USE OF COUPLED INCENTIVES TO IMPROVE ADOPTION OF ENVIRONMENT FRIENDLY TECHNOLOGIES

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ABSTRACT. We devise a system of coupled incentives that stimulate economic agents to coordinate their actions. The agents are price takers and their actions would be independent of one another ("uncoupled") if incentives were not implemented. The action coordination is expected to help a technology transition from the current one to a modern one. The latter is assumed environment friendlier than the former. With the incentives in place, the problem becomes one of a principal-(multi)agent game. In the game, the principal chooses instruments sufficient to generate an environment friendly agent reaction.

We define a specific *coupled incentive scheme* (CIS), in which individual agents are rewarded for their joint actions' effect, rather than for their own actions' accomplishments. We show that a technology switch can be realised through CIS for a budget, for which no technology change would be achieved if only individual agent actions were compensated. We show under which conditions the agents' game has a unique solution and what the principal's choices are for the solution's implementation.

KEYWORDS: coupled incentives, technology change, principal-agent problems
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1. INTRODUCTION

Implementation of environmental policies by the regional government often requires the use of taxes and/or subsidies. It is frequently so that *private* agents produce a *public* negative externality by using an outdated technology. However, it is usually only the government that perceives it as negative. Adoption of subsidies appears necessary if the government wants economic agents to switch from the old technology to a modern one, which will be environment friendly.

We introduce and discuss a coupled incentive model that can be useful to analyse various economic activities. The particular context for which it is developed is in agriculture and concerns *landscape changes*. For example, if farms are composed of pastures and of less cultivated areas (*e.g.*, woods), the regional government might be interested in grouping the less cultivated areas together. This would stimulate the biodiversity of the region as wild animals would likely reproduce faster in "aggregate wilderness" rather than in the scattered backwoods. From the farm economics viewpoint, either landscape arrangement may be equally profitable for each farm. However, the transition from the original field mix to the new one will be costly and the farmers will require incentives to perform an exchange. We will call the regime

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under which a farm produces, *technology.* It will be technology A under the original field mix and it will be a mixture of technologies A and B (or B only) after the field swap, which might be partial or total.

A technology switch is not easy for an economic agent because of the new technology capital expenditure cost. In an idealised model, the old technology, which is *already there*, can be maintained at no cost. If the change is going to happen the agents need to be helped by the government to meet the transition cost.

This paper is concerned with a mathematical model of a subsidy allocation scheme that should be useful for the government. The government pushes for a technology change and disposes of *limited* means. We define the *coupled incentive scheme* (CIS), in which individual agents are rewarded not only for their own actions but also for their joint actions' result. We show that, for the same cost incurred by the Regional Government, a higher goal fulfillment is achieved through CIS, than by a plan, in which only individual actions are compensated.

In proposing a specific CIS we will follow a couple of "natural" team building postulates formulated by Itoh [3] (also see [1] for an optimisation approach). In particular, our system will introduce *strategic interaction between agents* who otherwise would be action-decoupled and non cooperative. The incentive system will modify "their [agents'] attitudes toward performing [...] tasks" (see [3], p. 630). Also, the system will stimulate social interactions. In particular, faster (in transition) agents will be lobbying agents who are slow, at no cost to the principal. In fact, the agents' problem can be seen as an aggregative game often used to model collective actions and public good production (see [2]).

Economically, our system creates a public good produced by the agents' joint actions. Mathematically, we model the agent reactions to the principal's signals as a Nash equilibrium in a static non cooperative infinite game whose parameters are controlled by the principal. The principal's problem is one of mathematical programming with constraints, where the objective function is an environmental index that depends on agents' strategies. A constraint in the mathematical programme corresponds to the regional government's budget. We assume that the government can observe agent actions at negligible cost.

The paper is organised as follows. In Section 2, we present a multi-agent model and introduce a functional form for CIS. In Section 3, we concentrate on a game where the agents are symmetric and prove the solution uniqueness. This case is obviously simpler to solve than its non symmetric counterpart. However, in a competitive economy, farms in a region do not differ substantially in size. This means that a symmetric game solutions bear relevance to real life situations. For this case, we prove the existence of a Nash equilibrium for the agent problem and we analyse the cases where the equilibrium uniqueness is guaranteed. We show that our CIS can motivate agents to perform a technology switch even if the government disposes of a "small" budget only. In Section 4, we examine a non symmetric agent game. We collect the conclusions in the Concluding Remarks.

2. A MULTI-AGENT PROBLEM

2.1. **A coupled incentive scheme (CIS).** Consider N ($i = 1, 2, \ldots N$) agents, and a principal. The agents can cultivate their fields, each of surface s_i, using technology A (old) or B (new, environmental friendly) or both. A marginal revenue

m_i from using either technology is the same but might be different for each agent. A transition between the technologies is costly.

From the principal's point of view technology B is superior because it generates less of a negative externality. A local budget means M could be allocated to help agents to switch to technology B. We will devise an individual-and-collective incentive system (a "comprehensive" *coupled incentive scheme* or CIS) capable of inducing a maximal level of the technological change for a given level of M.

We will model the technology transition cost as the following quadratic function:

$$C_i(\tau_i, \tau_i^0, s_i) = \alpha_i\big((\tau_i - \tau_i^0)s_i\big)^2$$

where τ_i^0 is the current percentage of technology B in use, τ_i is the new level of use of technology B and $\alpha_i > 0$ is a coefficient. However, we will suppose that the current usage of technology B is negligible so $\tau_i^0 = 0$ [(1)].

The assumption of the quadratic cost seems generally justified for small to moderate τ_i. However, it appears defendable for $\tau_i \in [0, 1]$ in the *landscape change* context where re-cultivation of large areas does not need to exhibit the large scale effect. [(2)]

The agents are price takers and their payoff function can be written as follows:

$$\begin{aligned} f_i(\tau_i, \tau_{-i}; \mathbf{u}) &= (m_i(1-\tau_i) + m_i\tau_i)s_i - C_i(\tau_i, 0, s_i) + \Pi_i(\mathbf{u}; \tau_i, \tau_{-i}) \\ &= m_i s_i - \alpha_i\big(\tau_i s_i\big)^2 + \Pi_i(\mathbf{u}, \tau_i, \tau_{-i}) \end{aligned} \tag{1}$$

where a variable subscripted $_{-i}$ refers to the set of players that excludes player i. Function $\Pi_i(\mathbf{u}; \tau_i, \tau_{-i})$ models a subsidy that an agent will receive if they dedicate $\tau_i s_i$ of their field to the new technology and $\mathbf{u}$ is a collection of the principal's instrumental variables. The choice of $\Pi_i(\mathbf{u}; \tau_i, \tau_{-i})$ corresponds to an incentive scheme adopted by the government and is the subject of study in this paper.

A *natural* incentive function may be one in which an agent is reimbursed for the cost he incurs for implementing the new technology. As the cost is $\alpha_i\big(\tau_i s_i\big)^2$, the government may consider paying a fraction $u, u \in [0, 1]$ of this cost. Unfortunately,

$$(1-u)\alpha_i\big(\tau_i s_i\big)^2,$$

which is the remainder and the farmer's expenditure, is a convex function of τ_i minimised at $\tau_i = 0$. Therefore, if the government does not have enough funds to finance the full programme ($u = 1$), the optimal agent reaction is null.

Suppose that the principal would like *more* than S of the entire cultivated area $\sum_{i=1}^N s_i$ be dedicated to technology B. Presumably, this will be a substantially larger area than a single agent disposes of *i.e.*,

$$S >> s_j \quad \forall j, \quad S \leq \sum_{i=1}^N s_i.$$

Suppose that $\mathbf{u} = [u,\ w,\ S]$ where u, w are the principal's instruments that control incentive primes that the principal can award to agents for their individual and collective efforts, respectively. As above, symbol S denotes a minimum level of the desired technology transfer area. We will design a comprehensive incentive function

[(1)]If it was not, we would "re-scale" the problem and call the current technology mix a technology A′; we will say that technology B is introduced if the mix has been changed.

[(2)]A logistic cost function might need to be considered.

$\Pi_i(\mathbf{u}; \tau_i, \tau_{-i})$ such that for each agent *not only*

$$\frac{\partial \Pi_i}{\partial u} > 0 \quad \text{if} \quad \tau_i > 0$$

but also

$$\frac{\partial \Pi_i}{\partial w} > 0 \quad \text{if} \quad \tau_{-i} > 0 \quad \underline{\text{and}} \quad \sum_{i=1}^{N} \tau_i s_i > S. \tag{2}$$

Relationship (2) captures our objective to introduce a strategic interaction between agents. In particular, each agent will be interested in the other agents' participation in the technology transition programme. Henceforth, an incentive system relying on $\Pi_i(\mathbf{u}; \tau_i, \tau_{-i})$ will promote inter-agent communication, and lobbying for the new technology will eventuate without much of the principal's influence. In that sense, a system incorporating (2) will be more robust (*i.e.*, insensitive to model and information imperfections) than a purely individual incentive system.

We propose the following functional form for the incentive function Π_i where, as said, u, w, S are principal's instrumental variables:

$$\Pi_i(u, w, S; \tau_i, \tau_{-i}) = u\alpha_i(\tau_i s_i)^2 + w \max(0, \sum_{i=1}^{N} \tau_i s_i - S). \tag{3}$$

According to (3), an agent will be reimbursed a proportion u of their technology transfer cost and rewarded additionally if all agents' joint effort has exceed S. The function Π_i with parameters u, w and S will be announced to agents that will choose their optimal participation levels τ^* to maximise (1).

Notice that if $w \equiv 0$, formula (3) represents an agent's individual incentive function; for $w > 0$, it becomes an agent's coupled incentive function. Also, function (3) satisfies the postulates (2).

Result 2.1. *The choice of the incentive function (3) guarantees that the payoff to agent i grows in size of the area that agent j decided to cultivate in technology B, if enough agents have decided to do it.*

Obviously, the choice of an incentive function Π_i is non unique. The choice of (3) is "reasonable" in that it guarantees the above result. However, one could argue that small area agents can be given "disproportional" incentives because the second term of (3) is common for all agents. A few counter arguments could be put forward: large differences in area between farms are unlikely in a competitive economy; a large area agents will always receive a greater reward because of the first term; finally, even if some agents receive disproportional gains, who cares if this is done *pro publico bono.*

2.2. **The mathematical formulation of the incentive scheme.** Let $[\tau_1^*, \ldots, \tau_N^*]'$ denote the vector of the new technology *optimal* choices and let an environmental index J be defined as

$$J(\tau_1^*, \ldots, \tau_N^*; u, w, S) \equiv \sum_{i=1}^{N} \tau_i^* s_i. \tag{4}$$

The components of vector $[\tau_1^*, \ldots, \tau_N^*]'$ are agents' optimal reactions to the principal's instruments u, w, S and can be calculated as Nash equilibrium strategies in the following game:

$$\tau_i^* = \arg \max_{\tau_i \in [0,1]} f_i(\tau_i, \tau_{-i}^*; u, w, S) \quad i = 1, 2, \ldots N \tag{5}$$

We will show in Section 3 under what conditions the above equilibrium exists and is unique.

The principal's problem whose budget for the technology switch is $M > 0$ might be written as

$$\underline{\text{find}} \qquad (u^*, w^*, S^*) = \arg \max_{u \geq 0, w \geq 0, S \geq 0} J(\tau_1^*, \ldots, \tau_N^*; u, w, S) \tag{6}$$

$$\underline{\text{and}}\ \underline{\text{such}}\ \underline{\text{that}} \quad \sum_{i=1}^{N} \Pi_i(u^*, w^*, S^*; \tau_i^*, \tau_{-i}^*) \leq M \tag{7}$$

where $\tau_i^*,\ i = 1, 2 \ldots N$ satisfy (5) and

$$f_i(\tau_i^*, \tau_{-i}^*; u^*, w^*, S^*) > m_i s_i. \tag{8}$$

We will call the problem (6) subject to (7), (5) and (8) the principal's coupled incentive optimisation problem (PCIOP).

Obviously, there will be a trade-off between the level of J and budget M. So, the principal's problem could be formulated as a bi-criterial optimisation problems as follows:

$$\underline{\text{find}} \qquad (u^{\overline{JM}}, w^{\overline{JM}}, \overline{S}) \tag{9}$$

$$\underline{\text{such}}\ \underline{\text{that}} \quad J(\tau_1^*, \ldots, \tau_N^*; u^{\overline{JM}}, w^{\overline{JM}}, \overline{S}) \geq \overline{J} \quad \underline{\text{and}} \quad \sum_{i=1}^{N} \Pi_i(u^{\overline{JM}}, w^{\overline{JM}}, \overline{S}; \tau_i^*, \tau_{-i}^*) \leq \overline{M}$$

where $\overline{J}, \overline{M}$ would be the principal-defined satisfactory level of technology B implementation and an affordable budget, respectively. In other words, $(u^{\overline{JM}}, w^{\overline{JM}}, \overline{S})$ are the principal's instruments that determine an efficient boundary of (J, M). Computing this boundary would enable us to propose a collection of *satisfactory solutions* (see [4]) to the government problem.

Obviously, a different choice of the function Π_i would lead to different solutions to problems (6)-(7) and (9). In the remainder of this paper we will be concerned with the coupled incentive function (3), agent game (5) and the principal's problem (6), (7).

We shall request the solution to (5) be unique *i.e.*, the principal should know what the agents' reaction to u, w, S is. Otherwise the solution might be useless for the principal. However, we can distinguish between two "levels" of uniqueness:
(••) *strong* when the principal knows exactly what $\tau_i^* \quad \forall i$ is associated with a given set of instruments;
(•) *weak* when the principal knows only which aggregate area $\sum_i^N \tau_i^* s_i$ corresponds to a given set of instruments. Obviously,

$$(\bullet\bullet) \Longrightarrow (\bullet)$$

and

$$(\bullet\bullet) \Longleftrightarrow (\bullet)$$

if all agents are identical. Notice that weak uniqueness is sufficient for the principal to rest assured that a given set of instruments u, w, S generates an optimal value of

$$J(\tau_1^*, \ldots, \tau_N^*; u, w, S) = \sum \tau_i^* s_i.$$

3. The solution to a symmetric N-agent game

We first solve a *symmetric* N-agent game *i.e.*, when [3] $\alpha_i = \alpha = 1$, $s_i = s$, and $m_i = m$. The relevance of this case stems from the fact that, in competitive economies, no great variation among farms is expected in a geographic area.

3.1. A benchmark problem solution.

The solution to (5) will depend on the agent problem parameters (here, s only) and the principal's instruments u, w, S. Some combinations of the instruments may generate irrelevant solutions from the principal's point of view. To eliminate them from the discussion, we will first solve a *benchmark problem*.

The benchmark problem consists of finding how much M_b it would cost the principal to entice the agents to do the full technology switch if the individual incentive scheme only ($w = 0$) were applied. Any solution to (5) which would be more expensive than M_b or resulting, for the same M_b, in a smaller switch will be discarded.

Consider the individual incentive payoff function

$$f_i(\tau_i, \tau_{-i}; u, 0, 0) = m\,s + (u-1)\tau_i^2 s^2. \tag{10}$$

It is easy to see that payoff f_i is maximised at $\tau^* = 1$ (*i.e.*, the full technology switch will take place and $J = Ns$) if it is a convex and increasing function of $\tau \in [0, 1]$. This is true for $u = 1 + \varepsilon$ where $\varepsilon > 0$. The cost to the principal is

$$M_b = (1 + \varepsilon) N s^2\,. \tag{11}$$

Evidently, if $M < Ns^2$ and $w = 0$ then $\tau_i^* = 0$, which means that no technology switch is optimal. In the rest of this section, we will assume that the principal disposes of $M < Ns^2$ (and bear in mind that s is in hectares and M in dollars). We will show that, under CIS, a technology switch might be optimal.

3.2. Existence and uniqueness of Nash equilibrium in the agent problem.

In this section we prove several important facts about the equilibrium existence and uniqueness. We can prove the following lemma.

Lemma 3.1. *In the symmetric case, there exists a Nash equilibrium (5). For some combinations of the agent problem parameters and principal's instruments, the equilibrium is unique and relevant (i.e., $M \le Ns^2$ and $f_i > m\,s$).*

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[3] Notice that the coefficient $\alpha_i = 1$, which will disappear from notation, is not dimensionless. Its dimension is $\left[\frac{\$}{ha^2}\right]$, if the area is measured in hectares. To preserve notation consistence we will remember that the units of both u and $(1-u)$ are as above. If needed, we will use a coefficient $\kappa = 1$ whose dimension is $\left[\frac{\$}{ha^2}\right]$ to keep the "hidden" dimensionality of $\alpha_i = 1$ in a formula. A similar problem occurs when we later assume $s = 1$. In that case, we will understand that m in \$ rather than in $\left[\frac{\$}{ha}\right]$.

Proof. Remember that τ_i^* denotes the optimal choice of agent i. Let u, w and $S < \sum_{i=1}^{N} s_i = Ns$ be fixed by the principal. Agents want to compute τ_i^* such that it maximises their payoff function

$$f_i(\tau_i, \tau_{-i}; u, w, S) = ms + (u-1)\tau_i^2 s^2 + w \max\left(0, \sum_{k=1}^{N} \tau_k s_k - S\right). \tag{12}$$

1. Suppose $u > 1$. Function f_i is convex and increasing in $\tau_i \in [0, 1]$ so, $\tau_i^* = 1$ for all i and $f_i^* = m\,s + (u-1)s_i^2 + w(Ns - S) > ms$, however, $M = (us^2 + w(Ns - S))N > Ns^2$. Hence this solution is more expensive than the benchmark problem's.

2. Suppose $u = 1$.
   (a) If $w = 0$ then $\forall S$, $f_i^* = ms$, which is not sufficient to induce a technology switch. Moreover, τ_i^* can be any value $\in [0, 1]$ so, the choice would be non unique.
   (b) If $w > 0$ then $\tau_i^* = 1$, for all i; the payoff value is $f_i^* = ms + w(Ns - S) > m\,s$, however, $M = (s^2 + w(Ns - S))N > Ns^2$. Hence this solution is more expensive than the benchmark problem's.

3. Suppose $u < 1$. Here the study is more complicated because f_i is piecewise defined. We analyse $\tau_i \in [0, \frac{S}{Ns})$ and $\tau_i \in [\frac{S}{Ns}, 1]$ separately.
   (a) For all $\tau_i \in [\frac{S}{Ns}, 1]$, f_i is a concave function with derivatives:

$$\frac{\partial f_i}{\partial \tau_i} = 2(u-1)s_i^2 \tau_i + w s_i \tag{13}$$

   and

$$\frac{\partial^2 f_i}{\partial \tau_i^2} = 2(u-1)s_i^2 < 0. \tag{14}$$

   At

$$\overline{\tau} = \frac{w}{2(1-u)s} \tag{15}$$

   which solves $\frac{\partial f_i}{\partial \tau_i} = 0$, the payoff function is maximised. However, $\overline{\tau}$ can be more than 1 so, τ_i^* is the projection of $\overline{\tau}$ onto interval $[\frac{S}{Ns}, 1]$. Each agent's payoff at $\overline{\tau}$ is

$$f_i(\overline{\tau}; u, w, S) = m\,s + w\left(\frac{(2N-1)w}{4(1-u)} - S\right), \quad i = 1...N\ . \tag{16}$$

   At the left and right ends of $[\frac{S}{Ns}, 1]$ the payoffs are:

$$f_i\left(\frac{S}{Ns}; u, w, S\right) = m\,s - (1-u)\left(\frac{S}{N}\right)^2 < m, \quad i = 1...N\ , \tag{17}$$

$$f_i(1; u, w, S) = m\,s - (1-u)s^2 + w(Ns - S), \quad i = 1...N \tag{18}$$

   respectively. The latter can be greater than $m\,s$.
   (b) If $\tau_i \in [0, \frac{S}{Ns})$ the maximum of f_i is achieved at $\overline{\tau} = 0$ for all i and $f_i = m\,s$.

From "3a" and "3b" above we conclude that an equilibrium exists in the agent problem (5) and can be $\tau_i^* = \min(\overline{\tau}, 1)$ or $\tau_i^* = 0$.
In particular, if $\frac{(2N-1)w}{4(1-u)} - S > 0$ and $-(1-u)s^2 + w(Ns - S) > 0$, we obtain that the unique equilibrium is given by $\tau^* = \min(\overline{\tau}, 1)$ ($\tau = 0$ is eliminated because it gives an agent's profit $f_i(0; \cdot, \cdot, \cdot) = m\,s$ smiler than $f_i(\overline{\tau}^*; \cdot, \cdot, \cdot) > m\,s$).

□

We can draw several conclusions from the above lemma.

i. Only $0 \le u < 1$, $w > 0$ are the relevant instrument intervals. Indeed, $u > 1$ requires $M \ge M_b$ and $w = 0$ cannot generate $f_i > ms$.
ii. If the principal has a "small" budget $M \le M_b$ and proposes $u < 1, w > 0$ and $S < \dfrac{(2N-1)w}{4(1-u)}$ (but $S \ne Ns - \dfrac{(1-u)s^2}{w}$) there will be unique equilibria at $\tau_i^* = 1$, $\tau_i^* = 0$ or $\tau_i^* = \overline{\tau}$ depending on the parameter choice.
iii. Some of those equilibria will be relevant *i.e.*, $f_i > ms$ and $M < M_b$.

An illustration of how the optimal choice of a new technology can take place in case of a two player symmetric game (with $s = 1$) is presented in Figure 1.

The two surfaces in the top panel represent agent net gains $(f_i - m)$ from a technology switch. The dark plane is drawn at the zero gain level. The parameter u is fixed in this figure ($u = .5$). Another principal's instrument S equals .2 on the larger surface; $S = 1.5$ on the smaller surface. The coupled reward instrument w varies between 0 and 2 and constitutes an "independent" variable.

Given u, S and w, an agent maximises their net gain by choosing τ^*. Three principal's choices of w and the agent optimising τ^* are illustrated in Figure 1. The upper panel of Figure 1 shows these choices in 3D (where the axes are marked $w, \tau, f - m$) while the bottom panel presents them in a two dimensional space of τ and $f - m$. If $w = 0$, the agent chooses $\tau^* = 0$; if $w = .5$, the agent chooses $\tau^* = .5$; if $w = .9$, the agent chooses $\tau^* = 1$. We can see that depending on w (and, obviously, S and u) the equilibrium selected can be $0, 1$ or $\overline{\tau}$.

We have not included in this example the principal's constraint $M \le M_b$. It will restrict the u and w that the principal can propose to the agents. This constraint is part of the principal's problem and will be dealt with in the next section.

3.3. **A solution to the principal's problem.** It follows from Lemma 3.1 that if the principal disposes of $M > Ns^2$ then the technology transition is total with $u > 1$, $w = 0$ and S is irrelevant for this case. In other words, a rich principal does not need a coupled incentive scheme.

It is interesting to see that if $M < Ns^2$ then a coupled incentive scheme with (3), $0 \le u \le 1$, $w > 0$ and an appropriate S *guarantees* that a higher τ^* is realised than if, for the same M, an individual scheme with $0 \le u \le 1$ and $w = 0$ was implemented. Indeed, if $w > 0$ then $\tau^* > 0$ is given by (15); for $w = 0$, the non transfer $\tau^* = 0$ is optimal.

Let us see now which u, w, S have to be chosen by the principal to entice the agents to a technology switch. In other words, we are looking for the principal's instruments under which the agents modify their technologies, are better off ($f_i > m\,s$) and the principal's budget is $M < Ns^2$. From (15), (16) (see items 3.3a in Lemma 3.1's

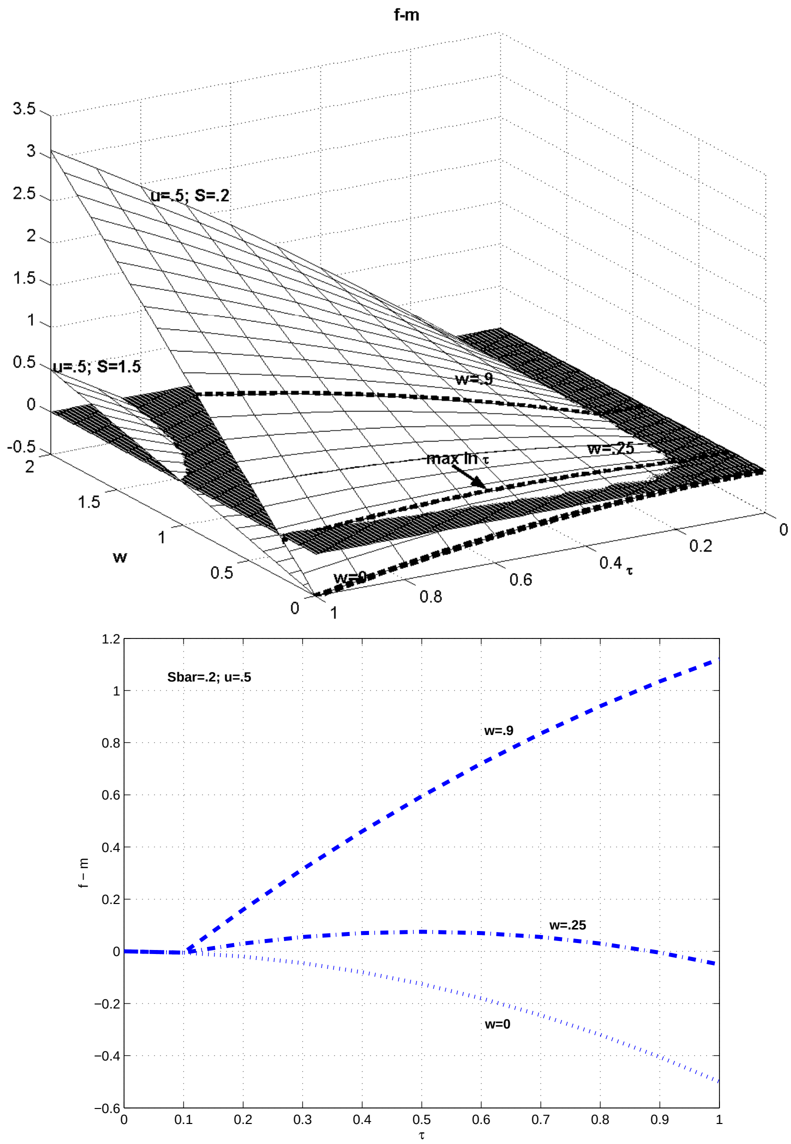

FIGURE 1. The choice of the maximising τ.

proof) we can see that the instruments u, w, S have to be such that

$$0 \leq u < 1, \tag{19}$$

$$\frac{S}{Ns} \leq \frac{w}{2(1-u)s} \leq 1, \tag{20}$$

$$N\left(u\left(\frac{w}{2(1-u)}\right)^2 + w\left(\frac{Nw}{2(1-u)} - S\right)\right) \leq M, \tag{21}$$

$$S < \frac{Nw - \frac{w}{2}}{2(1-u)}. \tag{22}$$

The principal's problem is then to maximise

$$J^* = \frac{Nw}{2(1-u)}. \tag{23}$$

subject to (19)-(22).

Maximisation of (23) subject to (19)-(22) is a rather complicated nonlinear programme, which does not need to possess a unique solution. Particular difficulties may arise because of the first (right hand) and last constraints, which are *strict* inequalities. It is possible that local maxima of (23) would occur if (19) and/or (22) were allowed to be active[(4)].

Definition 3.1. *A* satisfying solution *to the principal's problem is one for which all constraints are satisfied and the principal's utility is positive*[(5)].

To compute a satisfying solution a slack variable δ_1 needs to be added to (19) and another slack variable δ_2 to (22). So, the system that defines the feasibility region looks now as follows:

$$0 \le u + \delta_1 \le 1, \tag{24}$$

$$\frac{S}{Ns} \le \frac{w}{2(1-u)s} \quad \le \quad 1, \tag{25}$$

$$N\left(u\left(\frac{w}{2(1-u)}\right)^2 + w\left(\frac{Nw}{2(1-u)} - S\right)\right) \quad \le \quad M, \tag{26}$$

$$S + \delta_2 \le \frac{Nw - \frac{w}{2}}{2(1-u)} \tag{27}$$

where $\delta_1 > 0, \ \delta_2 > 0$.

While the slack variables may look arbitrary, they are two new principal's instruments and have a political-economic interpretation. The first slack variable δ_1 determines the maximum strength of the individual incentive $(1-\delta_1)\tau s$. This is the upper limit that can be paid to an agent because of his own commitment to a technology switch. The second slack variable δ_2 controls the minimum amount of the net gain a player will earn by participating in the programme. Indeed, from (16) and (27), it is evident that

$$f_i(\overline{\tau}; u, w, S) - m\, s \ge w\delta_2. \tag{28}$$

Once the principal has determined the instruments (including the slack variables), the solution to the symmetric agent game from Section 3.2, is *strongly* unique *i.e.*, the principal knows that payoff (23) is realised through the unique choice of agents' actions (15). We will now show what the relationship between instruments u, w, S has to be so that the principal's problem (max (23) s.t. (24))-(27)) possesses a solution.

(4) This happens to be a fact as shown by numerical experiments carried out in Example 3.1 further down in this section.

(5) Compare [4] .

It is reasonable to consider the situations where the principal uses the entire budget M to stimulate the agents *i.e.*,

$$N\left(u\left(\frac{w}{2(1-u)}\right)^2+w\left(\frac{Nw}{2(1-u)}-S\right)\right)=M. \tag{29}$$

Moreover, it is technically easier to solve the principal's problem (23,)(24)-(27) (with equality (29)) by substituting $\overline{\tau}$ from (15) and solving the problem in $u,\overline{\tau},S$. We get that $u,\overline{\tau},S$ have to satisfy

$$\begin{cases} 0\le u+\delta_1\le 1, \quad S+\delta_2\le \dfrac{2N-1}{2}\overline{\tau}s, \quad \overline{\tau}\le 1, \\ \\ (u+2N(1-u))\overline{\tau}^2s^2+2S(1-u)\overline{\tau}s=\dfrac{M}{N}. \end{cases} \tag{30}$$

From (29) we obtain that

$$S=\frac{(u+2N(1-u))\overline{\tau}^2s^2-M/N}{2(1-u)\overline{\tau}s}. \tag{31}$$

Using (31) we obtain that (30) is equivalent to

$$\begin{cases} 0\le u+\delta_1\le 1, \quad \overline{\tau}\le 1 \\ \\ \dfrac{(u+2N(1-u))\overline{\tau}^2s^2-M/N}{2(1-u)\overline{\tau}s}+\delta_2\le \dfrac{2N-1}{2}\overline{\tau}s \end{cases} \tag{32}$$

and, further, to

$$0\le u+\delta_1\le 1, \quad 2\delta_2\tau s(1-u)+\kappa\tau^2s^2\le\frac{M}{N} \tag{33}$$

where the coefficient κ has been added to stress that all variables have to be in specific units ($ and hectares, in this case).

As inequalities will be saturated we obtain:

$$u^*=1-\delta_1, \tag{34}$$

$$\overline{\tau}=\frac{1}{\kappa s}\left(\sqrt{\delta_1^2\delta_2^2+\kappa\frac{M}{N}}-\delta_1\delta_2\right). \tag{35}$$

It is evident from the above that $\overline{\tau}>0$ for δ_1,$\delta_2>0$. Moreover, for modest budgets, $\overline{\tau}<1$. If the optimal technology switch $\overline{\tau}<1$ then the optimal collective incentive parameter

$$w^*=\frac{2\delta_1}{\kappa}\left(\sqrt{\delta_1^2\delta_2^2+\kappa\frac{M}{N}}-\delta_1\delta_2\right) \tag{36}$$

while the optimal target area can be computed using (31). So, the principal's problem can be solved by calculating the optimal instruments (u^*,w^*,S^*) from (34), (36) and (31), respectively. They will induce $\tau^*=\min(\overline{\tau},1)$ $\times 100\%$ technology switch and the principal's utility level

$$J^*=\frac{N}{\kappa}\left(\sqrt{\delta_1^2\delta_2^2+\kappa\frac{M}{N}}-\delta_1\delta_2\right). \tag{37}$$

The above observations entitle us to formulae a rather general result.

Result 3.1. *For symmetric agents, for every pair of* $(\delta_1, \delta_2) \in (0,1) \times \mathcal{R}_+$ *and* $M < Ns^2$*, there exists a unique solution to the principal's coupled incentive optimisation problem* PCIOP *(defined on page 5).*

To get a feeling for which political variables δ_1, δ_2 enforce what solution, and through which instruments, we provide a numerical example.

Example 3.1. *Suppose that the principal needs to control two symmetric agents whose field areas are* $s = 1$*. The principal disposes of budget* $M = 1.8$*. (If* $M = 2$ *the total technology switch would be possible through an individual incentive programme.) What are the optimal instrument values and what percentage of the fields will be cultivated in the new technology after the technology switch?*

Three pairs of δ_1, δ_2 were selected. Remember that a low value of δ_1 corresponds to a strong individual incentive signal. A low value of δ_2 suggests that agents are sensitive to the difference $f_i - m$ and will react to small payoff improvements.

$$\delta_1 = .6, \quad \delta_2 = .1, \quad J^* = 1.781, \quad \tau^* = 89.06\%, \quad f_i - m = 0.1069,$$
$$[u, w, S] \;=\; [0.4000 \quad 1.0686 \quad 1.2358].$$

$$\delta_1 = .2, \quad \delta_2 = .1, \quad J^* = 1.8578, \quad \tau^* = 92.89\%, \quad f_i - m = 0.0372,$$
$$[u, w, S] \;=\; [0.8000 \quad 0.3715 \quad 1.2933].$$

$$\delta_1 = .2, \quad \delta_2 = .8, \quad J^* = 1.6042, \quad \tau^* = 80.21\%, \quad f_i - m = 0.2567,$$
$$[u, w, S] \;=\; [0.8000 \quad 0.3208 \quad 0.4031].$$

Assigning a larger budget ($M = 1.9$) generates a higher τ^* as follows:

$$\delta_1 = .2, \quad \delta_2 = .8, \quad J^* = 1.6554, \quad \tau^* = 82.77\%, \quad f_i - m = 0.2649$$
$$[u, w, S] \;=\; [0.8000 \quad 0.3311 \quad 0.4416].$$

In accordance with Result 3.1, even very small budgets can generate a technology switch. Take $M \leq .1$; this generates

$$\delta_1 = .2, \quad \delta_2 = .8, \quad J^* = 0.7224, \quad \tau^* = 36.12\%, \quad f_i - m = 0.0678$$
$$[u, w, S] \;=\; [0.8380 \quad 0.1170 \quad -0.0380].$$

The negative $S^* = -.038$ tells us that the agents would have been subsidised even if $\tau = 0$. However, that solution would not be optimal for the players and they would choose $\tau^* = 0.3612$. ◇

Parallel to the analytical solutions (34)-(37), (31) numerical solutions to the non-linear programme max (23) s.t. (24)-(27) were calculated[(6)]. The results obviously coincided, however, the numerical solution provided us with the information which constraints were active. As expected, the first (right hand) (24), third (26) and fourth constraint (27) were active while there was a slack on the other inequalities. An additional feature of a numerical solution is the computation of the Lagrange multipliers (not quoted here), which could help the principal to choose an appropriate pair of δ_1, δ_2.

[(6)]Matlab `fmincon` was used.

4. The solution to a two agent non symmetric game.

Anecdotal evidence speaks against existence of substantially different size farms in a geographic area where economic competition is allowed. However, our CIS could be used even if there were non symmetric agents. We will show how CIS will operate if there are two non symmetric agents (*i.e.*, $N = 2$). There is nothing that could stop us to apply CIS if $N > 2$, however, the analytical solutions become complicated.

4.1. **Uniqueness of Nash equilibrium in the agent problem.** We assume that there are two agents whose cultivation areas and cost coefficients are essentially different; for example consider:

$$s_1 < s_2, \qquad \alpha_1 > \alpha_2.$$

The above relationships reflect the fact that larger farms tend to have lower unitary costs. Considering other relations between cultivation areas and cost coefficients would not impact the qualitative results.

We assume that the joint action reward from CIS has been paid to agents so, $S < \sum_{i=1}^{2} \tau_i^* s_i$. This means that each agent $i = 1, 2$ maximises

$$f_i(\tau_i, \tau_{-i}; u, w, S) = m_i s_i + (u-1)\alpha_i \tau_i^2 s_i^2 + w\left(\sum_{k=1}^{2} \tau_k s_k - S\right) \tag{38}$$

at

$$\overline{\tau}_i = \frac{w}{2\alpha_i(1-u)s_i}. \tag{39}$$

Notice that the first and second order conditions are satisfied at $\overline{\tau}_i$.

We also assume that $u < 1$ for all $i = 1, 2$ which means that the principal does not dispose of a large budget to stimulate the agents individually only. Notice (see (39)) that $u < 1$ implies $\overline{\tau_i} \geq 0$. Moreover $\overline{\tau}_i = 0$ if and only if $w = 0$.

Suppose that the small budget excludes $\tau_i^* = 1$, $i = 1, 2$. If $w = 0$ then we have that $\tau_i^* = \overline{\tau}_i = 0$ is an optimal solution. For $w > 0$, we have that $\tau_i^* = \overline{\tau}_i$ if and only if CIS is profitable for each agent *i.e.*, $f_i(\overline{\tau}_i, \overline{\tau}_j, u, w, S) > m_i s_i$. This means that

$$f_i(\overline{\tau}_i, \overline{\tau}_j, u, w, S) = m_i s_i + \frac{w^2}{4\alpha_i(1-u)} + \frac{w^2}{2\alpha_j(1-u)} - wS > m_i s_i$$

for $i = 1, 2$ and $i \neq j$. Because of $\alpha_1 > \alpha_2$, these two conditions (for $i = 1, 2$) collapse[7] to

$$S < w\left(\frac{1}{4\alpha_2(1-u)} + \frac{1}{2\alpha_1(1-u)}\right). \tag{40}$$

Relationship (40) is a constraint that the principal's instruments have to satisfy.

[7] This is because

$$\frac{1}{4\alpha_1(1-u)} + \frac{1}{2\alpha_2(1-u)} > \frac{1}{4\alpha_2(1-u)} + \frac{1}{2\alpha_1(1-u)}.$$

4.2. **A solution to the principal's problem.** As in the symmetric agent benchmark problem in Section 3.1 we can establish what the budget is that would allow the principal to stimulate agents individually (*i.e.*, $w = 0$) to perform the technology switch. To achieve $\tau_i^* = 1, i = 1, 2$ and if $w = 0$, $u > 1$ is needed. In that case, the necessary budget would have to be $M > s_1^2 + s_2^2$. In the following we consider "small" budgets $M \leq \overline{M} \equiv s_1^2 + s_2^2$. So, the principal's problem (PCIOP) is to find u^*, w^*, S^* such that maximise

$$J = \sum_{i=1}^{2} \tau_i^* s_i = \sum_{i=1}^{2} \frac{w}{2\alpha_i(1-u)} \tag{41}$$

(because $\tau_i^* = \overline{\tau}_i$), subject to (40) and

$$\begin{cases} u < 1, \\ 0 < \dfrac{w}{2\alpha_i(1-u)s_i} \leq 1, \quad i = 1, 2 \\ u((\overline{\tau}_1 s_1)^2 \alpha_1 + (\overline{\tau}_2 s_2)^2 \alpha_2) + 2w(\overline{\tau}_1 s_1 + \overline{\tau}_2 s_2 - S) = M \leq \overline{M}. \end{cases} \tag{42}$$

Notice that $\nabla J \neq 0$ *i.e.*, gradient J cannot be zero. This means that there is no interior solution to the above maximization problem. Moreover, there is no boundary solution because some of the above constraints are strict inequalities.

However, we can easily see that a number of *satisfactory* solutions exist *i.e.*, such that (40) and (42) are satisfied and $J > 0$ (see Definition 3.1). As in the symmetric case we need to introduce slack variables to compute a satisfactory solution.

Before formally introducing slack variables let us identify a particular satisfactory solution that is simple yet economically interpretable.

Example 4.1. *Suppose no individual incentives will be paid to players so,* $u^* = 0$. *Consider* S^* *and* $w^* > 0$ *as follows*

$$S^* = \frac{w^*}{4}\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right). \tag{43}$$

This choice satisfies (40) because

$$\frac{w^*}{4}\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right) < \frac{w^*}{4}\left(\frac{1}{\alpha_1} + \frac{2}{\alpha_2}\right). \tag{44}$$

If we substitute $u^* = 0$ *and* S^* *in the budget equation, which is the last equation of (42), we obtain:*

$$M = \frac{(w^*)^2}{4}\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right)$$

so,

$$w^* = 2\sqrt{\frac{\alpha_1\alpha_2 M}{\alpha_1 + \alpha_2}} \quad \text{and} \quad S^* = \frac{\sqrt{M}}{2}\sqrt{\frac{1}{\alpha_1} + \frac{1}{\alpha_2}}. \tag{45}$$

Finally

$$\tau_i^* = \sqrt{\frac{\alpha_1\alpha_2 M}{(\alpha_1 + \alpha_2)\alpha_i^2 s_i^2}} = \sqrt{\frac{\alpha_j M}{(\alpha_1 + \alpha_2)\alpha_i s_i^2}} = \frac{1}{s_i}\sqrt{\frac{\alpha_j}{\alpha_i}}\sqrt{\frac{M}{\alpha_1 + \alpha_2}}. \tag{46}$$

Spending $M \in (0, \overline{M}]$ *guarantees* $0 < \tau_i^* < 1$.

This is an interesting solution. It tells us that, for a given budget M, players' efforts are inverse proportional to their own cultivation areas. The efforts depend also on the relative competitiveness of the players. For a given competitor's cost structure, the player's effort increases as his cost coefficient diminishes (*i.e.*, his competitiveness improves).

As in the symmetric case, we can introduce slack variables into the strict inequality constrains and solve the principal's problem for various levels of the slacks. We add $\delta_1 > 0$ to the left hand side of the first inequality in (42) to obtain

$$u + \delta_1 = 1 \tag{47}$$

with $0 < \delta_1 \leq \alpha_2$. The other slack variable is $\delta_2 > 0$ and is involved in satisfying (40). We put

$$S + \delta_2 = \frac{w}{4\alpha_2(1-u)} + \frac{w}{2\alpha_1(1-u)}. \tag{48}$$

As in Section 3, the slacks $\delta_1 > 0$, $\delta_2 > 0$ are the principal's policy variables. The first one determines the highest individual compensation and the second one controls the gain a player can obtain from participating in the programme.

We need to satisfy (40) *i.e.*,

$$0 \leq S < \frac{w}{4\alpha_2(1-u)} + \frac{w}{2\alpha_1(1-u)}.$$

Using (48) and dropping S yields

$$0 < \delta_2 \leq \frac{w}{4\alpha_2(1-u)} + \frac{w}{2\alpha_1(1-u)}.$$

Substituting (47) and (48) in the last equation of (42) (budget condition), we obtain:

$$w^2 \left(\frac{(1-\delta_1)(\alpha_1+\alpha_2) + 2\alpha_2\delta_1}{4\delta_1^2\alpha_1\alpha_2} \right) + 2\delta_2 w = M \tag{49}$$

Introducing

$$A \equiv \frac{(1-\delta_1)(\alpha_1+\alpha_2) + 2\alpha_2\delta_1}{4\delta_1^2\alpha_1\alpha_2} > 0$$

we obtain

$$u^* = 1 - \delta_1, \qquad w^* = \frac{-\delta_2 + \sqrt{\delta_2^2 + MA}}{A} \tag{50}$$

and S^* resulting from (48). The corresponding values of τ_i^* and J^* can be computed from (39) and (41); they are obviously greater than 0.

The above observations entitle us to formulae a result valid for a two player non symmetric PCIOP.

Result 4.1. *For two non symmetric agents, for every pair of $(\delta_1, \delta_2) \in (0, \alpha_2) \times \mathcal{R}_+$ and $M < s_1^2 + s_2^2$, there exists a unique solution to the principal's coupled incentive optimisation problem.*

Remark 4.1. *It is easy to see that if $\alpha_1 = \alpha_2 = 1$ we obtain*

$$A = \frac{1}{2\delta_1^2}, \quad w = 2\delta_1(\sqrt{\delta_2^2\delta_1^2 + M/2} - \delta_2\delta_1).$$

That is, Results 3.1 and 4.1 coincide for a two player symmetric PCIOP.

Remark 4.2. *Even if the principal could be satisfied with a weak solution (see page 5) a strong solution (39) for this asymmetric case was obtained. We conjecture that this is a particular feature of the incentive function* $\Pi_i(\dots)$ *(3).*

5. Concluding remarks

A team effort stimulating approach. We have presented a coupled incentive scheme (CIS) useful for a principal who disposes of scarce financial means and wants to induce a technological change among a group of agents (or "followers").

If the principal uses an amount of money to subsidise the agents and applies CIS, then their actions are coordinated and result in a higher utility to the principal and agents than if the same amount were spent on individual incentives only. CIS is practical in that it is based on three interpretable instruments (u, w, S) that can be easily communicated to the agents. The principal has also two policy variables (δ_1, δ_2) that might be chosen depending on idiosyncratic characteristics of the agents. Also, the scheme has an "economic" feature in that low cost agents react with greater efforts to a subsidy than their less competitive counterparts.

Implicit bilateral contracts. Because the collective reward depends on the other players' behaviour, agents who are "slow" in the transition process will be lobbied to speed up by their faster counterparts. In other words, bilateral contracts will spread from agent to agent, with no additional cost to the principal.

Asymmetric information extension. CIS has been presented for a *symmetric information* case. However, CIS can be useful even if the principal's knowledge of the agents' models is limited. In that case, an iterative approach becomes obvious. If a given instrument set brings no desired technology change, the principal will revise the agent models and recompute the policy instruments.

A dynamic game extension. It is easy to see that CIS is applicable in case the agent's payoff (1) comprises a future discounted profits $\frac{m_i s_i}{1-\varrho}$ where $0 < \varrho < 1$ is the discount factor. Indeed, this term is independent of the policy instruments and will not have an impact on the optimisation procedure.

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