Spatial evolution of social norms in a common-pool resource game

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Abstract

Conditions for the emergence of cooperation are studied in a spatial common-pool resource game. A fixed number of harvesters are located on a spatial grid. Harvesters choose among three strategies: defection; cooperation; and, enforcement. Individual payoffs are affected by both global factors: namely, aggregate harvest and resource stock level, and local factors, such as the imposition of sanctions on neighbors by enforcers. The evolution of strategies in the population is driven by social learning through imitation. Numerous equilibria exist in these settings. An important new finding is that clusters of cooperators and enforcers can survive among large groups of defectors. We discuss how the results contrast with the non-spatial, but otherwise similar, game of Sethi and Somanathan (1996).

Keywords: Common property, evolutionary game theory, local interactions game, self-organization, cooperation.
JEL-classification: C72; Q2.

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1. Introduction

The management of common-pool resources (CPRs), such as forests and groundwater basins, is characterized by a conflict between individual and social interests. While the collective interest of the group is to limit harvesting to a sustainable level, the combined actions of individual harvesters pursuing their own interest inevitably result in a suboptimal outcome, characterized by excessive exploitation of the resource (Dasgupta and Heal, 1979: Chapter 3). This dilemma can take the form of a game played by two types of harvesters: namely, defectors and cooperators, adopting high and low levels of harvest, respectively (Ostrom et al., 1994). In the finitely repeated CPR game, unilateral defection is the unique Nash equilibrium, and privatization of the resource remains often the preferred management issue.

In recent years, many studies have shown that cooperation between harvesters can be achieved in a self-organized or self-governed way (Ostrom, 1990). Accordingly, cooperative outcomes can be reached as long as norms or rules, such as trust, reward or punishment, prevail in the group. The imposition of sanctions on harvesters that adopt excessive exploitation levels has proved to be particularly effective in sustaining cooperation, even in the case of non-repeated interactions among unrelated individuals or in very large groups. Evidence from the field suggests that some agents voluntarily engage in 'altruistic punishment', i.e. in punishing free-riders, even if this implies an individual cost. Often, a small proportion of these altruistic punishers is sufficient to enforce cooperation in the group (Fehr and Gächter, 2002).

A rare theoretical analysis of the role of altruistic punishers or 'enforcers' in solving CPR dilemmas is presented in Sethi and Somanathan (1996). They consider an evolutionary CPR game played by a fixed population of players distributed among three strategies: defection; cooperation; and, enforcement. Payoffs of defectors are lowered by a sanction that depends on the number of enforcers in the population, while enforcers bear a cost which depends upon the total number of defectors. Players experience social learning and imitate the strategy that yields the highest payoffs. The evolution of strategies is captured by a replicator dynamics equation, which describes the increasing share of the best-performing strategy in the population as a result of agents imitating it. Sethi and Somanathan combine replicator and resource dynamics to show how changes in the resource stock affect harvesting behavior, and vice versa. They identify two equilibria in the system: namely, a final population composed of only defectors, and a cooperative equilibrium with only cooperators and enforcers.

The present model adds a major innovation to Sethi and Somanathan's work, i.e. spatial interactions between the agents. While Sethi and Somanathan focus mainly on aggregate population dynamics, our model more realistically emphasizes the role of locality. The approach is, therefore, unlike replicator dynamics, based on the explicit modeling of micro-interactions among individuals. The objective of the present paper is to find conditions for the emergence of cooperative equilibria in the spatial evolutionary CPR game. Particular attention is also given to the role of resource dynamics in the system. An implicit objective is then to test the robustness of Sethi and Somanathan's findings in an evolutionary game with spatial interactions. A major result of our model is that the management of CPR issues can be solved by the introduction of local punishment between the agents.

There are several reasons for emphasizing spatial aspects in resource issues. First, natural resources are intrinsically spatially distributed. Second, resource issues are characterized by spatial connectivity and interactions between the agents. Pollution, for instance, first spreads to adjacent areas. Farmers can also be connected through networks of irrigation pipes, so that the pumping of water in one location will affect nearby farmers. Third, the spatial proximity of the agents plays a major role in monitoring. In many cases, actual free-riders can only be observed by close neighbors. For instance, in the Mormon community around Salt Lake City, each irrigator receives water from a close neighbor and passes it on to another, so that each irrigator then monitors the next one (Smith, 2000). Similarly, in large forest areas, adjacent neighbors are the only ones who can adequately point out 'cheaters' who try to expand their fence area.

The present CPR game with spatial interactions has connections with two different bodies of literature. First, it relates to studies on local interactions games within the field of evolutionary game theory (Eshel et al., 1998; Lindgren and Nordahl, 1994; see Nowak and Sigmund, 1999, for an overview). A well-known result in this literature is that the introduction of local interactions between the agents lead to the survival of cooperation. The game presented here, however, combines both local and global interactions, leading to less immediate results. The structure of our game most closely relates to Nowak and May (1992). Just like in their model, agents are disposed on a grid and follow a fixed learning rule. Further similarities are the absence of mutations and the use of simulation techniques.

Second, the approach presented here relates to the wide body of experimental and theoretical work on the evolution of social norms in CPR games (Ostrom et al., 1994). In this literature, only a very few distinctive studies have formally analyzed an evolutionary CPR game with a variable resource stock (Akiyama and Kaneko, 2000; Janssen, 2001; Sethi and Somanathan, 1996). Among these studies, Sethi and Somanathan's analysis provides an attractive benchmark because of its simplicity and explanatory power. The present model adds insights to this literature as it illustrates how results from local interactions games can be applied to the management problem of common-pool resources.

The paper is organized as follows. Section 2 presents the benchmark neoclassical CPR game and its evolutionary spatial version. Section 3 identifies the equilibria of the system with a fixed resource level and shows how these contrast with Sethi and Somanathan's findings. In addition, we test for the effects of changes in parameters. Section 4 discusses the impact of adding resource dynamics to the evolutionary system. Section 5 concludes.

2. The Model

2.1 The CPR game

The benchmark neoclassical CPR game (Dasgupta and Heal, 1979: Chapter 3; Chichilnisky, 1994; Ostrom et al., 1994) is first presented briefly. This game relies on the assumption of maximizing behavior and thus contrasts sharply with evolutionary versions of the game that will be discussed further on.

A fixed population of m ($m > 1$) harvesters has access to a common-pool of resources. The individual effort level of harvester i ($i = 1, \ldots, m$) in period t $(t = 0, 1, \ldots)$ is denoted by x_{it}. The effort level reflects the intensity of the harvesting activity. Total effort of the population is simply the sum of all individual efforts:

$$X_t = \sum_{i}^{m} x_{it}. \tag{1}$$

Aggregate harvest H is a strictly concave and increasing function of total effort X_t and of the total stock of natural resources N_t. In a first stage, we ignore resource dynamics to simplify the analysis. We fix the resource stock N_t at N_0, so that the resource stock is constant at all times.[1] Suppressing N_0, the harvest rate can be written as a function of X only and we can ignore time subscripts for all variables. We define F as the total harvest rate, which is strictly concave and increasing with $F(0) = 0$, $F'(0) > w$, $F'(\infty) < w$, where w is the constant cost per unit of effort employed.

$$H(X, N_0) = F(X). \tag{2}$$

Figure 1 illustrates the CPR game. Each agent receives a share of total profits in proportion to the amount of effort invested. Individual profits are then given by:

$$\pi_i(x_i, X) = \frac{x_i}{X} F(X) - w x_i, \tag{3}$$

where it is assumed that the price of the harvested commodity is the numeraire. Individual payoffs are thus a function of global factors like the harvesting behavior of all agents. Indeed, as aggregate harvest is larger, or in other words as there are more defectors in the population, individual payoffs

[1] This assumption holds if the renegenerative capacity of the resource is large or if we consider only one-shot games.

decrease. Thus, the level of aggregate profits is:

$$\Pi = \sum_{i}^{m} \pi_i(x_i, X) = F(X) - wX. \tag{4}$$

At X_P, which is the Pareto efficient level of effort defined by $F'(X_P) = w$, total profits are maximized and the resource is used efficiently. When access to the resource is open to everyone, entry of harvesters continues until $F(X_O) = wX_O$, which is the point where no harvester enjoys positive profits.

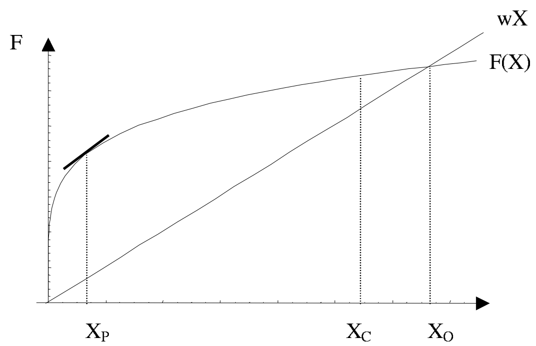

Figure 1. The CPR game.

In the case of a fixed population of agents, maximization of individual profits in the CPR game leads to a suboptimal outcome. This is a standard result that derives directly from the optimization problem. The Nash equilibrium X_C is characterized by $X_P \leq X_C \leq X_O$. It is unique and inefficient ($X_C \geq X_P$) but yields positive rents ($X_C \leq X_O$).

An evolutionary version of this standard framework has been studied by Sethi and Somanathan (1996). In an evolutionary game, harvesters are boundedly rational, i.e. they cannot solve the optimization problem. Instead, they rely on simple forms of social learning like imitation of the best-performing strategy. Diffusion of strategies occurs through the learning process and drives the evolution of strategies towards a common-pool equilibrium that falls between the benchmark equilibrium aggregate harvest rates X_P and X_O. In the remainder of the paper, we base our analysis on a similar evolutionary framework but introduce spatial interactions between the agents.

2.2 A spatial evolutionary CPR game

The spatial evolutionary CPR game embodies the following four main features:

(i) *Space*. A fixed population of m harvesters is distributed on a two-dimensional torus. A torus is a lattice whose corners are pasted together to ensure that all cells are connected, so that there are no edge-effects. Each single cell is occupied by a player and the position of each agent is fixed during the game. We define the neighborhood of each player as the set of the four closest neighbors, located North, South, East, and West of the player, as shown in Figure 2.

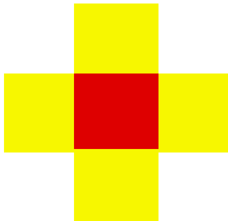

Figure 2. A neighborhood.

(ii) *Strategies*. Just as in Sethi and Somanathan (1996), we consider three possible different types of strategy for every player: defection; cooperation; and, enforcement. Enforcers punish defectors. The players have no understanding of the game, even though they interact repeatedly. In other words, they do not foresee that larger profits can be obtained when everyone cooperates.[2]

Both cooperators and enforcers choose a low level of effort x_L to avoid overexploitation of the resource, while defectors choose a high level of effort x_H, which yields higher profits ceteris paribus. Individual effort levels x_L and x_H are fixed such that:

$$X_P \leq mx_L \leq mx_H \leq X_O, \tag{5}$$

[2]This implies that the imposition of sanctions by enforcers is not motivated by their long run self-interest in obtaining larger payoffs by eliminating defectors. Instead, this behavior is initially motivated by moral values of altruism or preservation of the resource, or simply by negative feelings towards defectors.

where, as defined in the previous section, X_P is the socially optimal level of total harvest, X_O is the total level of harvest in an open-access situation, and mx_L (mx_H) is the total harvest when all agents harvest low (high).

In each round τ ($\tau = 0, 1, \ldots$) of the game, the aggregate effort X_τ is calculated according to the distribution of strategies in the population:

$$X_\tau = m_{D,\tau} x_H + (m_{E,\tau} + m_{C,\tau}) x_L, \tag{6}$$

where $m_{D,\tau}$, $m_{E,\tau}$ and $m_{C,\tau}$ are, respectively, the number of defectors, enforcers and cooperators present in the system in round τ.

Enforcers punish all defectors located in their neighborhood. Monitoring is thus conducted locally among close neighbors. This is a major difference with Sethi and Somanathan's model in which sanctions are imposed by the group of enforcers on the group of defectors at the aggregate level. Here, each enforcer punishing a defector bears a cost γ per defector, while each defector being punished by an enforcer pays a sanction δ. The maximum sanction falling on a defector is thus 4δ, when he is surrounded by four enforcers. Similarly, the maximum cost borne by an enforcer is 4γ.

These specific settings of the local monitoring system draw inspiration from examples of common-pool resource communities described by Ostrom (1990). In our community, an enforcing agent can bring charges against one or several of his defecting neighbors in front of a local court. To do so, he pays some administrative costs for each defector and, in return, the court decides on the fine to be paid by defectors. The total fine falling on a defector increases with the number of his neighbors who have brought charges against him.[3] Finally, costs and fines are paid directly to the court, implying that individual agents cannot extract any private monetary benefits from the fines collected.

Payoffs are calculated according to a player's current strategy, the aggregate effort X_τ

[3]Note that this is only one possible interpretation of the monitoring system. Another one is that fines falling on a defector increase with the probability of being caught.

and the strategies located in the neighborhood of each agent. We use the notations Dk to refer to a defector punished k times and thus paying the sanction $k\delta$. Similarly, El refers to an enforcer surrounded by l defectors. Payoffs are then formalized as follows:

$$\pi_{C,\tau} = \frac{x_L}{X_\tau}\left(H(X_\tau, N_0) - wX_\tau\right) \tag{7}$$

$$\pi_{Dk,\tau} = \frac{x_H}{X_\tau}\left(H(X_\tau, N_0) - wX_\tau\right) - k\delta \tag{8}$$

$$\pi_{El,\tau} = \frac{x_L}{X_\tau}\left(H(X_\tau, N_0) - wX_\tau\right) - l\gamma, \tag{9}$$

where $\pi_{j,\tau}$ denotes payoffs in round τ with strategy j ($j \in \{C, D, E\}$), k ($k \in \{0, 1, 2, 3, 4\}$) denotes the number of enforcers in the neighborhood of any given defector, and l ($l \in \{0, 1, 2, 3, 4\}$) the number of defectors in the neighborhood of any given enforcer. Thus, $\pi_{D3,2}$ refers to the payoffs of a defector surrounded by three enforcers in round $\tau = 2$. Obviously, $\pi_{Dh,\tau} > \pi_{Dh+1,\tau}$ and $\pi_{Eh,\tau} > \pi_{Eh+1,\tau}$ ($h \in \{0, 1, 2, 3\}$). In addition, from (5), (7), (8), and (9), we have $\pi_{D0,\tau} > \pi_{C,\tau} > \pi_{E1,\tau}$ and $\pi_{E0,\tau} = \pi_{C,\tau}$ for all τ.

We make one additional assumption regarding the level of punishment in the system, namely:

$$\pi_{D4,\tau} < \pi_{E0,\tau}, \quad \forall\tau. \tag{10}$$

This implies that a defector incurring the maximum sanction level earns lower payoffs than either any enforcer who is not punishing (E0) or a cooperator, who has no defecting neighbors. This is to ensure that enforcers can actually win over defectors.

In the spatial model, we further assume that $H(X, N_0)$ is a Cobb-Douglas production function:

$$H(X, N_0) = \alpha X^\beta N_0^{1-\beta} \qquad \alpha > 0, \quad 0 < \beta < 1. \tag{11}$$

As discussed in Section 2.1, we can solve $F'(X_P) = w$ and $F(X_O) = wX_O$ to find X_P and X_O, respectively. This gives:

$$X_P = (\frac{w}{\alpha\beta})^{\frac{1}{\beta-1}} N_0 \quad \text{and} \qquad X_O = (\frac{w}{\alpha})^{\frac{1}{\beta-1}} N_0. \tag{12}$$

(iii) *Learning*. Updating of strategies is driven by expectations of larger profits in the next period.[4] In each round, every player updates his current strategy by imitating the strategy that yields the highest payoffs in his neighborhood. Similar learning rules, where agents simply pick up the strategy with the largest score, were used by Axelrod (1984, Chapter 8) and Nowak and May (1992). When the best strategy in the neighborhood is identical to the player's current strategy, the player sticks to his current strategy. When multiple strategies other than the player's current strategy yield the largest (equal) payoffs in the neighborhood, i.e. when there is a tie between two strategies, the player randomizes among these strategies with probability $p = 0.5$.[5]

(iv) *Time*. We assume that agents exhibit synchronous behavior. In other words, interactions and learning occur simultaneously. Huberman and Glance (1993) have shown that adopting asynchronous learning may lead to different outcomes than when considering synchronous learning. In our model, however, the existence of a global 'clock' that governs the learning process is justified by adopting seasonal harvesting of the resource. The assumption that all harvesters modify their strategy at the beginning of each new season is thus realistic when dealing with resource issues.

The following sequence describes a round of the game:

[4]This implies that enforcers (defectors) can actually switch suddenly to defection (enforcement). According to Bowles and Gintis (2002), such strong deviations in behavior can be observed in reality and are often eased by the impact of emotions. Shame, for instance, can help to justify an extreme deviation from defection to enforcement. Similarly, we can interpret the deviation from enforcement to defection as being caused by feelings of frustration.

[5]There can only be two (equal) best strategies between which the player randomizes, given that there are three possible strategies in the system, and the player sticks to his current strategy when it yields the highest payoff.

1. Aggregate effort X_τ is computed given the distribution of strategies in the population.
2. Aggregate harvest $H(X_\tau, N_0)$[6] is calculated.
3. Individual payoffs are computed for all agents, given $H(X_\tau, N_0)$, the strategy chosen by each single agent and the distribution of strategies in his neighborhood.
4. Agents synchronously observe the performance of their neighbors' strategies and decide whether to stick to their current strategy or adopt a more successful one observed in their neighborhood. This learning process yields a new distribution of strategies in the population.

3. Spatial evolutionary dynamics

3.1 An example of spatial evolution

An example of the evolution of social norms in the spatial model is given in Figure 3. Here, the spatial population of 100 harvesters is initially distributed among 5 defectors, 5 enforcers and 90 cooperators.

[6] Recall that, for now, we ignore resource dynamics, so that the resource stock N_0 is available at the beginning of every round.

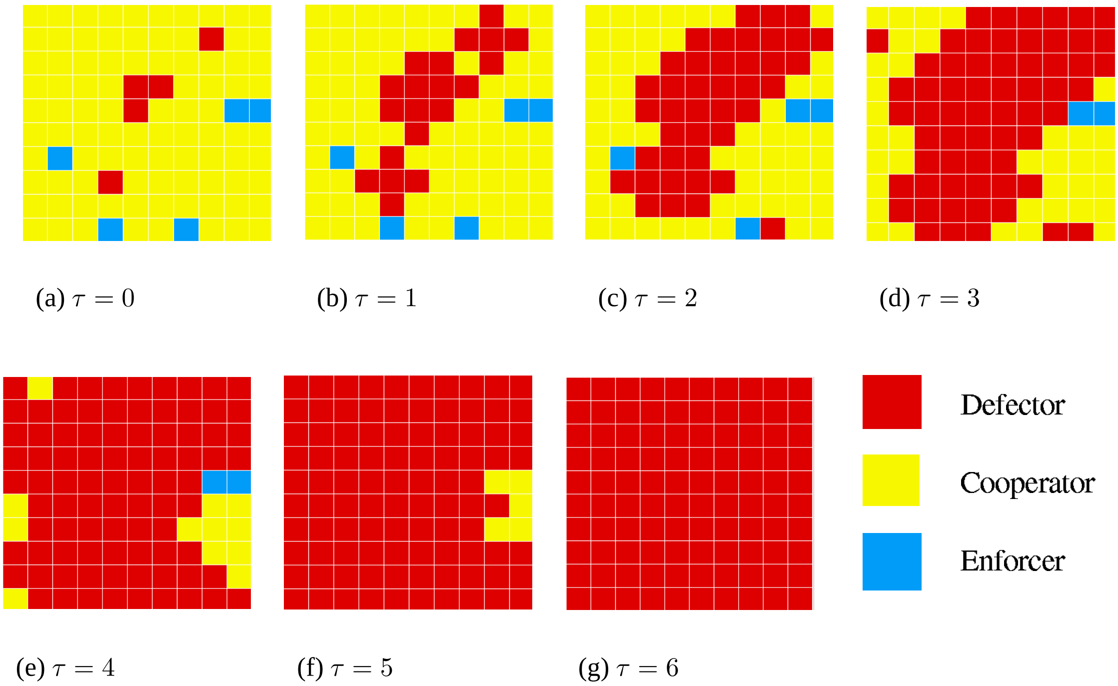

Figure 3. An example of spatial evolution of strategies.

Along the sequence of spatial grids, defectors first spread into their neighboring areas and progressively invade the whole grid. After 6 rounds, the population is left with only defectors and no further evolution of strategy is observed.

3.2 Definition of equilibria

In this section, we provide definitions of equilibria[7] in the system. We distinguish between local and global equilibria as follows:

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[7]Note that there are many diverse notions of equilibria in evolutionary game theory. One branch of the literature argues that considering stochastically-stable states allows a sharp reduction in the set of equilibria. The pioneering work of Foster and Young (1990) discusses stochastic games in which players have a small probability to make a mistake during the learning process. They define stochastically-stable states as states, or neighborhoods of states, in which the system will almost certainly settle as the noise tends to zero. Young (1993: Chapter 6) shows how stochastically-stable states can be identified in local interactions games. The literature on stochastic games is, however, beyond the scope of the present study since the purpose is to focus on Sethi and Somanathan's (1996) basic framework.

Definition 1 *A player is said to be in a local equilibrium when his current strategy earns no lower profits than any different strategy located in his neighborhood.*

Definition 2 *A global equilibrium is a spatial distribution of strategies in which all players are in a local equilibrium. Each global equilibrium is associated with a strategy equilibrium, defined as the final number of defectors, enforcers and cooperators in the system.*

In other words, a global equilibrium is a spatial distribution of strategies in which no player has an incentive to change strategy. Note that Definition 1 suggests that it is possible for a neighboring strategy to earn a larger profit than the player's current strategy whenever these strategies are identical. For example, enforcers $E1$ are in a local equilibrium, even if they earn the lowest payoffs, as long as their neighbor with the highest payoff is an enforcer $E0$. The same reasoning holds for defectors $D1$ and $D0$. This contrasts with Sethi and Somanathan's model, in which agents belonging to the same (sub)group always earn equal payoffs, since sanctions and costs falling on defectors and enforcers are determined at the aggregate level. Additional levels of information are thus available in the spatial game with micro-interactions.

3.3 Simulation results

3.3.1 Notation and parameter values

We use D, C, E to refer to equilibria composed of only defectors, cooperators or enforcers, respectively. In addition, DE, CE and CDE refer to equilibria composed of the corresponding mix of strategies.[8]

Which equilibrium emerges depends on three factors:

[8]Note that, in contrast with D, C and E-equilibria, many diverse strategy equilibria can be defined for each DE, CE or CDE-equilibria. The quantitative distinction between different strategy equilibria will only be relevant in Section 4 with the introduction of resource dynamics. In addition, note that there cannot be any CD-equilibrium for the simple reason that this corresponds to the case where there is no local punishment between the agents. In this case, defectors are never punished and spread quickly over the lattice.

1. The initial share of each strategy in the population. Strategy shares in round τ are denoted by $z_\tau = (\frac{m_{D,\tau}}{m}, \frac{m_{E,\tau}}{m}, \frac{m_{C,\tau}}{m})$, which corresponds to a population composed of a mix of m_D defectors, m_E enforcers and m_C cooperators in round τ. We will study how different initial shares (z_0) lead to diverse types of equilibria in Section 3.3.3.

2. The initial spatial distribution of strategies. Initially, strategies either form clusters or are scattered irregularly. Section 3.3.2 studies how different equilibria can be reached by varying the initial spatial arrangement of the strategies, while initial shares remain fixed at z_0.

3. Parameter values. Most of the simulations were performed on a 10x10 ($m = 100$) spatial grid. A simulation run corresponds to 50 time steps, which appears to be sufficiently long for the system to settle into an equilibrium. The following parameters were used:

$$\begin{array}{ll} \alpha = 0.2 & \beta = 0.2 \\ N_0 = 500 & w = 0.2 \\ x_H = 4 & x_L = 2 \\ \delta = 0.4 & \gamma = 0.1. \end{array} \tag{13}$$

   Given the other parameter values, the levels of harvest x_H and x_L satisfy (5).[9] The level of sanction $\delta = 0.4$ satisfies condition (10).[10] A sensitivity analysis of critical parameters is conducted in Section 3.3.4.

[9] $X_P = 67$ and $X_O = 500$ according to (12).

[10] Condition (10) is rewritten as:

$$x_H \left(\frac{\alpha X_\tau^\beta N_0^{1-\beta}}{X_\tau} - w \right) - 4\delta < x_L \left(\frac{\alpha X_\tau^\beta N_0^{1-\beta}}{X_\tau} - w \right).$$

After simplification, it follows that this is satisfied for all X_τ, i.e. in every round τ, whenever the following condition holds:

$$\left[\frac{1}{\alpha} \left(\frac{4\delta}{x_H - x_L} + w \right) \right]^{\frac{1}{\beta-1}} N_0 < m x_H.$$

3.3.2 The effects of the initial spatial distribution of strategies

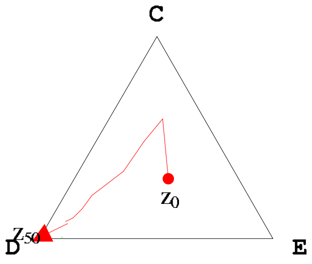
In this section, we study the impact of the initial spatial arrangement of strategies on convergence to equilibria. For this purpose, we use a procedure that uniformly randomizes[11] the spatial distribution of the strategies, while keeping the shares of each strategy in the population fixed.[12] We use simplex representations to present our results. In a simplex, each point corresponds to a three-dimensional coordinate.

In each graph in Figure 4, the system starts near the center[13] of the simplex at point $z_0 = (0.30; 0.40, 0.30)$, i.e. with an initial population composed of 30 defectors, 40 enforcers and 30 cooperators. Each simplex corresponds to a different random initial spatial arrangement of the strategies. Arrows indicate the evolution of strategy shares over time and σ denotes the number of rounds after which an equilibrium is reached.

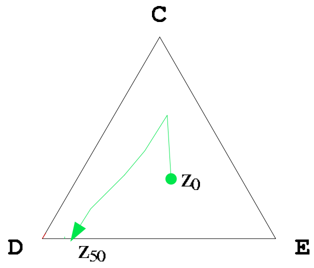
[11]This means that we randomize according to the uniform distribution, where each spatial distribution has an equal probability.

[12]Since the process is random, it is possible to obtain the same spatial settings twice. This is, however, unlikely to happen for a large population (100 agents) and a limited number of runs (100 runs), as conducted in our model.

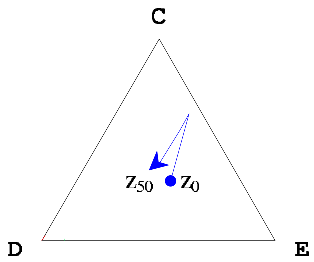
[13]It will be shown in Section 3.3.3 that choosing initial coordinates situated in the center of the simplex, while varying the spatial distribution, leads to the possible emergence of all types of equilibria (except C and E-equilibria).



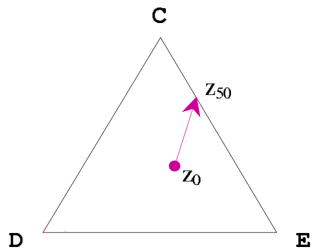
(a) D-equilibrium,
$z_{50} = (1, 0, 0)$,
$\sigma_D = 13$



(b) DE-equilibrium,
$z_{50} = (0.91, 0.09, 0)$,
$\sigma_{DE} = 10$



(c) CDE-equilibrium,
$z_{50} = (0.37, 0.29, 0.34)$,
$\sigma_{CDE} = 9$



(d) CE-equilibrium,
$z_{50} = (0, 0.26, 0.74)$,
$\sigma_{CE} = 3$

Figure 4. Evolution of strategy shares over time for four different initial spatial distributions of strategies, with $z_0 = (0.30, 0.40, 0.30)$.

The evolution of strategy shares is governed by two forces. First, defectors who are subject to severe sanctions imitate enforcers located in their neighborhood. In some sense, defectors are ‘eliminated’ by enforcers, causing the number of defectors to decrease. Second, enforcers who punish at least one defector switch to cooperation when cooperators are located in their neighborhood. In other words, cooperators ‘eliminate’ enforcers and the number of cooperators increases. Accordingly, in all graphs of Figure 4, the initial change in strategy shares corresponds to a sharp increase in the number of cooperators and a drop in the number of defectors and enforcers. The final spatial outcome depends on which force dominates. If cooperators quickly eliminate many enforcers, ultimately, defectors will either spread completely (Figure 4a) or partially (Figure 4b and 4c) through the spatial grid. If, instead, enforcers quickly eliminate defectors, the system will converge to a cooperative equilibrium with only cooperators and enforcers left (Figure 4d).

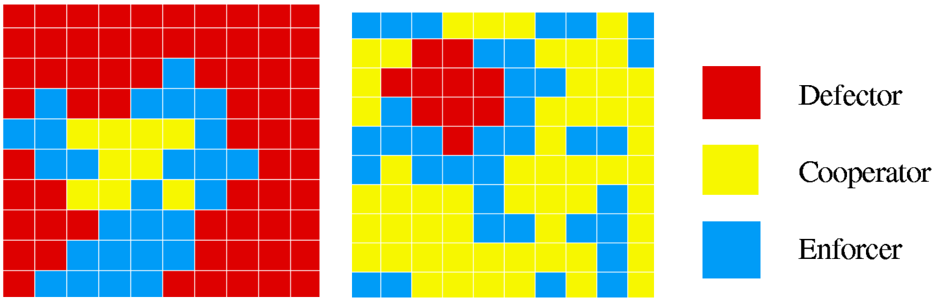
As shown by the position of the point z_{50}, the system can potentially converge to D, DE, CE or CDE-equilibria.[14]ote that the CE-equilibrium (just like the C or E-equilibrium) is not stochastically stable in the sense defined by Foster and Young (1990) (see footnote 7). Indeed, a single perturbation of the system, due for instance to a single harvester switching to defection by mistake, would make such an equilibrium quickly collapse and converge to a D-equilibrium. This, however, is not necessarily true for DE and CDE-equilibria. This suggests that which equilibrium actually emerges depends crucially on the initial spatial location of the strategies on the lattice. Note that both the type of equilibrium and the number of rounds after which an equilibrium is reached (indicated by σ) differ.

In general, the coexistence of several strategies in the long run is favored by the formation of 'clusters'. A cluster is a spatial configuration composed of a central agent and its 4 immediate neighbors. It can be shown intuitively that a 'defecting cluster' composed of 5 defectors can always survive in a large population of cooperators and enforcers. Indeed, the central defector $D0$ in some sense 'protects' his neighboring defectors. Even when they are subject to severe sanctions, they will stick to defection, since their $D0$ neighbor always earns the largest payoffs. Similarly, the survival of enforcers and cooperators in a group of defectors is facilitated by the formation of 'cooperative clusters', i.e. a group of 4 enforcers or cooperators around a central enforcer $E0$. In such a cluster, neighboring enforcers who punish many defectors are protected by the presence of the central enforcer $E0$. Similarly, neighboring cooperators stick to their strategy, since enforcers $E0$ and cooperators earn equal payoffs, as long as defectors located next to cooperators are themselves punished and earn less than the central $E0$. This suggests that the survival of cooperators is strongly related to the number of enforcers located in their neighborhood.

Defecting or cooperative clusters can concatenate to form larger groups. In large cooperative groups, cooperators hold a central position and are then protected by a shell of cooperative clusters formed around enforcers $E0$. An example of a large cooperative group coexisting in a population

[14]N

of defectors is given in Figure 5a. Similarly, Figure 5b shows a defecting group coexisting in a population of cooperators and enforcers.



(a) Cooperating group in a large population of defectors

(b) Defecting group in a large population of cooperators and enforcers

Figure 5. Examples of clustering in CDE-equilibria, at $\tau = 50$, with $z_0 = (0.30; 0.40; 0.30)$.

These spatial considerations show that the inclusion of a spatial dimension, or a more micro approach, leads to major differences when compared with Sethi and Somanathan's results. While in their model only two equilibria, D and CE, occur, in the spatial model additional equilibria can be observed: namely, DE and CDE-equilibria. In Sethi and Somanathan's game, enforcers always earn lower profits than cooperators as long as there are some defectors in the population, so that, ultimately, they will be completely eliminated. Instead, in the spatial game with micro-interactions, enforcers $E1$ to $E4$ can 'survive' in the long run by forming clusters. This is a crucial element for the occurrence of DE and CDE-equilibria.

3.3.3 The effects of initial strategy shares in the population

In this section, we study the effects of variation in initial strategy shares. To reduce the number of runs necessary to cover the whole simplex, only initial strategy shares that are multiples of 0.05 are considered. The set of initial coordinates $Z = \{(1; 0; 0), (0.95; 0.05; 0), \ldots, (0.30; 0.40; 0.30), \ldots,$

$(0; 0.05; 0.95)(0; 0; 1)\}$ is composed of 231 coordinates z_0. For every $z_0 \in Z$, we compute 100 runs of 50 rounds, each run corresponding to a random spatial distribution. In total, therefore, 23,100 runs are necessary to cover the set Z.

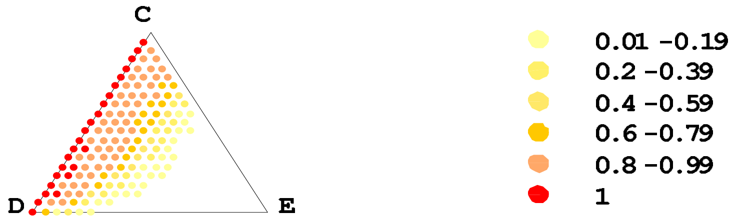
Figure 6 provides results about the frequencies of occurrence of each equilibrium for each initial strategy share, where every $z_0 \in Z$ is represented by a dot. The grey-black scale indicates the frequency of occurrence of each type of equilibrium out of 100 random spatial distributions. A black colored coordinate indicates that, starting with the respective z_0, all runs converge to the given type of equilibrium.

As expected, Figure 6a shows that D-equilibria are more easily achieved for initial populations with few enforcers and, inversely, CE-equilibria are more likely to be reached for initial populations composed of many enforcers (Figure 6b). This is consistent with Sethi and Somanathan's findings. Second, DE-equilibria are most likely to be achieved for middle-range initial shares with a slight majority of defectors (Figure 6c), while CDE-equilibria are most frequently achieved for middle-range initial shares with a slight majority of enforcers (Figure 6d). This makes sense intuitively, since a large number of enforcers need to surround cooperators to allow CDE-equilibria to emerge, as explained in Section 3.3.2. When there are more defectors in the system, cooperators find it difficult to survive, and DE-equilibria emerge.

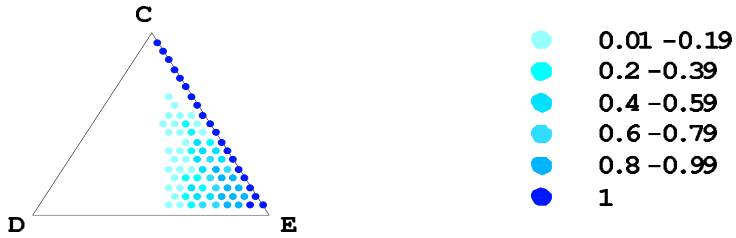
In this example, we find that, on average, 41% of the runs (out of 23,100) converge to a D-equilibrium, 18% converge to a CE-equilibrium, 18% to a DE-equilibrium, 20% to a CDE-equilibrium, 3% to a E-equilibrium[15] and 0.4% to a C-equilibrium.[16]

[15]Note that E and C equilibria are not studied by Sethi and Somanathan as, in their analysis, initial shares are distributed among the three strategies.

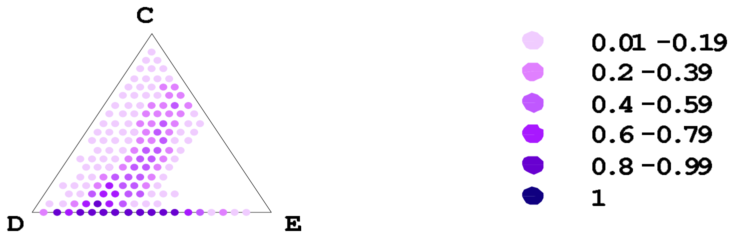
[16]Frequencies do not add up to 1 due to rounding off.



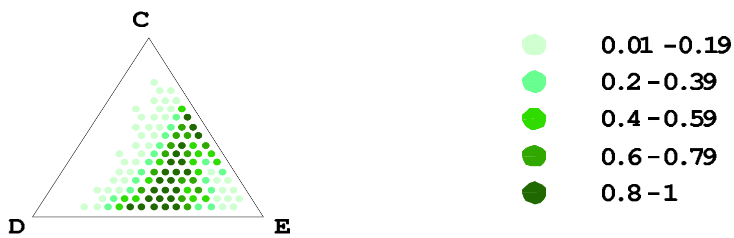
(a) Frequencies for initial coordinates converging to D-equilibria



(b) Frequencies for initial coordinates converging to CE-equilibria



(c) Frequencies for initial coordinates converging to DE-equilibria



(d) Frequencies for initial coordinates converging to CDE-equilibria

Figure 6. Frequency of occurrence of each type of equilibrium for different initial strategy shares, with $\delta = 0.4$, $n = 100$, $\alpha = 0.2$.

3.3.4 Sensitivity analysis

In this section, we perform a sensitivity analysis of three central parameters which were also studied by Sethi and Somanathan: namely, the level of sanctions δ; the parameter α that reflects harvest technology or resource price; and, finally, the population size m. Parameters are varied as follows: (1) the level of sanctions is varied between $\delta = 0.1$ and $\delta = 1.5$ in 14 steps of 0.1; (2) the parameter α is varied between $\alpha = 0.1$ and $\alpha = 0.7$ in 6 steps of 0.1; and (3), finally, the population size is varied between $m = 36$ and $m = 121$, using 5 different grids of 6x6, ..., 11x11.[17]

For each possible parameter configuration, we performed 23,100 simulation runs as in Section 3.3.3. The results are given in terms of the average frequency of occurrence of a given equilibrium. Contour lines in Figure 7 delimit the set of parameters converging to D-equilibria at a given frequency. For instance, the set of parameter values for which D-equilibria are reached at an average frequency larger or equal to 90% is located below or on the contour line 0.9.

According to Figure 7a, D-equilibria are more easily achieved for low levels of sanctions. These findings make sense intuitively, and are similar to those of Sethi and Somanathan. Further, D-equilibria are more likely reached for a large α. An increase in the net return to harvesting, due to, for instance, an improvement in technology or an increase in the resource price, causes a rise in profits and gives an extra advantage to defectors. As a consequence, the set of D equilibria expands. This is also in line with Sethi and Somanathan's findings.

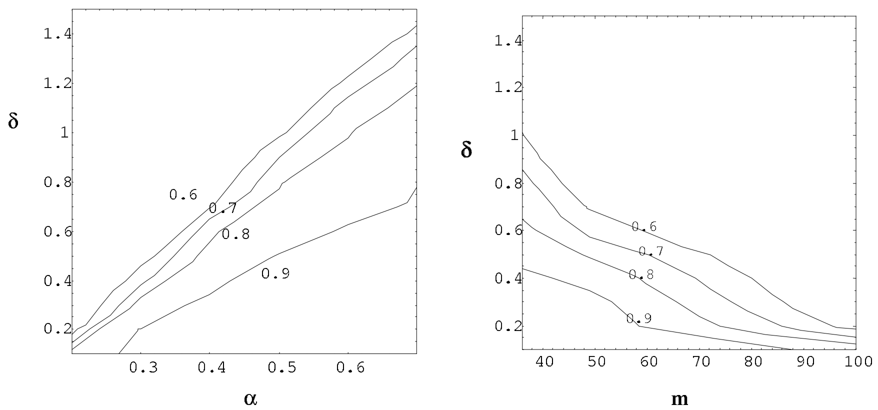
In Figure 7b, D-equilibria are best reached for a low level of sanctions and a small population size. As population size falls, i.e. as there are fewer harvesters with the same amount of resource, total effort decreases. This effect translates into a rise in net return to harvesting, causing the size of the set of D-equilibria to expand.

Sethi and Somanathan suggest that population growth negatively affects the level of sanctions. Their motivation is that as the group gets larger, it becomes more difficult for enforcers to detect

[17] We arbitrarily set maximum values for δ and α. As shown further, an increase in δ reduces the set of D-equilibria, while an increase in α enlarges it. Accordingly, varying *both* parameters δ and α excludes finding maximum parameter values leading to extreme situations with 0% or 100% of D-equilibria. This sensitivity analysis thus serves only for illustrative purposes. Parameter values for population size m are restricted by (5).

defectors. They coin this as an 'anonimity' effect which reduces the impact of individual sanctions. Since monitoring occurs locally in the spatial model, defectors cannot possibly 'hide' from their direct neighbors. As a consequence, there is no anonimity effect in our model and thus no connection between the level of sanctions and population size.

Contour graphs, as in Figure 7, can be obtained for all D, CE, CDE and DE-equilibria, where the same parameter effects are observed. To sum up, parameter conditions that facilitate the occurrence of cooperative equilibria in the spatial model are: (1) a high level of sanctions; (2) a low net return to harvesting; and (3) a large population size.



(a) Parameter space (α, δ), $m = 100$.

(b) Parameter space (m, δ), $\alpha = 0.2$.

Figure 7. Range of convergence to D-equilibria (contour lines denote average frequencies of occurrence).

4. Spatial evolutionary and resource dynamics

4.1 Definition of resource stock equilibria

Following Sethi and Somanathan, we introduce resource dynamics in the system. Surprisingly, the role of resource dynamics on harvesting behavior is often neglected in the literature on common-pool issues. Experiments and games developed by Ostrom et al. (1994) ignore resource issues. In real-world situations, however, harvesters often modify their behavior when new information about the level of resource becomes available. Feedback effects are observed from harvesting activities to the natural resource and vice versa.

The growth rate $G(N)$ of the resource stock follows a logistic pattern:

$$G(N_t) = rN_t\left(1 - \frac{N_t}{N_K}\right), \tag{14}$$

where r is the intrinsic growth rate of the resource, and N_K its carrying capacity. $G(N)$ reaches a maximum at $N_{MSY} > 0$, which is the maximum sustainable yield, so that $G' > 0$ for $N < N_{MSY}$ and $G' < 0$ for $N > N_{MSY}$. At the end of each round, the resource stock is increased according to the replenishment rate (14) and depleted by the total harvest, i.e. the sum of individual harvests:

$$N_{t+1} = N_t + G(N_t) - H(X_t, N_t). \tag{15}$$

An additional fifth step is introduced in the time sequence of the game, as defined in Section 2.2. The resource stock is given by (15), where the time subscript is replaced by the round subscript τ. Additionally, in the second step of the time sequence, the function $H(X_\tau, N_0)$ is replaced by $H(X_\tau, N_\tau)$.

For each N_t, there exists a corresponding $X_P(N_t)$ and $X_O(N_t)$, which represent the efficient and open-access levels of exploitation of the resource, respectively. These are calculated by replacing N_0 by N_t in (12). In addition, we follow Sethi and Somanathan by assuming that individual effort is positive and increasing in N_t. The intuition behind this assumption is that harvesters will intensify their effort level as the resource becomes more abundant. The effort function exhibits decreasing returns to scale in the resource stock:

$$x_{H,t}(N_t) = \lambda x_H N_t^\theta \tag{16}$$

$$x_{L,t}(N_t) = \lambda x_L N_t^\theta. \tag{17}$$

Here x_H and x_L are the fixed individual efforts used in Section 3, $\lambda > 0$ and $0 < \theta < 1$. Condition (5) is rewritten as:

$$X_P(N_t) \leq m x_{L,t}(N_t) \leq m x_{H,t}(N_t) \leq X_O(N_t), \quad \forall \quad 0 \leq N_t \leq N_K. \tag{18}$$

Profits are affected by changes in resource stocks according to:

$$\pi_{C,t} = x_{L,t}(N_t) \left(\frac{H(X_t, N_t)}{X_t} - w \right) \tag{19}$$

$$\pi_{Dk,t} = x_{H,t}(N_t) \left(\frac{H(X_t, N_t)}{X_t} - w \right) - k\delta \tag{20}$$

$$\pi_{El,t} = x_{L,t}(N_t) \left(\frac{H(X_t, N_t)}{X_t} - w \right) - l\gamma. \tag{21}$$

In these settings, global factors enters the individual payoffs function through both X_t and N_t. Just like in the previous section, a large number of defectors, i.e. a large X_t, decreases individual payoffs. Nevertheless, a large level of resource stock N_t brings up larger payoffs for all agents. The evolution of N_t, however, is governed by the levels of X_t and thus by the evolution of strategies. Hence, an intrinsic feature of the dynamics is that the evolution of the resource stock jointly determines the strategy equilibrium, and inversely.

Proposition 1 below states that, as the system converges to a strategy equilibrium, defined in terms of the final number of defectors, cooperators and enforcers, the resource stock will also converge to an equilibrium level.

This main result following from the introduction of resource dynamics into the system is summarized as follows:

Proposition 1 *To each element of the set of strategy equilibria there corresponds a unique resource stock steady-state N^*. This steady-state is found at the intersection of the curves $H(X^*, N)$ and $G(N)$, where X^* is the aggregate effort level associated with a given strategy equilibrium. The resource stock steady-state is locally stable for all $N_0 > \bar{N}$, where $\bar{N}$ is the intersection between the curves $H(m x_H, N)$ and $G(N)$.*

As the strategy equilibrium converges progressively to a D-equilibrium, aggregate harvest increases and the resource stock is lowered accordingly. By contrast, convergence towards a CE-equilibrium results in low harvesting of the resource. Intuitively,the final resource stock achieved under a CE-equilibrium will thus be larger than the final resource stock in a D-equilibrium.

This intuition can be appreciated by looking at the steady-states pictured in Figure 8. In Figure 8, every curve $H(X, N)$ intersects with $G(N)$ at two different points. The stability results of these intersections can be observed intuitively. Arrows in Figure 8 show the direction of resource dynamics.[18] Looking at the directions of the arrows, it is clear that, for each curve H, only the second intersection point is locally stable. Thus, in Figure 8, starting from $N_0 > \bar{N}$, it can be seen that the resource stock will always converge to a stable state.

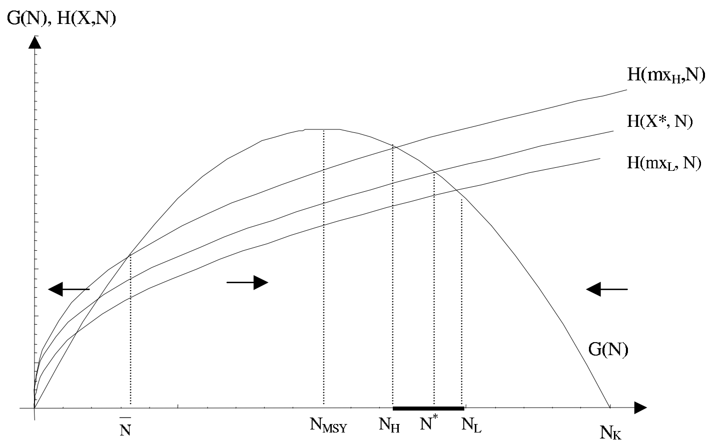


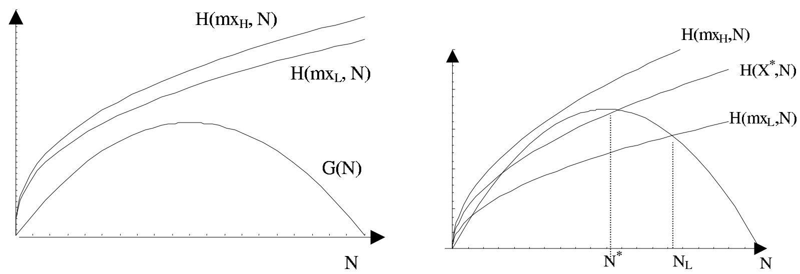
Figure 8. Steady-states with evolutionary and resource dynamics.

The total harvest achieved in a D-equilibrium is given by the curve $H(mx_H, N)$ and the corresponding equilibrium resource stock is given by $N^* = N_H$. Similarly, in CE, C or E-equilibria, the total harvest is pictured by the curve $H(mx_L, N)$ and the stock will converge to the corresponding equilibrium resource stock $N^* = N_L$. Accordingly, $N_L > N_H$ implies that the equilibrium stock is larger for a cooperative equilibrium than for a defecting one. The same reasoning applies to DE and CDE-equilibria. As shown in Figure 8, a given DE or CDE-equilibrium corresponds to a

[18] A left (right) pointing arrow indicates a decrease (increase) in the resource stock. This holds for intervals where $G(N) < (>) H(X, N)$.

particular curve $H(X^*, N)$, where $mx_L < X^* < mx_H$. The directions of the arrows show that the resource stock will converge to N^* with $N_H < N^* < N_L$. Obviously, all equilibrium stocks for CDE and DE-equilibria fall within the range defined by N_H and N_L.

Under some conditions, no positive equilibrium resource stock can be reached. Figure 9 shows some special cases. In Figure 9a, the resource stock will be inevitably depleted in the long run since the aggregate harvest always exceeds the replenishment rate of the resource for all stock levels. In Figure 9b, a D equilibrium will always result in overexploitation of the resource, but there exist a DE or a CDE-equilibrium with a harvest level equal to $H(X^*, N)$ and a positive resource stock. Thus, the resource stock affects the final strategy equilibrium. Indeed, with a low initial stock or a low replenishment rate, only a limited set of strategy equilibria can be reached.



(a) $H(X, N) > G(N)$ (b) $H(X_H, N) > G(N)$

Figure 9. Some possible configurations of equilibria.

4.2 Simulation results

To show the diversity of possible stable resource stocks, we perform simulations with the same parameters as in Section 3.3. In addition, we set $r = 0.5$, $N_K = 1000$, $\lambda = 0.05$, and $\theta = 0.5$, such that (18) holds. These parameters also satisfy condition (10).[19] Table 1 gives the resource stock

[19]With resource dynamics, condition (10) is satisfied for all X_τ, i.e. in every round τ, whenever:

$$\left[\frac{1}{\alpha}\left(\frac{4\delta}{x_H - x_L} + w\right)\right]^{\frac{1}{\beta-1}} N_K < mx_H.$$

equilibria for 100 random runs, starting with $z_0 = (0.30; 0.40; 0.30)$.

The resource equilibria for D and CE-equilibria are $N_H = 616.96$ and $N_L = 669.25$, respectively. Equilibrium resource stocks for DE and CDE-equilibria vary within this range. On average, equilibrium resource stocks for CDE-equilibria are larger than equilibrium stocks for DE-equilibria. This makes sense, since CDE-equilibria are often characterized by a larger number of cooperative players. This diversity of stock equilibria contrasts with Sethi and Somanathan's results. Indeed, in their model, only N_H and N_L are possible equilibrium resource stocks, given that only two types of equilibra, D and CE, can be achieved.

Table 1. Final stock levels at $\tau = 50$ for different equilibria, with $z_0 = (0.30; 0.40; 0.30)$.

	N_D	N_{DE}	N_{CDE}	N_{CE}
Max	616.96	628.60	644.37	669.25
Min	616.96	618.61	622.40	669.25
Average	616.96	621.30	627.3	669.25
Std Dev	0	2.22	4.58	0

The parallel convergence of strategies and resource stock is depicted in Figure 10. The upper graphs show the evolution of the three strategies over time, while the lower graphs illustrate the resource stock dynamics. Starting from $N_0 = 500$, resource levels increase steadily until the equilibrium state. Several observations can be made. First, as expected, the evolution of strategies in the population is characterized by an early increase (decrease) in the number of cooperators (defectors), and the stable stock achieved in a CDE-equilibrium is larger than the stable stock corresponding to a D-equilibrium. Second, variations in strategy shares are non-monotonic and resource dynamics exhibit different degrees of early overshooting.

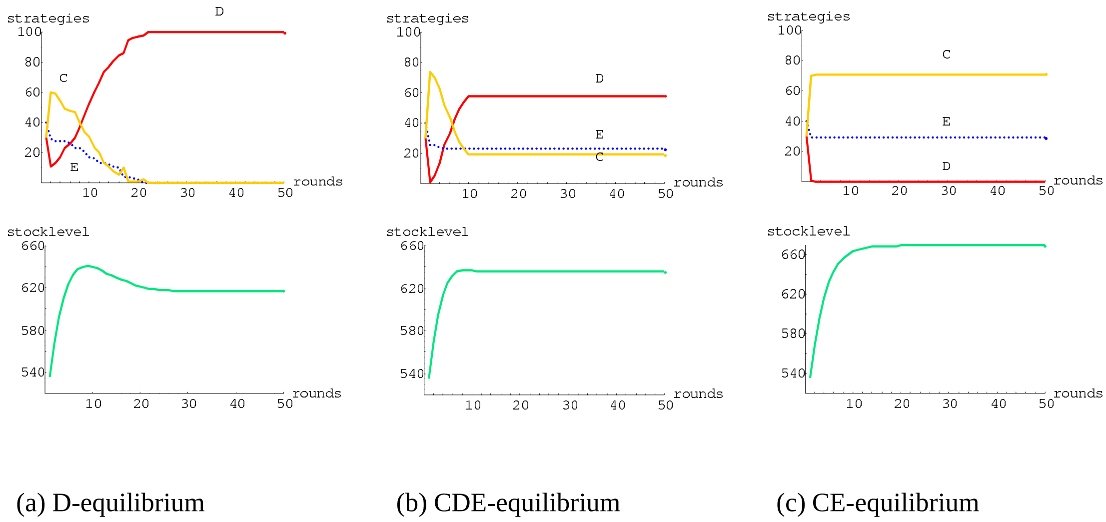


Figure 10. Evolution of strategies and resource stock dynamics toward an equilibrium.

5. Conclusion

The analysis here contributes to the literature on the evolution of social norms in a common-pool resource game. This type of analysis supports the idea that sustainable resource use can be achieved by self-organized systems. A major innovation of our approach is the introduction of spatial interactions between the harvesters. This means that agents are modeled explicitly and the analysis is conducted at the micro-level. Using numerical simulation, we have shown that results differ greatly from the non-spatial or aggregate game, as studied by Sethi and Somanathan (1996).

All (strategy) equilibria of Sethi and Somanathan are generated by our model. Two additional types of equilibria are identified: namely, one that corresponds to a final population composed of defectors and enforcers, and one in which all three strategies coexist in the long-run. The occurrence of these equilibria is explained by two characteristic features of the spatial model. First, enforcers punishing defectors can survive in the long run as long as no cooperators are located in their neighborhood. Second, cooperators are protected by the formation of clusters of cooperators and enforcers around enforcers who do not punish any defectors. These results stress a general insight: namely, that spatial interactions favor diversity.

Conditions for the emergence of cooperative equilibria are identified by performing a range of sensitivity analyses. Initial populations composed of a large number of enforcers lead to more cooperative outcomes. Similar results follow from an increase in the level of sanctions and a reduction of the net return to harvesting. Population growth is shown to enlarge the set of cooperative equilibria, because it negatively affects the net return to harvesting.

Finally, we have shown that the diversity of equilibria is not affected by the introduction of resource dynamics. An equilibrium resource stock exists for every possible strategy equilibrium of the system. Since the set of strategy equilibria is wider than in Sethi and Somanathan's model, the range of possible resource equilibria is wider as well.

Further research can examine the impact of more complex resource dynamics, additional types of social norms, involving, for instance, a sanction that depends on the size of the resource, and more refined learning processes in which agents improve their understanding of the game.

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