$$

$$\Delta_{\rho_i^0,0\to\rho_{max}}(Z_i(I^0,S^0,\rho_j^0=\rho_{max})) > 0. \tag{33}$$

$$\begin{aligned}
\Delta_{\rho_i^0,0\to\rho_{max}}(Z_i(I^0,S^0,\rho_j^0=0)) &= (\Delta_{\rho_i^0}\Pi_i^1 + \delta\Delta_{\rho_i^0}W_i^2) \\
&= \rho_{max}I_i^0((v_i-u_i)r - u_i - c_r) - c_f \\
&+ \delta((v_i-u_i)(\\
&I_i^0\rho_{max}r \\
&- \rho_{max}I_j^0 r' r \\
&- \rho_{max}I_j^0(1+r)r' \\
&) \\
&- u_i I_i^0 \rho_{max} \\
+(u_i+c_r)(\quad \rho_{max}(I_i^0(1+r)+I_j^0 r')) \\
&+ c_f \\
&).
\end{aligned}$$

$$\begin{aligned}
\Delta_{\rho_i^0,0\to\rho_{max}}(Z_i(I^0,S^0,\rho_j^0=\rho_{max})) &= (\Delta_{\rho_i^0}\Pi_i^1 + \delta\Delta_{\rho_i^0}W_i^2) \\
&= \rho_{max}I_i^0((v_i-u_i)r - u_i - c_r) - c_f \\
&+ \delta(\\
(v_i-u_i)(\\
&I_i^0\rho_{max}r \\
&+ I_i^0\rho_{max}r(1+r) \\
&+ I_i^0\rho_{max}r'^2 \\
&) \\
&- u_i I_i^0\rho_{max} \\
&).
\end{aligned}$$

Conditions for observing multiplicity in the coordination game 4 (see figure ??)

$$I_i^0 \in [\frac{\alpha}{1+r+r'}, \frac{\alpha}{1+r+r'(1-\rho_{max})}] \tag{34}$$

$$\Delta_{\rho_i^0,0\to\rho_{max}}(Z_i(I^0,S^0,\rho_j^0=0)) < 0 \tag{35}$$

$$\Delta_{\rho_i^0,0\to\rho_{max}}(Z_i(I^0,S^0,\rho_j^0=\rho_{max})) > 0. \tag{36}$$

$$
\begin{aligned}
\Delta_{\rho_i^0,0\to\rho_{max}}(Z_i(I^0,S^0,\rho_j^0=0)) &= (\Delta_{\rho_i^0}\Pi_i^1+\delta\Delta_{\rho_i^0}W_i^2) \\
&= \rho_{max}I_i^0((v_i-u_i)r-u_i-c_r)-c_f \\
&+\delta((v_i-u_i)(\\
& I_i^0\rho_{max}r \\
& -\rho_{max}I_j^0r'r \\
& -\rho_{max}I_j^0(1+r)r' \\
&) \\
& -u_iI_i^0\rho_{max} \\
+(u_i+c_r)(\quad \rho_{max}(I_i^0(1+r)+I_j^0r')) \\
& +c_f \\
&).
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{\rho_i^0,0\to\rho_{max}}(Z_i(I^0,S^0,\rho_j^0=\rho_{max})) &= (\Delta_{\rho_i^0}\Pi_i^1+\delta\Delta_{\rho_i^0}W_i^2) \\
&= \rho_{max}I_i^0((v_i-u_i)r-u_i-c_r)-c_f \\
&+\delta(\\
(v_i-u_i)(\\
& I_i^0\rho_{max}r \\
& +I_i^0\rho_{max}r(1+r) \\
& +I_i^0\rho_{max}r'^2 \\
&) \\
& -u_iI_i^0\rho_{max} \\
&).
\end{aligned}
$$

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