

Biodiversity and economic growth: stabilization versus preservation of the ecological dynamics

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Abstract

This work examines the impact that economic growth can have on biodiversity and on the ecological dynamics that would naturally emerge in the absence of human activity. The loss of biodiversity may induce policy-makers to implement defensive actions that prevent single species from extinction. These defensive actions, however, may deeply alter the natural dynamics of interaction between species, leading to an ecological equilibrium that is completely different from the one that would exist in the absence of human intervention. This suggests that there might exist a conflict between preserving biodiversity (through stabilization of the ecological system) and preserving the intrinsic features of the ecological dynamics. To investigate this issue more deeply, we analyze the impact that different objective functions and defensive technologies can have on the natural ecological dynamics, and show that human action can modify the stability of the ecological fixed points. From the simple analytical formulations adopted in the paper, it emerges that it is possible to stabilize the ecological fixed point and consequently to avoid the extinction of a species, even in the absence of defensive expenditures specifically finalized at the protection of that species. The “stabilizing” effect of human intervention, however, turns out to be enhanced when specific defensive expenditures are implemented. Finally, numerical simulations suggest that human activity can have an even deeper impact on the ecological dynamics, substantially modifying not only the stability of the fixed points, but also their number.

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1 Introduction

In recent years, world-wide ecological problems, such as global warming and ozone depletion, have played an increasingly central role in the scientific debate on the state of the environment. Among these problems, special attention has been devoted to the loss of biodiversity of the ecosystem, mainly for two reasons. In the first place, because many researchers believe that this loss is currently occurring at an unparalleled rate in human history.¹ In the second place, because biodiversity is considered essential for human survival, and in many cases cannot be replaced by man-made physical capital. These two categories of reasons help to explain the attention recently devoted to this subject in both political and academic spheres. As regards the political debate, at the Johannesburg Summit of 2002 the conservation of biodiversity was recognized as a fundamental prerequisite for the achievement of sustainable development and as one of the absolute priorities of future environmental policies. As to the academic debate, many contributions have sought to make an economic evaluation of biodiversity. Certain authors (Montgomery et al., 1999) have proposed a theoretical framework from which the value to be attributed to biodiversity could be derived in order to guide the decisions of the policy-makers. Other authors, instead, have criticized the traditional approach of economic theory based on the identification of the correct market price for natural resources, emphasizing the fact that the exchange value of biodiversity is only a tiny portion of its total value (Gowdy, 1997) and that neither very high nor very low market prices can ensure the survival of a particular species (McDaniel and Gowdy, 1998). In more general terms, several contributions (von Amsberg, 1995) question the intertemporal efficiency of the markets in guiding investment decisions under risk and argue that the excessive reduction of biodiversity can be seen as a specific example of this general result.²

Alongside this strand of literature dealing with the market's capacity to evaluate biodiversity, a growing interest has also been devoted to the relationship between biodiversity, ecological stability and the sustainability of economic development. More specifically, several authors (Perrings 1995, Atkinson et al. 1999) have focused attention on the link between sustainable development and resilience (Holling, 1973), which is an indicator of the stability of the ecosystem and can be closely approximated by the biodiversity existing within the ecosystem.³ Using an optimal control model, Li and Lofgren (1998) have examined the effect that the interaction between human activity and biodiversity can have on the stability of the economic and ecological systems. Cabo et al. (1999) have analyzed the relation between biodiversity and growth through a

¹Although similar episodes of destruction of biodiversity have occurred in the past (as in the case of the extinction of the dinosaurs) the present loss of biodiversity appears to be related to human activity and not to exogenous natural phenomena (Heal, 1994).

²Gowdy and McDaniel (1995) go even further in their criticism of the market system, claiming that the organizational principles governing the market are inherently in conflict with the self-regulatory principles of the ecosystems.

³See Common and Perrings (1992) for precise mathematical definitions of stability and resilience.

model in which the North transfers capital to the South. Tisdell (1999) has investigated the effect of growth on biodiversity through the analysis of a case study regarding China. Barbier and Schulz (1997) have examined the way in which the inclusion of the biodiversity value modifies the results of both closed and open economies in a traditional model of growth and resource exploitation.

The present study intends to contribute to this strand of research, analyzing the impact that human activity can have on the defence of biodiversity and on the natural ecological dynamics, namely, on the ecological dynamics that would naturally emerge in the absence of human interference.

The natural law of evolution can obviously lead to the extinction of some species. However, as Barney (1980) has pointed out, most of the recent biodiversity loss is due to the impact of human activity. To counterbalance this trend, policy makers can implement defensive actions that try to prevent single species from extinction.⁴ Such defensive actions can be specifically aimed at preserving particular species or generically designed to protect the habitat where species live. For instance, providing to an endangered species the food it needs to survive is an example of specific defensive expenditure, whereas introducing costly policy measures to reduce polluting activities in a biodiversity rich area can be interpreted as a generic defensive expenditure. All defensive expenditures, whether generic or specific, may help preserving biodiversity, but they may also deeply modify the natural dynamics of interaction between the species. Human action can therefore lead to an ecological dynamics that ensures species persistence, but turns out to be completely different from that which would exist in the absence of mankind. These considerations raise the kind of questions that we intend to address in the present paper, namely: what is the impact of human intervention on the stability and the number of ecological equilibria? Can defensive expenditures succeed in preserving biodiversity and to what extent? Can generic defensive expenditures be sufficient for this purpose?

To answer these questions, in what follows we will consider a stylized ecosystem in which the ecological dynamics is given by the interaction between two species, species x and species y . More specifically, for the sake of simplicity we shall assume that the ecological dynamics is represented by the following linear system:

$$\begin{aligned}\dot{x} &= \beta_1 + \rho_1 x + \gamma_1 y \\ \dot{y} &= \beta_2 + \rho_2 y + \gamma_2 x\end{aligned}\tag{1}$$

where $\beta_1, \beta_2 > 0$ while the other parameters may assume positive or negative value depending on the type of interaction occurring within each species as well as between them (symbiotic, prey-predator etc.). We have deliberately chosen to keep the stylized ecosystem as simple as possible to allow for more

⁴As some authors have argued (Shogren and Crocker, 1999; Shogren, 2000; Perrings, 2003), decision makers can respond to exogenously induced changes in the natural ecological dynamics in two ways: through mitigation and adaptation. In this paper we will consider defensive expenditures of the first kind, that try to reduce the likelihood of the event rather than simply adapt to it.

complex dynamics that could arise in the integrated model from the interaction between the economic and the ecological components.⁵ System (1), however, can be interpreted as the linearization around a fixed point of a more complex, non-linear system (such as Lotka-Volterra equations). Since in this study we will concentrate on the local stability analysis of the systems, notice that the linearity hypothesis does not constitute here any loss of generality. The dynamics (1) can be thought of as the evolution that species would naturally follow if we let Nature take its course and will be our benchmark throughout the paper. The objective of our study is to analyze how economic growth and the underlying agents decisions can modify the stability characteristics of a fixed point of the ecological dynamics (1) (provided this exists). For this purpose, we will examine a simple growth model that links the ecological dynamics (1) to a physical capital accumulation dynamics. The interdependencies between the economic growth and the ecological dynamics (1) will be given by the negative ecological effects of aggregate consumption, and by the positive ecological effects of the defensive environmental choices. The negative impact of aggregate consumption on the evolution of the species can be interpreted as the reduction in animal and vegetal populations generated by anthropogenic polluting activities. Thus, for instance, production and consumption activities can contribute to global warming that is considered as the main culprit for the collapse of several species.⁶ The defensive and consumption choices depend on the objective function maximized by the representative agent as well as on the defensive “technology” at disposal. To analyze the impact that different objective functions and different defensive technologies have on the ecological dynamics, we shall take three different theoretical formalizations into account. In model 1 we shall assume an objective function in which both species x and y are essential (i.e. the marginal utility of each species tends to infinite as the number of individuals of the species approaches zero) and a defensive technology which prevents discrimination between the two species. In model 2 we will analyze the same objective function, but in this case it will be assumed that it is possible to implement specific defensive expenditures for species x and for species y . Finally, in model 3 we will hypothesize that the objective function depends on an aggregate measure of x and y , as implicitly assumed in models in which environmental resources are represented by a single state variable. In this case, therefore, the representative agent does not care for biodiversity and makes generic defensive expenditures.

In the following analysis we will show that when x and y are both sufficiently high at the fixed point, the stability features of this point are completely preserved. On the contrary, when x and y are sufficiently low at the fixed point, human intervention may cause a stabilization of the fixed point that is necessary

⁵Chen (1997), for instance, has shown that chaotic dynamics can arise in a globally combined climatic-economic system even though none of the two systems behaves chaotically on its own. See also the interesting paper by Rosser (2001) for further discussion and analysis of complex dynamics in economic-ecological systems.

⁶In this sense, in line with several works in the literature (e.g. Brock, 1977; Forster, 1973; Tahvonen and Kuuluvainen, 1993) the model can be seen as an optimal growth model with pollution rather than an optimal harvesting model.

to prevent the extinction of one or both species. This suggests that the decision makers may be compelled to choose between “stabilization” (preservation of biodiversity) and the preservation of the intrinsic features of the dynamics generated by the interaction between species.

The structure of the paper is as follows. Sections 2, 3 and 4 investigate each of the three models in turn, focusing on the local stability analysis of the fixed points to describe how results are affected by changes in the objective function, in the defensive technology and in the parameter values. Section 5 summarizes the main findings of the paper and draw some concluding remarks on the conflict between stabilization and preservation of the natural ecological dynamics that emerge in the paper.

2 Model 1

Let us assume that there is a representative agent in the economy whose utility depends on consumption (c) and on the amount of the two species x and y . Following Li and Löfgren (1998), it can be argued that the two species enter the utility function both for their amenity value and for their optional uses in the future. We assume a logarithmic utility function in each argument, so that the representative agent suffers an infinite welfare loss from zero consumption as well as from the extinction of each species:

$$U = \ln x + a \ln y + b \ln c \quad (2)$$

where $a, b > 0$.

Suppose there is a single good in the economy that is produced by capital alone (k). The corresponding output is used for capital accumulation, consumption and defensive expenditures (d):

$$\dot{k} = k^\alpha - c - d \quad (3)$$

where $0 < \alpha < 1$.

Human activity influences the natural ecological dynamics through aggregate consumption and defensive expenditures. The impact of defensive expenditures on the species evolution is assumed to be a decreasing function of the expenditures, so that their effectiveness decreases as they increase. We consider here the case in which there are no specific defensive expenditures, namely, the defensive technology cannot distinguish between the two species:

$$\dot{x} = \beta_1 + \rho_1 x + \gamma_1 y - \epsilon_1 c + \sigma_1 d^\mu \quad (4)$$

$$\dot{y} = \beta_2 + \rho_2 y + \gamma_2 x - \epsilon_2 c + \sigma_2 d^\mu \quad (5)$$

where $\sigma_1, \sigma_2, \epsilon_1, \epsilon_2 > 0$ and $0 < \mu < 1$.

The representative agent thus chooses consumption and defensive expenditures so as to maximize the present discounted value of the utility function subject to the physical capital accumulation dynamics and the “modified” ecological dynamics, that is, the natural ecological dynamics (1) as modified by human intervention. The optimization problem thus becomes:

$$\begin{aligned} \max_{c,d} \int_0^{\infty} (\ln x + a \ln y + b \ln c) e^{-rt} dt \\ \text{subject to equations (3)-(5)} \end{aligned} \quad (6)$$

where r indicates the subjective discount rate ($r > 0$).

The current value Hamiltonian function is:

$$\begin{aligned} H = \ln x + a \ln y + b \ln c + \lambda(k^\alpha - c - d) \\ + \theta(\beta_1 + \rho_1 x + \gamma_1 y - \epsilon_1 c + \sigma_1 d^\mu) \\ + \eta(\beta_2 + \rho_2 y + \gamma_2 x - \epsilon_2 c + \sigma_2 d^\mu) \end{aligned}$$

where λ , θ and η are respectively the multipliers of k , x and y , and have the usual interpretation of shadow “prices”.

From the maximum principle, the first-order necessary conditions for optimality are:

$$\frac{\partial H}{\partial c} = \frac{b}{c} - \lambda - \epsilon_1 \theta - \epsilon_2 \eta = 0 \quad (7)$$

$$\frac{\partial H}{\partial d} = -\lambda + \mu \sigma_1 d^{\mu-1} \theta + \mu \sigma_2 d^{\mu-1} \eta = 0 \quad (8)$$

$$\dot{x} = \frac{\partial H}{\partial \theta} = \beta_1 + \rho_1 x + \gamma_1 y - \epsilon_1 c + \sigma_1 d^\mu \quad (9)$$

$$\dot{y} = \frac{\partial H}{\partial \eta} = \beta_2 + \rho_2 y + \gamma_2 x - \epsilon_2 c + \sigma_2 d^\mu \quad (10)$$

$$\dot{k} = \frac{\partial H}{\partial \lambda} = k^\alpha - c - d \quad (11)$$

$$\dot{\theta} = r\theta - \frac{\partial H}{\partial x} = (r - \rho_1)\theta - \gamma_2 \eta - \frac{1}{x} \quad (12)$$

$$\dot{\eta} = r\eta - \frac{\partial H}{\partial y} = (r - \rho_2)\eta - \gamma_1\theta - \frac{a}{y} \quad (13)$$

$$\dot{\lambda} = r\lambda - \frac{\partial H}{\partial k} = \lambda(r - \alpha k^{\alpha-1}) \quad (14)$$

Substituting the optimal choices of the control variables c and d in the equations (9)-(14) and setting $\dot{x} = \dot{y} = \dot{k} = \dot{\theta} = \dot{\eta} = \dot{\lambda} = 0$, we obtain the following system of six differential equations whose solution provides the fixed point of the integrated economic-ecological system described in the model:

$$\beta_1 + \rho_1 x + \gamma_1 y - \frac{\epsilon_1 b}{\lambda + \epsilon_1 \theta + \epsilon_2 \eta} + \frac{\sigma_1(\sigma_1 \theta + \sigma_2 \eta)}{2\lambda} = 0 \quad (15)$$

$$\beta_2 + \rho_2 y + \gamma_2 x - \frac{\epsilon_2 b}{\lambda + \epsilon_1 \theta + \epsilon_2 \eta} + \frac{\sigma_2(\sigma_1 \theta + \sigma_2 \eta)}{2\lambda} = 0 \quad (16)$$

$$k^\alpha - \frac{b}{\lambda + \epsilon_1 \theta + \epsilon_2 \eta} - \left(\frac{\sigma_1 \theta + \sigma_2 \eta}{2\lambda}\right)^2 = 0 \quad (17)$$

$$(r - \rho_1)\theta - \gamma_2 \eta - \frac{1}{x} = 0 \quad (18)$$

$$(r - \rho_2)\eta - \gamma_1 \theta - \frac{a}{y} = 0 \quad (19)$$

$$\lambda(r - \alpha k^{\alpha-1}) = 0 \quad (20)$$

In what follows we will refer to the solution of this system as the fixed point of the economic growth dynamics to distinguish it from the fixed point of the natural ecological dynamics that solves system (1).

2.1 Stability analysis of model 1

Let us now turn to the local stability analysis of the economic growth dynamics and of the natural ecological dynamics (1) to compare the stability of the fixed point with and without human action. The analysis of the natural ecological dynamics is straightforward. The Jacobian matrix of the natural system is:

$$J = \begin{pmatrix} \rho_1 & \gamma_1 \\ \gamma_2 & \rho_2 \end{pmatrix}$$

therefore the characteristic polynomial proves to be:

$$z^2 - (\rho_1 + \rho_2)z + \rho_1\rho_2 - \gamma_1\gamma_2 \quad (21)$$

and the fixed point is a saddle point if $\rho_1\rho_2 - \gamma_1\gamma_2 < 0$, is locally attractive if $\rho_1\rho_2 - \gamma_1\gamma_2 > 0$ and $\rho_1 + \rho_2 < 0$ and is repulsive if $\rho_1\rho_2 - \gamma_1\gamma_2 > 0$ and $\rho_1 + \rho_2 > 0$.

Before analyzing the local stability of the fixed points of model 1, observe that if the Jacobian matrix admits three negative (and hence three positive) eigenvalues, then the fixed point can be achieved, namely, for any initial state (x_0, y_0, k_0) sufficiently close to the steady state it is possible to determine the initial values of the co-state variables $(\theta_0, \eta_0, \lambda_0)$ such that the economic growth trajectory eventually converges to the fixed point. If the number of negative eigenvalues is strictly less than three, this is no longer possible a part for a zero measure set of parameter values.⁷ Therefore, if the fixed point of system (1) is attractive, then the stability properties of the natural dynamics will be preserved if the fixed point of model 1 has three or more negative eigenvalues. Similarly, if system (1) has a repulsive fixed point, its stability properties will be preserved with human intervention provided the fixed point of model 1 has less than three negative eigenvalues.

The Jacobian matrix for model 1 is given by:

$$Jc := \begin{pmatrix} \rho_1 & \gamma_1 & 0 & H_{\theta,\theta} & H_{\theta,\eta} & H_{\theta,\lambda} \\ \gamma_2 & \rho_2 & 0 & H_{\theta,\eta} & H_{\eta,\eta} & H_{\eta,\lambda} \\ 0 & 0 & \frac{\alpha}{k^{1-\alpha}} & H_{\theta,\lambda} & H_{\eta,\lambda} & H_{\lambda,\lambda} \\ \frac{1}{x^2} & 0 & 0 & r - \rho_1 & -\gamma_2 & 0 \\ 0 & \frac{a}{y^2} & 0 & -\gamma_1 & r - \rho_2 & 0 \\ 0 & 0 & \frac{\alpha(1-\alpha)\lambda}{k^{2-\alpha}} & 0 & 0 & r - \alpha k^{\alpha-1} \end{pmatrix}$$

where:⁸ $H_{\theta,\theta} := \frac{\epsilon_1^2 b}{\phi^2} + \frac{\sigma_1^2}{2\lambda}$, $H_{\theta,\eta} := \frac{\epsilon_1 b \epsilon_2}{\phi^2} + \frac{\sigma_1 \sigma_2}{2\lambda}$, $H_{\theta,\lambda} := \frac{\epsilon_1 b}{\phi^2} - \frac{\sigma_1 \psi}{2\lambda^2}$, $H_{\eta,\eta} := \frac{\epsilon_2^2 b}{\phi^2} + \frac{\sigma_2^2}{2\lambda}$, $H_{\eta,\lambda} := \frac{\epsilon_2 b}{\phi^2} - \frac{1}{2} \frac{\sigma_2 \psi}{\lambda^2}$, $H_{\lambda,\lambda} := \frac{b}{\phi^2} + \frac{1}{2} \frac{\psi^2}{\lambda^3}$ and $\phi := \epsilon_1 \theta + \epsilon_2 \eta + \lambda$, $\psi := \sigma_1 \theta + \sigma_2 \eta$.

Observe that from (20) $r - \alpha k^{\alpha-1} = 0$ in the fixed point. Also observe that if the values of x and y are very high in correspondence of the fixed point,

⁷Thus, for instance, if there is only one negative eigenvalue out of six, this implies that only one trajectory will converge to the fixed point in a six dimensional space, which can obviously occur only for very particular values of the state and co-state variables.

⁸Notice that $H_{i,j}$ ($i, j = \theta, \eta, \lambda$) indicates the second partial derivative of the Hamiltonian (first with respect to i and then with respect to j) in correspondence of the fixed point.

the $\frac{1}{x^2}$ and $\frac{a}{y^2}$ terms in the Jc matrix assume values close to zero and are therefore negligible. In other words, for sufficiently elevated x and y values, the eigenvalues of the Jc matrix have the same sign as those of the Jc' matrix that has been obtained from Jc by setting $r - \alpha k^{\alpha-1}$, $\frac{1}{x^2}$ and $\frac{a}{y^2}$ all equal to zero:⁹

$$Jc' := \begin{pmatrix} \rho_1 & \gamma_1 & 0 & H_{\theta,\theta} & H_{\theta,\eta} & H_{\theta,\lambda} \\ \gamma_2 & \rho_2 & 0 & H_{\theta,\eta} & H_{\eta,\eta} & H_{\eta,\lambda} \\ 0 & 0 & \frac{\alpha}{k^{1+\alpha}} & H_{\theta,\lambda} & H_{\eta,\lambda} & H_{\lambda,\lambda} \\ 0 & 0 & 0 & r - \rho_1 & -\gamma_2 & 0 \\ 0 & 0 & 0 & -\gamma_1 & r - \rho_2 & 0 \\ 0 & 0 & \frac{\alpha\lambda}{2k^{1+\alpha}} & 0 & 0 & 0 \end{pmatrix}$$

Let us now analyze the characteristic polynomial of this simplified matrix Jc' . After several Gauss transformations, we obtain the following characteristic equation:

$$EQ_1(z)EQ_2(z)EQ_3(z) = 0$$

where

$$EQ_1(z) := z^2 - (\rho_1 + \rho_2)z + \rho_1\rho_2 - \gamma_1\gamma_2$$

$$EQ_2(z) := z^2 + (\rho_1 + \rho_2 - 2r)z + \rho_1\rho_2 - \gamma_1\gamma_2 - r(\rho_1 + \rho_2) + r^2$$

$$EQ_3(z) := z^2 - rz - \frac{\alpha\lambda}{2k^{1+\alpha}}H_{\lambda,\lambda}$$

Notice that $EQ_1(z)$ coincides with the characteristic polynomial (21) of the natural dynamics (1). Using this property, it is possible to prove the following proposition:

Proposition 1 *When x and y are sufficiently high, the stability properties of the fixed point of the natural dynamics are entirely transferred to the fixed point of the economic growth dynamics.*

Proof. See the Appendix ■

Therefore, if both species are sufficiently abundant in correspondence of the fixed point, the economic activity and the defensive expenditures do not alter the stability properties of the fixed point of the natural dynamics. Thus, for

⁹This can be easily proved by solving the problem for the following set of parameter values: $\alpha = \mu = \rho_1 = 0.5$; $\rho_2 = -0.3$; $\gamma_1 = -0.1$; $\gamma_2 = 1.55$; $\beta_1 = 1000$; $\beta_2 = 4.5$; $r = 0.65$; $a = b = \epsilon_1 = \epsilon_2 = \sigma_1 = \sigma_2 = 10$. In this case the values of the two species at the fixed point are $x = 4020.48$ and $y = 30025.61$ and it can be shown that the signs of the coefficients of the characteristic polynomial are maintained when passing from Jc to Jc' .

instance, if system (1) has an attractive (repulsive) fixed point, the economic growth trajectories of model 1 will also converge (not converge) to their own fixed point.

So far we have examined the case where x and y are sufficiently high at the fixed point. But what if we consider the more general case in which the two variables can assume any possible (positive) value?

In this case the analysis of the Jacobian matrix Jc turns out to be excessively complicated, so that it is necessary to resort to numerical examples. We therefore proceed with a number of simulations, considering in the first place the case in which the fixed point of the natural dynamics is repulsive.

2.2 Simulations of the model

The values used in the natural dynamics are as follows: $\beta_1 = 4, \beta_2 = 0.1, \rho_1 = 0.18, \rho_2 = -0.15, \gamma_1 = -0.1, \gamma_2 = 0.27114$. The co-ordinates of the corresponding fixed point turn out to be:

$$\bar{x} = 5244 \text{ and } \bar{y} = 9480$$

while the eigenvalues are:

$$\lambda_1 = 0.025607 \text{ and } \lambda_2 = 0.004393.$$

For the economic growth dynamics we use the following additional parameters: $\alpha = \mu = 0.5; a = b = \epsilon_1 = \epsilon_2 = \sigma_1 = \sigma_2 = 10; r = 0.2$.

At the fixed point of the economic growth dynamics the state and control variables take the following values:

$$\bar{x} = 208.066576, \bar{y} = 301.201335, \bar{k} = 6.25, \bar{c} = 1.904725, \bar{d} = 0.595275$$

with eigenvalues:

$$\lambda_1 = \lambda_2 = -0.021273, \lambda_3 = \lambda_4 = 0.221273, \lambda_5 = -0.102566, \lambda_6 = 0.302566.$$

From the simulation results, it emerges that the values of x and y in correspondence of this fixed point are much lower than at the fixed point of the natural dynamics. Moreover, as suggested by the signs of the eigenvalues, while the natural ecological dynamics show a repulsive fixed point that cannot therefore be reached by the correspondent trajectories (both eigenvalues being positive), the economic growth trajectories can reach the fixed point (3 negative eigenvalues). Therefore, differently from Proposition 1, if x and y are not sufficiently high at the fixed point the economic activity and the defensive expenditures produce a “stabilizing” effect that alters the stability properties of the fixed point of the natural dynamics. This seems consistent with our a priori expectations. Since utility is logarithmic in x and y , agents would suffer an infinite welfare loss from the extinction of one or both species (i.e. the loss of biodiversity). Therefore, when x and y are sufficiently low at the fixed point, agents have an incentive to increase their defensive expenditures in order to stabilize the fixed point and thus avoid the biodiversity loss that could derive from oscillations around an equilibrium with low values of x and y or from trajectories that move away from that equilibrium. It is interesting to notice that this stabilizing effect exists even though the monitoring activity does not discriminate between the two species. Thus, for example, if a share of income is directed to cleaning up the sea, this may lead to an ecological equilibrium in which both the species x and

y under consideration (e.g., sharks and sardines) are preserved, although the defensive expenditures for the protection of the sea do not discriminate between the forms of life which populate it. However, the reduction in x and y at the fixed point seems to suggest that this stabilization effect may come at the cost of a reduction in the number of individuals for each population, thus generating a trade-off between the stabilization of the species and their richness.

We then performed some comparative static analysis to examine how the sign of the eigenvalues (and thus the stability of the fixed point) changes with the variation of x and y at the economic growth fixed point. For this purpose, it is sufficient to vary (coeteris paribus) the parameter β_1 . Figure 1 shows how the state and control variables change as β_1 changes in the interval $[0.1, 10]$ with a rate of 0.1, keeping the values of the other parameters as in the previous analysis.

The vertical line indicates the threshold level of β_1 ($\beta_1 = 7.1$) that separates the area on the left where the fixed point can be reached by the economic growth trajectories from the area on the right where it cannot be reached. The values of \bar{x} and \bar{y} at the threshold level are $\bar{x} = 379.90$ and $\bar{y} = 552.38$.

As shown in the diagram, when the value of β_1 decreases, the values of x and y at the fixed point also decrease. Therefore, in line with what one could reasonably expect, the defensive expenditures increase with the reduction of β_1 to reduce the risk of extinction of the species and the resulting disutility loss for the agents. This alters the stability of the fixed point, making it now reachable from the growth trajectories (when $\beta_1 < 7.1$), but reducing x and y at a much lower level than in the case of an unstable fixed point (when $\beta_1 > 7.1$).

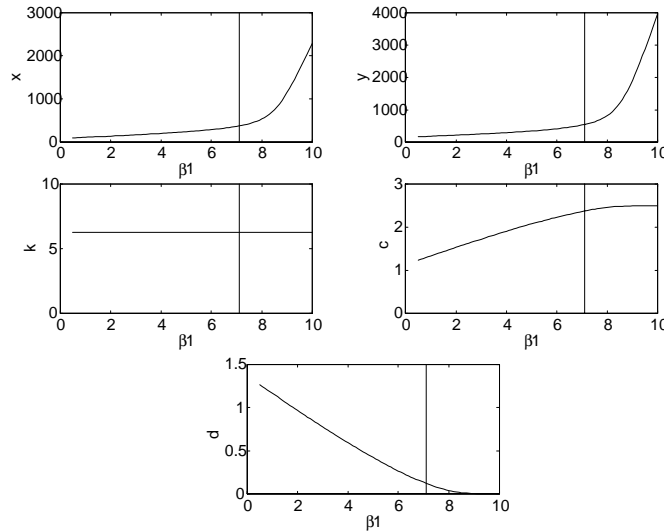


Figure 1

Also observe that k is constant and takes on the value $\frac{1}{(2r)^z} = 6.25$. From equation (20), in fact, it derives that $k = \left(\frac{r}{\alpha}\right)^{\frac{1}{\alpha-1}}$, therefore the capital level is

independent of β_1 at the fixed point. Moreover, from (11) $k = (c + d)^{\frac{1}{\alpha}}$, so that it must be $(c + d) = (\frac{x}{\alpha})^{\frac{\alpha}{\alpha-1}}$ at the fixed point. Therefore, recalling that $r = 0.2$ and $\alpha = 0.5$ in the simulations, for each value of β_1 consumption and defensive expenditures always sum up to the constant level 2.5 in the diagram, so that -as figure 1 shows- an increase in defensive expenditures will come at the cost of a reduction in consumption and viceversa.

We then performed comparative static analysis with respect to the intertemporal discount rate, by varying the parameter r in the interval $[0.2, 0.75]$ with a rate of 0.1. The outcome is represented below using a bar diagram for the species x (figure 2) and for the species y (figure 3). The two figures show for each value of r on the vertical axis, the correspondent values of x and y that delimit the passage from a reachable to an unreachable fixed point, each bar indicating the maximum value of x (y) above which the fixed point cannot be reached.

As one can see, the threshold values of x and y increase with a reduction in the intertemporal discount rate r . Therefore, if the economic agents are relatively more “patient”, they exert a stronger stabilizing effect, intervening already at high values of x and y to ensure that the fixed point can be reached by the growth trajectories. The mechanism which determines this result is the following: the more patient the agents, the greater is their level of accumulation and therefore the higher is the level of (stabilizing) defensive expenditures which they can afford. Moreover, the more patient the agents (the greater the importance they give to the future), the greater the disutility which they derive from oscillations around the fixed point that may eventually lead to biodiversity loss in the future.

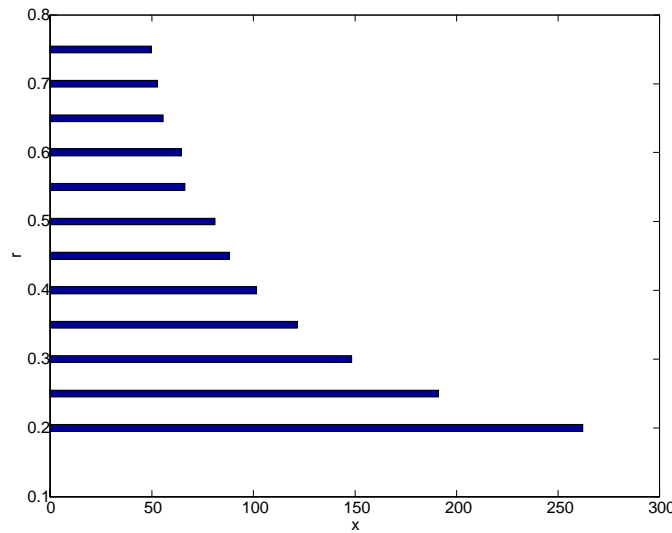


Figure 2

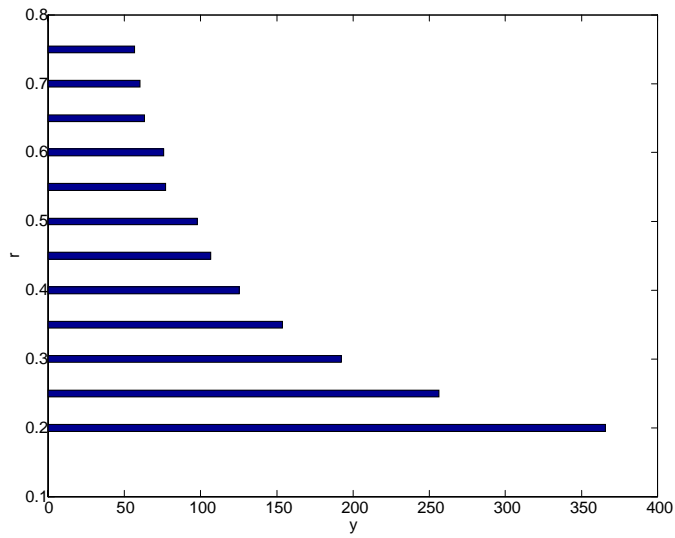


Figure 3

Some interesting results can also be obtained by varying the environmental impact of consumption (ϵ) in the interval $[1, 100]$ with a rate of 1. Figure 4 shows the correspondent trends of the state and control variables of the model, assuming -for the sake of simplicity- an equal impact of aggregate consumption on the evolution of the two species ($\epsilon_1 = \epsilon_2$). The vertical line (at $\epsilon = 6.5$) separates the area where the fixed point can be reached (on the right) from the area where it cannot be reached (on the left).¹⁰

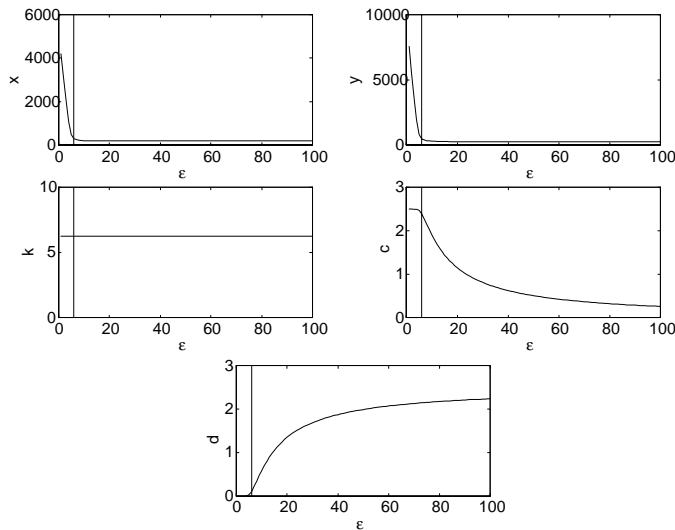


Figure 4

¹⁰Notice that the two areas are now reversed with respect to figure 1. From the system (1), in fact, increases in ϵ and β have opposite effects on the evolution of the two species.

As figure 4 shows, in this case an increase in ϵ_i ($i = 1, 2$) provokes the passage from the area where the fixed point cannot be reached to the area where economic growth trajectories converge to it. An increase in the environmental impact of consumption, in fact, reduces x and y at the fixed point, provoking an increase in defensive expenditures to stabilize the fixed point. The same variation holds true for changes in σ_1 and σ_2 , where an increase in the parameters boosts the efficacy of the defensive expenditures.¹¹

Figure 5 shows a simulation of the trajectories converging to the fixed point in the integrated economic-ecological model 1 with the variation of the initial values over time.

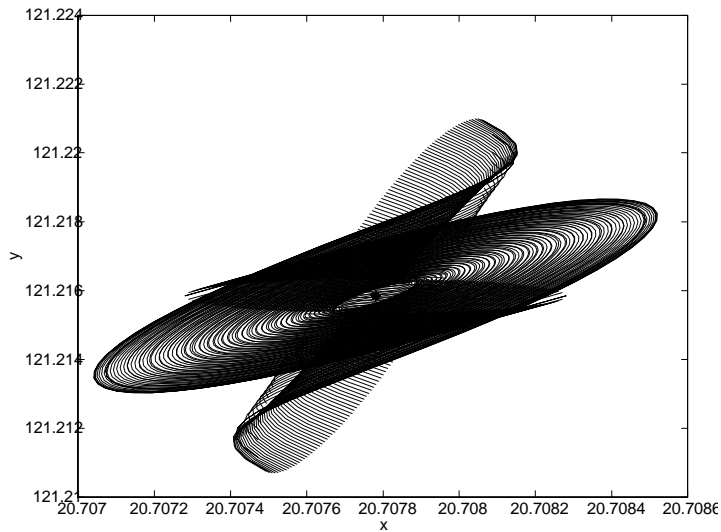


Figure 5

Let us now analyze a numerical example concerning the case in which the fixed point of the natural dynamics is attractive. To make this example comparable with the previous ones, we use a similar set of parameter values as before, the only difference concerning the natural dynamics parameters ρ_1 and ρ_2 that have now been set equal to 0.15 and -0.18 , respectively.¹² In this case, the co-ordinates of the natural dynamics fixed point are:

$\bar{x} = 90$ and $\bar{y} = 480$, with eigenvalues $\lambda_1 = -0.029289$ and $\lambda_2 = -0.170711$.

whereas the coordinates of the fixed point in the growth dynamics are:

$x = 171.2727, y = 210.5766, k = 6.25, c = 1.7368, d = 0.1774$

with eigenvalues:

$\lambda_1 = \lambda_2 = -0.5641, \lambda_3 = -0.11097, \lambda_4 = \lambda_5 = 0.25641, \lambda_6 = 0.31097$.

From the simulations made varying the parameters, it emerges that the negative sign of the three eigenvalues persists even with variations of $\epsilon_1, \epsilon_2, \sigma_1, \sigma_2, \beta_1,$

¹¹Results are available from the authors upon request.

¹²Notice that this change in the parameter values is necessary to satisfy the conditions for an attractive fixed point in the natural dynamics, i.e. $\rho_1 + \rho_2 < 0$ and $\rho_1\rho_2 - \gamma_1\gamma_2 > 0$

β_2 . This suggests, therefore, that if a fixed point is attractive for the natural dynamics, its stability properties are preserved by human intervention as the fixed point of the integrated model can always be reached by the growth trajectories, independently of the parameter values.

The same applies when the fixed point of the natural dynamics is a saddle point, as it can be easily verified through the use of numerical examples.¹³

2.3 Multiplicity of fixed points

A particularly interesting result is that a multiplicity of fixed points can arise in the economic growth model, even though the natural dynamics, being linear, only allows at most one steady state. This can be easily proved by solving the system (15)-(20) for the following set of parameter values:

$$\beta_1 = 30, \rho_1 = 0.003, \gamma_1 = -0.18, \beta_2 = 0.5, \rho_2 = -0.005, \gamma_2 = 1.55e^{-4}, \alpha = \mu = 0.5, a = b = \epsilon_1 = \epsilon_2 = \sigma_1 = \sigma_2 = 10, r = 0.008.$$

In this case, we discover the contemporary presence of three fixed points that have, respectively, three, two and only one negative eigenvalue. As pointed out above, only the fixed point with three negative eigenvalues can be reached by the growth trajectory. As we see, starting from a very simple natural dynamics (linear and therefore with at most one fixed point) the integrated model generates a fairly complex dynamics of economic growth.

Figure 6 shows the simulation results obtained by varying β_1 in the interval [20, 200], with a rate of 1. The two curves in the diagram connect changes in the sign of the eigenvalues to changes in x and β_1 . The upper branch of the curve represents the set of pairs (x, β_1) corresponding to the fixed point with 2 negative eigenvalues, whereas the lower branch refers to the fixed point with 3 negative eigenvalues. The two branches meet in correspondence of $x = 22640.72$ and $\beta_1 = 183.088$. An analogous representation can be made for the species y .

The curve which represents the fixed point with 1 negative eigenvalue is not shown in figure 6 for scale reasons. To overcome this drawback, we also report a diagram that shows how changes in β_1 modify the number of simultaneously existing equilibria. For this purpose, figure 7 represents the range of values of β_1 for which each of the three fixed points exists. As the figure shows, when β_1 is above 183.088 (i.e. the meeting point of the two branches in figure 6), only the fixed point with 1 negative eigenvalue exists. However, when β_1 falls below that threshold level, also the other two fixed points emerge from the analysis, rising from one to three the possible steady state equilibria of the model. When β_1 falls even further (approximately around $\beta_1 = 60$), the fixed point with three positive eigenvalues disappears.¹⁴ This changes again the number of simultaneously existing equilibria from three to two, none of which can be locally reached by the growth trajectories.

¹³Simulation results are available from the authors upon request.

¹⁴This is represented by the discontinuity of the lower branch of the curve in figure 6 and by the discontinuity of the lower line in figure 7.

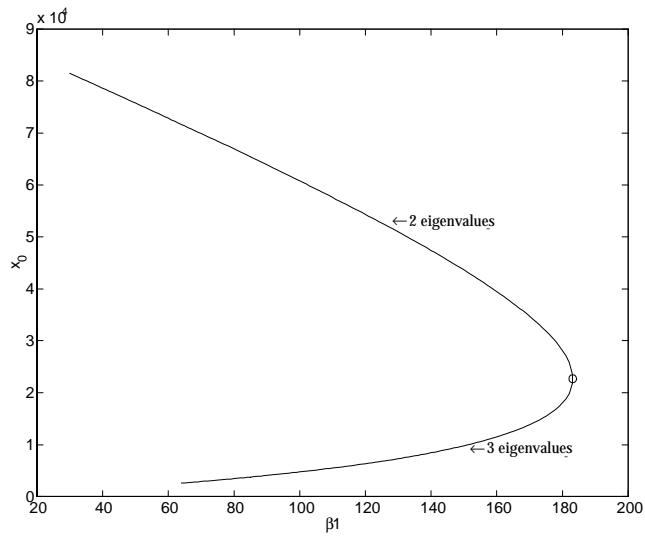


Figure 6

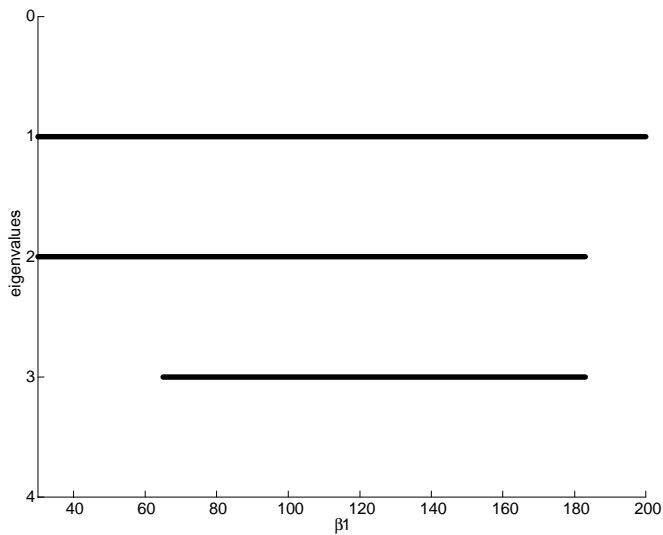


Figure 7

Finally, we also performed a simulation, varying the initial values of the two species, at the converge point $x = 22640.72$ of the two branches with 3 and 4 positive eigenvalues. This simulation (shown in figure 8) enables us to “glimpse” a dynamics of considerable interest, that seems to suggest the possible existence of fairly complex dynamics around the fixed point.¹⁵

¹⁵The analysis of the dynamics out of the equilibrium, however, goes beyond the scope of the present paper and is therefore left for future research.

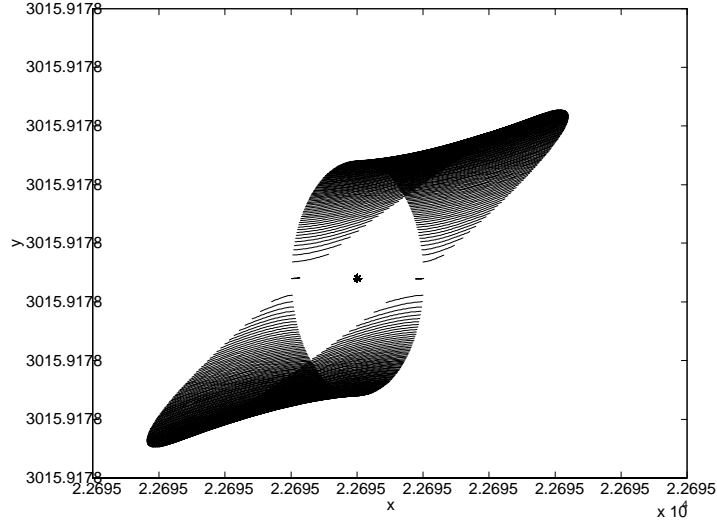


Figure 8

3 Model 2

Let us now examine a variant of model 1 in which the defensive expenditures are diversified, that is some are specifically allocated to sustaining species x , others to species y . Assuming that the representative agent has the same objective function as before, the optimization problem will now look as follows:

$$\max_{c, d_1, d_2} \int_0^{\infty} (\ln x + a \ln y + b \ln c) e^{-rt} dt \quad (22)$$

subject to:

$$\dot{k} = k^\alpha - c - d_1 - d_2 \quad (23)$$

$$\dot{x} = \beta_1 + \rho_1 x + \gamma_1 y - \epsilon_1 c + \sigma_1 d_1^\mu \quad (24)$$

$$\dot{y} = \beta_2 + \rho_2 y + \gamma_2 x - \epsilon_2 c + \sigma_2 d_2^\mu \quad (25)$$

where defensive expenditures d_1 are specific to species x , while d_2 are specific to species y .

The current value Hamiltonian function is:

$$H = \ln x + a \ln y + b \ln c + \theta(\beta_1 + \rho_1 x + \gamma_1 y - \epsilon_1 c + \sigma_1 d_1^\mu) + \eta(\beta_2 + \rho_2 y + \gamma_2 x - \epsilon_2 c + \sigma_2 d_2^\mu) + \lambda(k^\alpha - c - d_1 - d_2)$$

From the maximum principle, we have:

$$\frac{\partial H}{\partial c} = \frac{b}{c} - \lambda - \epsilon_1\theta - \epsilon_2\eta = 0 \quad (26)$$

$$\frac{\partial H}{\partial d_1} = -\lambda + \mu\sigma_1 d_1^{\mu-1}\theta = 0 \quad (27)$$

$$\frac{\partial H}{\partial d_2} = -\lambda + \mu\sigma_2 d_2^{\mu-1}\eta = 0 \quad (28)$$

$$\dot{x} = \frac{\partial H}{\partial \theta} = \beta_1 + \rho_1 x + \gamma_1 y - \epsilon_1 c + \sigma_1 d_1^\mu \quad (29)$$

$$\dot{y} = \frac{\partial H}{\partial \eta} = \beta_2 + \rho_2 y + \gamma_2 x - \epsilon_2 c + \sigma_2 d_2^\mu \quad (30)$$

$$\dot{k} = \frac{\partial H}{\partial \lambda} = k^\alpha - c - d_1 - d_2 \quad (31)$$

$$\dot{\theta} = r\theta - \frac{\partial H}{\partial x} = (r - \rho_1)\theta - \gamma_2\eta - \frac{1}{x} \quad (32)$$

$$\dot{\eta} = r\eta - \frac{\partial H}{\partial y} = (r - \rho_2)\eta - \gamma_1\theta - \frac{a}{y} \quad (33)$$

$$\dot{\lambda} = r\lambda - \frac{\partial H}{\partial k} = \lambda(r - \alpha k^{\alpha-1}) \quad (34)$$

Substituting to the control variables c , d_1 and d_2 from (26)-(28) into (29)-(34), the fixed points of the model are the solutions of the following system of six equations:

$$\beta_1 + \rho_1 x + \gamma_1 y - \frac{\epsilon_1 b}{\lambda + \epsilon_1\theta + \epsilon_2\eta} + \frac{\sigma_1^2\theta}{2\lambda} = 0 \quad (35)$$

$$\beta_2 + \rho_2 y + \gamma_2 x - \frac{\epsilon_2 b}{\lambda + \epsilon_1\theta + \epsilon_2\eta} + \frac{\sigma_2^2\eta}{2\lambda} = 0 \quad (36)$$

$$k^\alpha - \frac{b}{\lambda + \epsilon_1\theta + \epsilon_2\eta} - \left(\frac{\sigma_1\theta}{2\lambda}\right)^2 - \left(\frac{\sigma_2\eta}{2\lambda}\right)^2 = 0 \quad (37)$$

$$(r - \rho_1)\theta - \gamma_2\eta - \frac{1}{x} = 0 \quad (38)$$

$$(r - \rho_2)\eta - \gamma_1\theta - \frac{a}{y} = 0 \quad (39)$$

$$\lambda(r - \alpha k^{\alpha-1}) = 0 \quad (40)$$

3.1 Stability analysis of model 2

The local stability analysis of the natural dynamics is obviously the same as in model 1, therefore we focus here on the local behavior of the economic growth dynamics of model 2.

The Jacobian matrix of model 2 is:

$$Jc = \begin{pmatrix} \rho_1 & \gamma_1 & 0 & \frac{\epsilon_1^2 b}{\phi^2} + \frac{\sigma_1^2}{2\lambda} & \frac{\epsilon_1 b \epsilon_2}{\phi^2} & \frac{\epsilon_1 b}{\phi^2} - \frac{\sigma_1^2 \theta}{2\lambda^2} \\ \gamma_2 & \rho_2 & 0 & \frac{\epsilon_1 b \epsilon_2}{\phi^2} & \frac{\epsilon_2^2 b}{\phi^2} + \frac{\sigma_2^2}{2\lambda} & \frac{\epsilon_2 b}{\phi^2} - \frac{\sigma_2 \eta}{2\lambda^2} \\ 0 & 0 & \alpha k^{\alpha-1} & \frac{\epsilon_1 b}{\phi^2} - \frac{\sigma_1^2 \theta}{2\lambda^2} & \frac{\epsilon_2 b}{\phi^2} - \frac{1}{2} \frac{\sigma_2^2 \eta}{\lambda^2} & \frac{b}{\phi^2} + \frac{\sigma_1^2 \theta^2}{2\lambda^3} + \frac{\sigma_2^2 \eta^2}{2\lambda^3} \\ \frac{1}{x^2} & 0 & 0 & r - \rho_1 & -\gamma_2 & 0 \\ 0 & \frac{\alpha}{y^2} & 0 & -\gamma_1 & r - \rho_2 & 0 \\ 0 & 0 & \frac{\alpha(1-\alpha)\lambda}{k^{2-\alpha}} & 0 & 0 & r - \alpha k^{\alpha-1} \end{pmatrix}$$

where, as before, $\phi := \epsilon_1 \theta + \epsilon_2 \eta + \lambda$.

Given the similarity between this matrix and the corresponding Jacobian in model 1, the observations made on that model using the simplified matrix Jc'' hold true even in this case. Model 2 behaves in a substantially similar manner to the previous model. The only significant difference is that with discriminating defensive expenditures the “stabilization” effect is amplified, that is, we obtain fixed points with three negative eigenvalues (i.e. that can be reached by the growth trajectories) for higher values of x and y . In the case of specific defensive expenditures, therefore, agents intervene to stabilize the fixed point at an earlier stage than before, without awaiting a further decline in the stock of the two species as in the case of generic defensive expenditures. This can be seen from figure 9 that compares the simulation results in the two models for r which varies in the interval $[0.2, 0.75]$ with a rate of 0.1. As the figure shows, for any given value of r the threshold level of x and y is higher in model 2 than in model 1.¹⁶ Observe that this difference diminishes with the increase in the value of r . In fact, the more impatient the agents (the higher r), the lower the capital accumulation level and thus also the correspondent defensive expenditures. A lower level of defensive expenditures reduces, therefore, their capacity to stabilize the fixed point, no matter whether they are generic or specific, hence decreasing also the difference in the effects between the two kinds of intervention.

¹⁶Recall that the threshold level of x and y represents the value of the two variables at the fixed point above which we pass from 3 to 1 negative eigenvalues, that is, from a reachable to an unreachable fixed point.

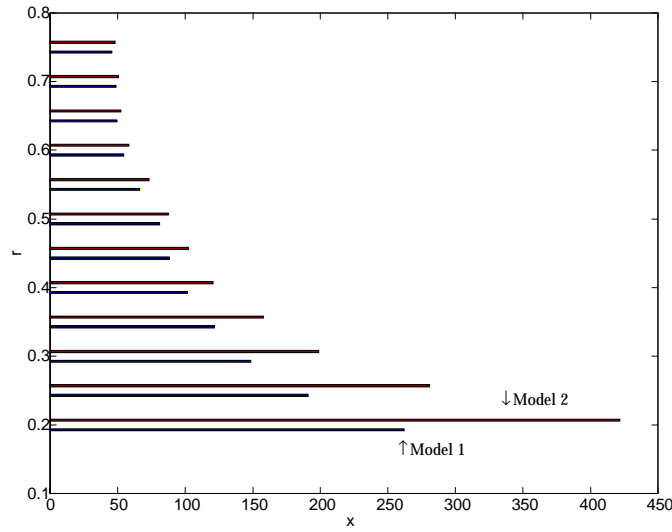


Figure 9

In the following section we shall examine the variant in which the utility depends on an “aggregate” measure of the two species; more specifically, we shall assume that x and y are perfect substitutes with a marginal substitution rate equivalent to a . As we shall see, in this case we do not have a stabilization effect for low values of x and y . Instead what happens is that with low values of x and y there are no longer fixed points for the growth dynamics.

4 Model 3

In this case we assume that the objective function of the representative agent comprises the global stock of environmental resources (here represented by the weighted sum of the two species) and not the number of individuals of each species. The optimization problem thus becomes:

$$\max_{c,d} \int_0^{\infty} [\ln(x + ay) + \ln c] e^{-rt} dt \quad (41)$$

under the constraints (3)-(5) of model 1. The correspondent current value Hamiltonian function is:

$$H = \ln(x + ay) + \ln c + \theta(\beta_1 + \rho_1 x + \gamma_1 y - \epsilon_1 c + \sigma_1 d^\mu) + \eta(\beta_2 + \rho_2 y + \gamma_2 x - \epsilon_2 c + \sigma_2 d^\mu) + \lambda(k^\alpha - c - d)$$

From the maximum principle it follows that the dynamics of x , y and k is defined by the equations (9)-(11) while the dynamics of the co-state variables is given by the following equations:

$$\dot{\theta} = r\theta - \frac{\partial H}{\partial x} = (r - \rho_1)\theta - \gamma_2\eta - \frac{1}{(x + ay)}$$

$$\dot{\eta} = r\eta - \frac{\partial H}{\partial y} = (r + \rho_2)\eta + \gamma_1\theta - \frac{a}{(x + ay)}$$

$$\dot{\lambda} = r\lambda - \frac{\partial H}{\partial k} = r\lambda - \alpha\lambda k^{\alpha-1} = \lambda(r - \alpha k^{\alpha-1})$$

where c and d must be chosen in such a way as to satisfy:

$$\begin{aligned} \frac{\partial H}{\partial c} &= \frac{1}{c} - \lambda - \epsilon_1\theta - \epsilon_2\eta = 0 \\ \frac{\partial H}{\partial d} &= -\lambda + \mu\sigma_1 d^{\mu-1}\theta + \mu\sigma_2 d^{\mu-1}\eta = 0 \end{aligned}$$

The fixed points are given by the solutions of the following system:

$$\begin{aligned} \beta_1 + \rho_1 x + \gamma_1 y - \frac{\epsilon_1}{\lambda + \epsilon_1\theta + \epsilon_2\eta} + \frac{\sigma_1(\sigma_1\theta + \sigma_2\eta)}{2\lambda} &= 0 \\ \beta_2 + \rho_2 y + \gamma_2 x - \frac{\epsilon_2}{\lambda + \epsilon_1\theta + \epsilon_2\eta} + \frac{\sigma_2(\sigma_1\theta + \sigma_2\eta)}{2\lambda} &= 0 \\ k^\alpha - \frac{1}{\lambda + \epsilon_1\theta + \epsilon_2\eta} - \left(\frac{\sigma_1\theta + \sigma_2\eta}{2\lambda}\right)^2 &= 0 \\ (r - \rho_1)\theta - \gamma_2\eta - \frac{1}{(x + ay)} &= 0 \\ (r - \rho_2)\eta - \gamma_1\theta - \frac{a}{(x + ay)} &= 0 \\ \lambda(r - \alpha k^{\alpha-1}) &= 0 \end{aligned}$$

4.1 Stability analysis of model 3

Since the natural dynamics is obviously unchanged, like in model 2 it is here sufficient to analyze the local behavior of the economic growth dynamics to compare the stability properties of the systems with and without human action.

The Jacobian matrix of model 3 is:

$$Jc = \begin{pmatrix} \rho_1 & \gamma_1 & 0 & \frac{\epsilon_1^2 b}{\phi^2} + \frac{\sigma_1^2}{2\lambda} & \frac{\epsilon_1 b \epsilon_2}{\phi^2} + \frac{\sigma_1 \sigma_2}{2\lambda} & \frac{\epsilon_1 b}{\phi^2} - \frac{\sigma_1 \psi}{2\lambda^2} \\ \gamma_2 & \rho_2 & 0 & \frac{\epsilon_1 b \epsilon_2}{\phi^2} + \frac{\sigma_1 \sigma_2}{2\lambda} & \frac{\epsilon_2^2 b}{\phi^2} + \frac{\sigma_2^2}{2\lambda} & \frac{\epsilon_2 b}{\phi^2} - \frac{\sigma_2 \psi}{2\lambda^2} \\ 0 & 0 & \alpha k^{\alpha-1} & \frac{\epsilon_1 b}{\phi^2} - \frac{\sigma_1 \psi}{2\lambda^2} & \frac{\epsilon_2 b}{\phi^2} - \frac{1}{2} \frac{\sigma_2 \psi}{\lambda^2} & \frac{b}{\phi^2} + \frac{1}{2} \frac{\psi^2}{\lambda^3} \\ \frac{1}{(x+ay)^2} & \frac{a}{(x+ay)^2} & 0 & r - \rho_1 & -\gamma_2 & 0 \\ \frac{a}{(x+ay)^2} & \frac{a^2}{(x+ay)^2} & 0 & -\gamma_1 & r - \rho_2 & 0 \\ 0 & 0 & \frac{\alpha(1-\alpha)\lambda}{k^{2-\alpha}} & 0 & 0 & r - \alpha k^{\alpha-1} \end{pmatrix}$$

where ϕ and ψ have the same values as before. This matrix is almost identical to the Jacobian in model 1, the only difference concerns the first two terms in the fourth and fifth rows.¹⁷ In model 1 all these terms became negligible if x and y were *both* sufficiently high. In this case, instead, it is sufficient that x or y gets sufficiently high for this property to apply. Therefore, the observations made in model 1 using the simplified matrix Jc' in this case hold true even if only one of the two species is very high. This means that -differently from models 1 and 2- we do not have here a “stabilization” effect even if, for example, x is very high but y is very low (and therefore at risk of extinction). Since the representative agent cares for the aggregate level of the two species, the potential extinction of one of them does not necessarily provide disutility to the agent as long as its reduction (and eventual disappearance) is compensated by the increase in the other species.

Figure 10 shows the simulation results obtained by varying β_1 as in the previous models. When $\beta_1 > 9.8$ it turns out that there exists a fixed point but it cannot be reached by the growth trajectories, whereas when $\beta_1 < 9.8$ there are no fixed points, as suggested by the discontinuity of the curves in the figure. Therefore, with the objective function of model 3, the fixed points with low x and y not only fail to be “stabilized” but actually cease to exist.

Observe that, like in model 1, $k = (c+d)^{\frac{1}{\alpha}}$ and $(c+d) = (\frac{r}{\alpha})^{\frac{\alpha}{\alpha-1}}$ at the fixed point. Even in this case, therefore, capital as well as the sum of consumption and defensive expenditures are constant as β_1 changes.

¹⁷Recall that in model 1 these terms were $1/x^2$ and 0 in the fourth row, 0 and $1/y^2$ in the fifth row of the Jacobian.

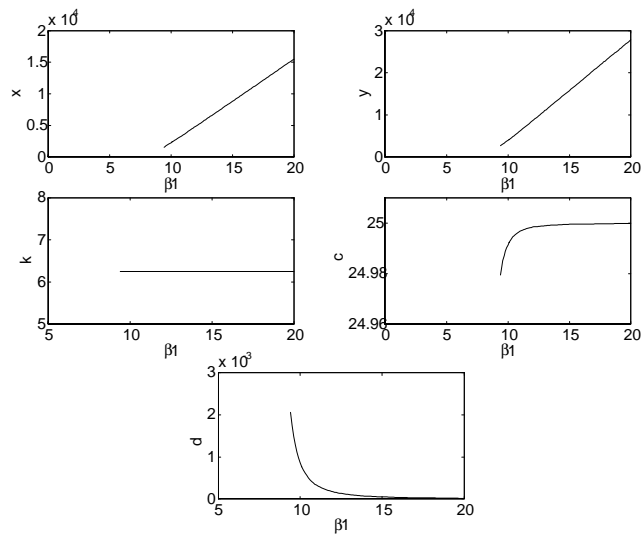


Figure 10

5 Concluding remarks

The objective of the present study was to analyze how human activity can alter the natural ecological dynamics. To this end, focusing attention on local stability analysis, we examined the impact which economic growth and defensive expenditures can have on the stability of the fixed point of the ecological dynamics. Although this study is merely a preliminary and not exhaustive analysis of the complex relations existing between economic activity and ecological evolution, some interesting results emerge from the simple analytical formulation adopted here that can contribute to stimulate the current debate on the role of man in the defence of biodiversity. In the first place, the solution of model 1 suggests that it is possible to stabilize the fixed point of the ecological dynamics and consequently to avoid the extinction of a species, even in the absence of defensive expenditures specifically finalized at the protection of that species.¹⁸ This stabilization of the fixed point takes place only when x and y are sufficiently low. As a matter of fact, in this case the agents have an incentive to make defensive expenditures in order to avoid the loss of biodiversity (i.e. the extinction of one or both species) that could derive from oscillations around an equilibrium with low values of x and y . Nevertheless, comparing model 1 (in which defensive expenditures do not distinguish between the two species) with model 2 (in which they are specific for each species), we observe that in the latter case the stabilization of the fixed point occurs for higher values of x and y .

¹⁸This is what has actually happened, for instance, in the Marquesas where twenty years ago the local authority decided to prohibit access to an island to protect its ecosystem. This generic defensive action (that had a clear opportunity cost in terms of foregone tourism) has led to the stabilization of the population of grey partridges that build their nests in that island (National Geographic, 2003, p.123).

Thus, for instance, if the agents can implement activities specifically designed to protect the dolphins (e.g. providing them with the necessary food, prohibiting fishing with “cast nets”¹⁹, etc...) their protection from the risk of extinction occurs at an earlier stage, without awaiting a further reduction in their number as occurs in the case of defensive expenditures that protect the sea in a generic sense. The introduction of specific defensive expenditures tends, therefore, to “anticipate” the stabilizing effect of human intervention.

The above result appears to be coherent with the opposite effect which emerges in model 3 in which the representative agent cares for the set of existing species as a whole, but does not consider any of them essential in her utility function. In this case the fixed points are not stabilized even when there is an endangered species, because the extinction of that species does not necessarily provide disutility to the agent as long as its reduction (and eventual disappearance) is compensated by the increase in the other species.

From the analysis of the models, moreover, it emerges that human activity can have an even deeper impact on the ecological dynamics, substantially modifying not only the stability of the fixed points, but also their number. This is what happens, for example, in model 1, where the introduction of human activity into the analytical formalization increases from one to three the number of the equilibria of the ecological dynamics.

The results obtained in this work seem to suggest that, although the present model is very simple, it may provide some testable hypothesis on the effects that different policies may have on the stability of an ecosystem and on the number of its equilibria. The preliminary analysis developed in this paper can be extended in various directions in the future. In the first place, while the analysis conducted so far is of a purely local nature, it would be interesting to carry out also a global analysis to verify the possible existence of limit cycles or even more complex dynamics beyond the equilibrium point, as some simulations performed in the paper would seem to suggest (see figures 5 and 8). In the second place, the conflict between stabilization and preservation of the natural dynamics that emerge in the paper suggests some extensions of the analysis that would be interesting to examine in the future. As we have seen in our formalizations, human intervention can lead to the stabilization of the fixed points, even when these were not stable for the natural dynamics. The tendency to modify the natural dynamics can be even more accentuated in reality by human preferences and species’ behaviors. As to human preferences, people may be more willing to pay to protect from extinction a species that is perceived as a defenceless prey (e.g. the panda) rather than some aggressive and dangerous predator (e.g. the shark). As a consequence, the social planner may finance more the interventions that defend one species than another. Similarly, in the management of a natural park, the managing body may have a specific ecological model in mind (that differs from that of the natural ecological dynamics) in which one species may be preferred to another because it attracts more tourists to the park. In this

¹⁹It is estimated that this method of fishing (that is very popular in the eastern tropical Pacific) has caused the death of over seven million dolphins and the extinction of two species of dolphins in the last thirty years (Wallach and Sforza, 2000).

case, the social planner may be induced to stabilize the fixed point at a higher number of individuals of one species than another. In both examples, therefore, human preferences may help to preserve single species from extinction, but they also deeply modify the natural interaction dynamics among different species.

Moreover, defensive expenditures may alter the natural ecological dynamics even because they change species' behavior. Suppose, for instance, that x and y represent respectively the number of sardines and of tuna fish, and that the defensive intervention takes the form of supplying man-made food to the two species. In this case, the sardines and tuna fish would increase in number, but the tuna fish would partially lose their typical predatory instinct towards the sardines, thus altering what would have been the natural evolution of the two species. Feeding species to avoid their extinction can therefore preserve biodiversity, but this may come at the cost of an irreversible loss in the dynamics of the ecosystem.

In conclusion, biodiversity plays an essential role in both human survival (favoring the ecological equilibrium) and in anthropic activities (providing, for example, active principles which can be exploited for the production of medicinal products in the pharmaceutical industry). The loss of biodiversity therefore constitutes a significant risk for mankind, resulting in the potentially irreversible loss of the genetic information comprised within it. For this reason the protection of biodiversity is extremely important to avoid entering the shadow area in which risks are uncertain and potentially irreversible. However, as we have tried to demonstrate in this study, the attempt to protect biodiversity may alter the natural ecological dynamics, generating a conflict between the capacity to preserve biodiversity for the benefit of future generations and the possibility of leaving them the natural dynamics of the ecosystems. The need to harmonize these different requirements raises the problem of the criterion governing the choice of the objective function on the part of the policy maker. Far from pretending to provide exhaustive answers to such a complex argument, this paper has simply tried to highlight this dilemma of choice so as to bring to the fore an aspect which has up to now been frequently ignored in the debate on the protection of biodiversity.

6 Appendix

We can distinguish three cases depending on whether the dynamics (1) has a repulsive fixed point, an attractive fixed point and a saddle point.

6.1 Repulsive fixed point for the natural dynamics

The equation $EQ_1 = 0$ produces two eigenvalues with positive real part. The equation $EQ_2 = 0$ generates 2 real positive eigenvalues, having the known term always positive and the coefficient of z always negative. Recall that from (18) r is always greater than ρ_1 . Finally, the equation $EQ_3 = 0$ features a negative and

a positive solution. To sum up, the representation of the sign of the eigenvalues is the following

$$\begin{array}{l} EQ_1 \quad + \quad + \\ EQ_2 \quad + \quad + \\ EQ_3 \quad + \quad - \end{array}$$

From the above representation, we observe that there is only one negative eigenvalue; this means that the fixed point cannot (generically) be reached by the economic growth trajectory. Consequently, the property of instability of the fixed point of the natural dynamics is transferred unaltered to the economic growth dynamics.

6.2 Attractive fixed point for the natural dynamics

The equation $EQ_1 = 0$ generates two eigenvalues with negative real part so that the fixed point of the natural dynamics is a sink. The equation $EQ_2 = 0$ has two positive real eigenvalues, and the equation $EQ_3 = 0$ two eigenvalues of the opposite sign. In synthesis, the following picture emerges:

$$\begin{array}{l} EQ_1 \quad - \quad - \\ EQ_2 \quad + \quad + \\ EQ_3 \quad + \quad - \end{array}$$

Consequently we have three negative eigenvalues and the fixed point can be reached from the growth trajectory; this means that the maximum values of the variables x and y , as well as that of the variable k , are constant. We would recall that, as far as the variables x and y are concerned, this also occurred in the natural dynamics.

6.3 Saddle-point for the natural dynamics

It is easy to observe that in this case the configuration of the signs of the eigenvalues is given by

$$\begin{array}{l} EQ_1 \quad + \quad - \\ EQ_2 \quad + \quad + \\ EQ_3 \quad + \quad - \end{array}$$

Consequently, in this case too the fixed point is not generically reached by the growth trajectory.

7 Bibliographical references

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