UFZ-Discussion Papers

**Departments of
Economics, Sociology and Law (OEKUS)
and Ecological Modelling**

4/2003

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May 2003

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Abstract
Recent political developments and academic debate indicate that future political commitment to the protection of natural resources is uncertain. This political uncertainty is particularly problematic when the danger of irreversible damage such as the extinction of species looms. Accordingly, policies are needed which can ward off such damage. This paper analyses a particular policy regarding a situation where the survival of an endangered species depends on certain types of biodiversity-enhancing land-use measures being carried out regularly, yet due to uncertain political commitment the periodical availability of a budget to finance these measures is not guaranteed. To insure against future underfunding for conservation, a fund is established which allows money to be saved for conservation in later periods. To maximise the long-term survival of the endangered species, it has to be decided in each period whether to spend the available money now or to allocate it to the fund for future use. The paper provides an ecological-economic model for this dynamic optimisation problem.

Key words: Species conservation, ecological-economic modelling, political uncertainty, environmental policy

JEL: Q20

1. Introduction[1]

Over the last thirty years, the need to protect natural resources and the environment has been increasingly recognised. This has resulted in a wide range of corresponding policies at both a national and an international level. However, recent political developments in some countries show signs of this trend being reversed. One prominent example is the Bush administration's decision to abandon the 1997 Kyoto Protocol in March 2001. The new tendency in some countries' policies is mirrored in the intellectual debate on environmental issues. Bjørn Lomborg's book "The sceptical environmentalist", for instance, has received great attention. Lomborg basically argues that the dangers of natural resource depletion and environmental destruction often are exaggerated. However, whether this new political and intellectual trend is merely short-lived or alternatively marks the beginning of a sustained rollback in environmental policy is not yet apparent. In other words, future political developments are shrouded by uncertainty.

Political uncertainty may pose a particular problem if the danger of irreversible damage looms. One area where this is particularly relevant and which is addressed in this paper is the conservation of endangered species. For example, the policy of a government currently in power to protect such a species may irrevocably founder if a subsequent government fails to continue this policy and the later reintroduction of conservation measures is too late to save the species from extinction. Therefore, governments concerned by the long-term prevention of species loss need to examine how it can be ensured that their current policy aims will be achieved without having to worry about future political uncertainty.

This paper addresses this task with regard to a situation which is typical of the conservation of many endangered species. Such a situation is characterised by the need to regularly carry out certain types of biodiversity-enhancing land-use measures to ensure the survival of the species and political uncertainty hanging over the regular availability of a budget to finance these measures. More specifically, we are concerned with a situation where the current budget is reasonably high and future conservation budgets are expected to decline in the medium term (although the size of these budgets is not known with any certainty).

Such a situation may arise when a future government's preferences are highly likely to shift away from biodiversity conservation – for example if sentiments to this effect are expressed by an opposition with a good chance of electoral success. Though primarily motivated by political uncertainty, the paper's analysis is generally relevant to any situation where a decrease in funding for conservation can be anticipated. One example of such a situation is when an economic downturn looms, which may prompt cuts in public spending and consequently – assuming the government at this time considers conservation a normal good – result in less public funding for conservation. Another example is that of a foundation or NGO which finances conservation measures and which expects a decline in donations.

A government that intends to take the danger of a dwindling budget into account into its conservation policies has to establish an institutional framework to ensure the money will be available for future conservation regardless of subsequent governments' preferences. One way of doing this is to establish a conservation trust fund which is independent of any future government's decisions. Below, we assume that this option is taken by the present government. Thus, the government has to decide in each period whether to spend the money available for conservation now or to allocate it to the trust, which will use it to compensate for any funding cuts and to finance conservation measures in the future.

[1] Comments by Stefan Baumgärtner on a previous version of this paper are highly appreciated.

We assume that the government is unable to give any more money to the fund than what it regularly spends on conservation. This is a realistic assumption because such a measure may require raising taxes or increasing the public debt and may therefore be unpopular with voters, jeopardising the government's chances of re-election. If the assumption is not met and for some reason the government is able to allocate additional money to the fund, this will obviously ease the problem of political uncertainty and thus need less attention. Therefore, under the assumption above, the government is faced with the problem of efficiently allocating a conservation budget over time (spending now or allocating it to the trust for future use) to maximise the survival probability of the endangered species.

As both economic and ecological parameters influence the efficient allocation of the conservation budget over time, an ecological-economic modelling approach has to be chosen. Recently, a growing number of studies have integrated ecological and economic knowledge into the assessment of conservation policies at both a conceptual (e.g. Wu and Bogess 1999, Drechsler and Wätzold 2001, Baumgärtner 2003) and an applied level (e.g. Ando et al. 1998, Richards et al. 1999, Polasky et al. 2001, Johst et al. 2002).

Most of these studies evaluate conservation policies from a static viewpoint. Exceptions include Richards et al. (1999), who develop optimal dynamic fire management strategies for a nature reserve, and Johst et al. (2002), who develop an ecological-economic modelling procedure to determine efficient compensation payments for species protection which are differentiated in not only space but also time. Costello and Polasky (2002) address a different dynamic optimisation problem. They evaluate different choice rules for a conservation agency which receives a budget in each period and has to decide which sites of heterogeneous ecological benefits to purchase first to maximise coverage by different species at the end of a number of periods. The agency has to take into account that sites not included in the reserve system are threatened by the prospect of development occurring with various probabilities.

Our analysis differs from the above studies as it focuses on the problem of uncertainty about future political commitment to protect biodiversity as well as how best to deal with it from the angle of a current government committed to species conservation. To our knowledge, the newly evolving problem of political uncertainty has not yet been addressed in the environmental economics literature with respect to the protection of natural resources and the environment in general or with particular focus on species conservation.

The paper is structured as follows. The basic setup of the model is explained in Section 2. The model solutions are derived in Section 3 using stochastic dynamic programming and interpreted in Section 4. The paper concludes with a discussion of the results in Section 5.

2. The model

2.1 Ecological benefits, economic costs and the objective function

This paper is concerned with a situation where the present government aims to protect an endangered species not only today, but also in the future. We divide time into discrete periods, e.g. years, and assume the government wishes to protect the species over a number of T+1 periods, where T is arbitrary within the scope of this paper. As population dynamics and extinction processes are random processes, extinction cannot be avoided with certainty, although its risk can be minimised. The aim of the government then is to maximise the species' population survival probability P_T over the T+1 periods of interest. To develop a conceptual framework for this optimisation problem we have to find some formal rules that relate biodiversity-enhancing land-use measures with the survival probability of a population in a general way.

In theoretical ecology, it is commonly assumed that the survival probability η of a single population decreases exponentially at a constant rate known as the "population extinction rate" (Levins 1969, Wissel et al. 1994, Hanski 1999).[2] Then the survival probability of the population over a time interval of length Δt is

$$\eta(\Delta t) = \exp(-\nu \Delta t) \tag{1}$$

where ν is the extinction rate. Due to the Markovian nature of population dynamics (Footnote 1), the survival probability in a particular period is not related to the survival probability of the preceding, following or any other periods. Using this, the survival probability over T+1 periods each of length Δt can be written as

$$\eta_T = \prod_{t=0}^{T} \exp(-\nu_t \Delta t) = \exp(-\Delta t \sum_{t=0}^{T} \nu_t) \tag{2}$$

where ν_t is the population extinction rate in period t. Here we assume that the population extinction rate is constant within each period but may vary between periods, depending on land use (see below). Lande (1993) and Wissel et al. (1994) develop general models to describe the stochastic dynamics of endangered populations and to establish how the population extinction rate (ν_t) depends on the population growth rate and the habitat capacity (K_t). The habitat capacity measures the number of individuals the habitat can sustain and is a function of habitat characteristics such as habitat size and quality. According to the two models cited, the extinction rate in period t is given by

$$\nu_t = \frac{a}{K_t^{\alpha}} \tag{3}$$

where a is a species-specific parameter. The exponent α is positive:

$$\alpha > 0, \tag{4}$$

and inversely related to the stochastic variation in the population growth rate (Wissel et al., 1994; Drechsler and Wätzold 2001).

We assume that without biodiversity-enhancing land-use measures the habitat capacity has a value of $K^{(0)}$. If certain biodiversity-enhancing land-use measures are carried out during period t, the habitat capacity is increased from its original value $K^{(0)}$ by an amount κ_t to $K^{(0)}+\kappa_t$. We assume that this gain is maintained only for the time of the measures being carried out, i.e. for the duration of period t. If in the following period no measures are carried out, the habitat capacity is assumed to drop back to its original value $K^{(0)}$. The biodiversity-enhancing land-use measures impose opportunity costs c_t. We assume that the gain in habitat capacity is proportional to these costs with some proportionality factor b,

$$\kappa_t = bc_t \tag{5}$$

i.e. marginal costs are considered to be constant. With Eq. (3) we obtain for the extinction rate in period t:

$$\nu_t = \frac{a}{(K^{(0)} + bc_t)^{\alpha}} = \frac{ab^{-\alpha}}{(C + c_t)^{\alpha}} \tag{6}$$

[2] The basic mathematical assumption is that the population dynamics can be described by a stochastic Markovian birth and death process (e.g. Goel and Richter-Dyn 1974; Nisbet and Gurney 1982) with constant birth and death rates. For the mathematical deduction of Eq. (1), including a discussion of the underlying assumptions, see Wissel et al. (1994) and Drechsler and Wissel (1997).

where

$$C = \frac{K^{(0)}}{b} \tag{7}$$

The present government's conservation aim can now be expressed as the maximisation of the (T+1) period survival probability P_T with ν_t given by Eq. (6). As the numerator of the right-hand side of Eq. (6) as well as the period length Δt (Eq. 2) is the same for all periods t=0…T, this is achieved by maximising the objective function

$$S = \sum_{t=0}^{T} Z_t \qquad \text{where}$$

$$Z_t = -\frac{1}{(C + c_t)^{\alpha}} \tag{8}$$

2.2 The dynamics of the financial budget and the conservation agency's management options

Both present and future governments provide a certain grant in each period for conservational purposes. We denote the grant in period t as g_t and assume that it consists of a deterministic component h_t and – taking into account uncertainty about future economic and political developments – a random component ε_t, such that

$$g_t = h_t + \varepsilon_t \tag{9}$$

for all t=0…T. The ε_t are assumed to be uniformly distributed in the interval

$$\varepsilon_t \in \left[-\sigma, +\sigma\right] \ . \tag{10}$$

such that the probability density function of ε_t is

$$f(\varepsilon_t) = \begin{cases} (2\sigma)^{-1} & \text{for } -\sigma \le \varepsilon_t \le \sigma \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

It is plausible to that the relative variation in the grants is less than 100%, i.e. σ/h_t<1. These governmental grants are considered as exogenous in our model. They constrain the decisions of the conservation agency, which forms a different body and independently decides in each period how the available money should be allocated between present and future conservation. This decision is based on the objective function (Eq. 8), i.e. the aim of maximising the survival probability of an endangered species over time. The agency has to consider that in each period t it can give payments p_t to land-users to increase the habitat capacity in that period by κ_t. It is assumed that land-users will carry out biodiversity-enhancing land-use measures when their costs are covered by the payments (c_t=p_t). In other words, the payments p_t induce biodiversity-enhancing land-use measures of costs c_t=p_t which increase the habitat capacity by κ_t=bc_t=bp_t (cf. Eq. 5). The Z_t from the objective function Eq. (8) then assumes the value

$$Z_t = -\frac{1}{(C + c_t)^{\alpha}} = -\frac{1}{(C + p_t)^{\alpha}} \tag{12}$$

If the payment (p_t) is less than the grant in period t (g_t), the remaining amount (g_t-p_t) can be put into a conservation fund for future use. In this way, the conservation fund increases the independence of the conservation agency from a future decrease in governmental grants. The size of the fund in period t is denoted as F_t with the initial fund size being F_0. It is assumed that the money in the fund does not earn interest (cf. Discussion). Then for all periods t=1…T, the size of the fund is given by the recursive equation

$$F_t = F_{t-1} + g_{t-1} - p_{t-1} \tag{13}$$

In each period t the agency can select and spend an amount

$$0 \le p_t \le \bar{p}_t = F_t + g_t, \quad t = 0...T \tag{14}$$

and save the amount $F_t+g_t-p_t$ for the next period t+1 ($\bar{p}_t$ is the amount of money available in period t).

One problem for the agency is that the grant g_t may vary over time and may do so randomly according to Eq. (9). This means that the agency is faced with a decision under uncertainty. It also means that the objective function, Eq. (8), is random. We assume that the agency's goal is to maximise the expected value of S, i.e.

$$\langle S \rangle = \left\langle \sum_{t=0}^{T} Z_t \right\rangle = -\left\langle \sum_{t=0}^{T} \frac{1}{(C + p_t)^\alpha} \right\rangle \to \max \tag{15}$$

where $\langle \ \rangle$ denotes the mean over all possible future developments of the g_t, subject to the stochastic equation of motion, Eq. (13) and the constraints, Eq. (14).

3. Model analysis

The conservation agency is facing a multi-period decision problem under uncertainty with the standard approach of solution being stochastic dynamic programming (SDP) (cf. e.g. Clark 1990, Dixit and Pindyck 1994, Richards et al. 1999, Costello and Polasky 2002). In SDP the optimal management strategy is determined backwards, i.e. first the optimal decision for the terminal period T is determined. Then – assuming that in T the optimal decision is made – the optimal decision in the preceding period T-1 is determined, and this process is continued until the first period t=0 is reached.

Following Clark (1990, section 11.1), to perform the SDP we introduce the value function

$$J(p_t, t) = \max_{p_t} \left\langle \sum_{j=t}^{T} Z_j \right\rangle \tag{16}$$

which sums the components of the objective function Eq. (15) for periods j=t…T. Maximising the right-hand side of Eq. (16) delivers the optimal payments, p_t. We start with the terminal period t=T and find for the optimal payment[3]

$$p_T{}^* = \bar{p}_T = F_T + g_T \tag{17}$$

(i.e., all money is spent) with the corresponding value

$$J(p_T, T) = -\frac{1}{(C + F_T + g_T)^\alpha} \tag{18}$$

The optimal payment p_{T-1}* in period t=T-1 is given by the Bellman equation

[3] The calculations from Eqs. (17)-(21) are shown in detail in Appendix A.

$$J(p_{T-1},T-1)=\max_{p_{T-1}}\{J(p_{T-1},T)+Z_{T-1}\} \tag{19}$$

whose solution is

$$p_{T-1}^{*}=\min(\overline{p}_{T-1},\hat{p}_{T-1}) \tag{20}$$

where $\overline{p}_{T-1}=F_{T-1}+g_{T-1}$ and

$$\hat{p}_{T-1}=\frac{F_{T-1}+g_{T-1}+h_T}{2}-\frac{\sigma^2(\alpha+2)/6}{F_{T-1}+g_{T-1}+h_T+2C}. \tag{21}$$

If $\hat{p}_{T-1}<\overline{p}_{T-1}$ (*case a*), we have an interior solution where only part of the available budget $\overline{p}_{T-1}$ is spent. If alternatively $\hat{p}_{T-1}\geq\overline{p}_{T-1}$ *(case b*), we have a corner solution where the entire available budget is spent. If we insert Eqs. (18), (20) and (21) into Eq. (19) (cf. Eq. A5 in Appendix A), the corresponding value J for cases (a) and (b) is

$$J(p_{T-1},T-1)=\begin{cases}-\dfrac{1}{(C+0.5(F_{T-1}+g_{T-1}+h_T))^{\alpha}}\left[2+\dfrac{\sigma^2\alpha(\alpha+1)(\alpha+2)/6}{(C+0.5(F_{T-1}+g_{T-1}+h_T))^2}\right] & \text{(case a)}\\ -\dfrac{1}{(C+F_{T-1}+g_{T-1})^{\alpha}}-\dfrac{1}{(C+h_T)^{\alpha}}\left[1+\dfrac{\sigma^2\alpha(\alpha+1)(\alpha+2)/6}{(C+h_T)^2}\right] & \text{(case b)}\end{cases} \tag{22}$$

Now turn to period t=T-2. The structure of the corresponding Bellman equation

$$J(p_{T-2},T-2)=\max_{p_{T-2}}\{Z_{T-2}+J(p_{T-2},T-1)\} \tag{23}$$

which determines the optimal payment p_{T-2}* depends on whether $J(p_{T-2},T-1)$ in Eq. (23) is given by case (a) or by case (b) of Eq. (22). This in turn depends on the relative magnitudes of $\hat{p}_{T-1}$ and $\overline{p}_{T-1}$, which hinge on g_{T-1} and other variables. Unfortunately, g_{T-1} is not yet known in the present period t=T-2. Whether case (a) or (b) applies can only be decided probabilistically and $J(p_{T-2},T-1)$ is the weighted sum of the two quantities in Eq. (22) with the weights being the probabilities of case (a) and case (b), respectively. Consequently, Eq. (23) is considerably more complex than Eq. (19), and cannot be solved analytically. To be able to proceed, we assume for the moment that the agency is not completely ignorant of the future grant g_{T-1} but knows whether case (a) or case (b) applies. Then Eq. (22) has the same structure as Eq. (19) (except that it now depends on the fund F_{T-2} and the grants g_{T-2}, g_{T-1} and g_T). The solution is analogous to that of Eq. (19), and again we have to distinguish between two cases: a case (a) where an interior solution exists and only part of the available budget is spent on conservation: $\hat{p}_{T-2}<\overline{p}_{T-2}=F_{T-2}+g_{T-2}$, and a case (b) where a corner solution exists and all the money, $\overline{p}_{T-2}$, is spent. We can proceed in the same way until we have reached the first period, t=0.

We find that the optimal payment p_{T-k}* in a particular period t=T-k depends on whether in this and the following periods an interior solution $\hat{p}_{T-k+n}<\overline{p}_{T-k+n}$ exists. Let $l\leq k$ be the maximum number of consecutive periods starting from the current period t=T-k, such that for each period an interior solution exists. In mathematical terms:

$$l=\max\{0,\max\{\lambda\in\{1,2,...,k\}\mid\hat{p}_{T-k+n}<\overline{p}_{T-k+n},\forall n\in\{0,1,2,...\lambda-1\}\}\} \tag{24}$$

(in the following, we will speak of a "*chain of length l*"). If for instance we had five periods with the periods t=T-k, T-k+1, T-k+2 and T-k+4 having an interior solution and period t=T-

k+3 having only a corner solution, then we would have a chain of length 3 containing periods $t=T-k$, $T-k+1$ and $T-k+2$. Below we will see that if the payments undergo no variation ($\varepsilon_t=0$), then l is the maximum number of consecutive periods which allow an even allocation of the budget across them. If we assume for a moment l as given, then the optimal payment in period $T-k$ is

$$p_{T-k}*(l) = \frac{1}{l+1}\left(F_{T-k} + g_{T-k} + \sum_{n=1}^{l} h_{T-k+n}\right) - \frac{\sigma^2}{\frac{1}{l+1}\left(F_{T-k} + g_{T-k} + \sum_{n=1}^{l} h_{T-k+n}\right) + C} \cdot \frac{(\alpha+2)}{6} \cdot \frac{\sum_{n=1}^{l} 1/n}{l+1} \tag{25}$$

(proof by induction in Appendix B). To give an example, consider a two-period problem, k=1, which is equivalent to the above solution for period $t=T-1$. If $\hat{p}_{T-1} < \overline{p}_{T-1}$ (case a as above), then Eq. (24) leads to l=max{0,1}=1 and $p_{T-1}* = \hat{p}_{T-1}$ with $\hat{p}_{T-1}$ given by Eq. (21). If $\hat{p}_{T-1} \geq \overline{p}_{T-1}$ (case b as above), Eq. (24) becomes l=max{0,Ø}=0 and Eq. (25) reduces to $p_{T-1}* = \overline{p}_{T-1} = F_{T-1} + g_{T-1}$.

The problem now is that l depends on the future grants, and if these are uncertain, l is uncertain, too. To eliminate this uncertainty from the optimal payment $p_{T-k}*$, a sensible option is to find the mean of all possible chain lengths. For this we need the probability distribution $P(l)$ of the chain length, which is determined in Appendix C. If the $P(l)$ are known the optimal payment in period $t=T-k$ may be approximated[4] by

$$\left\langle p*_{T-k}\right\rangle = \sum_{l=0}^{k} P(l)\, p*_{T-k}(l) \tag{26}$$

Equation (26) means that the optimal payment is the sum of the payments for all l, each payment being weighted by the probability $P(l)$ that the chain length is l.

4. Interpretation of the general solution

To gain a general understanding of the model behaviour, we proceed in a stepwise manner and interpret Eq. (26) by way of three examples. In the first example we investigate the deterministic behaviour of the model. The grants may vary but do so in a deterministic (i.e., predictable) manner, i.e., stochasticity is absent ($\varepsilon_t=0$). In the second example we add stochasticity to the grants ($\varepsilon_t \in [-\sigma,+\sigma]$), but assume that changes in the grants do not exhibit any deterministic changes: $h_t=h$, $F_0=0$. Lastly, in the third example the variation in the grants has both a deterministic and stochastic component. In particular, we assume that the grants have a negative deterministic trend such that the deterministic component of the grant in the first period, h_0, is reduced to $h_0-\delta$ in the first and $h_0-t\delta$ in period t. In addition to this, there is a stochastic variation in the grants described by $\varepsilon_t \in [-\sigma,+\sigma]$. This example takes up

[4] The exact solution for $p_{T-k}*$ would have to be calculated from the Bellman equation for period $T-k$, $J(p_{T-k}, T-k) = \max\limits_{p_{T-2}} Z_{T-k} + J(p_{T-k}, T-k+1)$, where $J(p_{T-k}, T-k+1)$ is a linear combination of the values J for all possible chain lengths l (cf. discussion of Eq. 22). Although an analytical solution of this complex Bellman equation cannot be obtained, calculations indicate that the exact solution is, similar to Eq. (26), a weighted sum of the $p*_{T-k}(l)$ with the weights being the probabilities $P(l)$ multiplied by some correction factors.

the theme of the paper by adopting a 'pessimistic view' of the future political situation regarding species conservation.

Example 1: Deterministic grants, ε_t=0

With all ε_t being zero, Eq. (25) reduces to

$$p_{T-k}*(l) = \frac{1}{l+1}\left(F_{T-k} + \sum_{n=0}^{l} h_{T-k+n} \right) \tag{27}$$

which is just the average of the grants over all l+1 periods from t=T-k to t=T-k+l. Note that without loss of generality, the fund F_{T-k} may be set to zero, as a non-zero value can be considered in the first grant, h_{T-k}. For the moment we assume the chain length l as given.

The fund in the following period is $F_{T-k+1} = F_{T-k} + h_{T-k} - p*_{T-k}$. By setting σ=0 in Eq. (C6) of Appendix C we obtain the optimal payment $p_{T-k+1}*(l) = p_{T-k}*(l)$. Proceeding forwards in time, the same result is obtained for all further periods:

$$p_{T-k+n}*(l) = p_{T-k}*(l), \quad n = 1...l \tag{28}$$

In other words, if l is the chain length, the optimal decision is to spend the same amount of money in each period and this amount is the average of all grants over the l+1 periods.

Now we determine the chain length l. With ε_t=0, Eq. (C10) in Appendix C reduces to

$$\Delta_{T-k+n}(l) = F_{T-k} + \sum_{i=0}^{n} h_{T-k+i} - \frac{n+1}{l+1} d_l > 0 \tag{29}$$

and Eq. (24) can be rewritten as

$$l = \max\{0, \max\{\lambda \in \{1,2,...,k\} \mid \Delta_{T-k+n}(l) > 0, \forall n \in \{0,1,2,...\lambda - 1\}\}\} \tag{30}$$

What do Eqs. (29) and (30) mean? As explained above, without loss of generality we can assume F_{T-k}=0. Then, using Eq. (C4) in Appendix C, Eq. (29) can be rewritten as

$$\frac{1}{n+1}\sum_{i=0}^{n} h_{T-k+i} > \frac{1}{l+1}\sum_{j=0}^{l} h_{T-k+j} \tag{31}$$

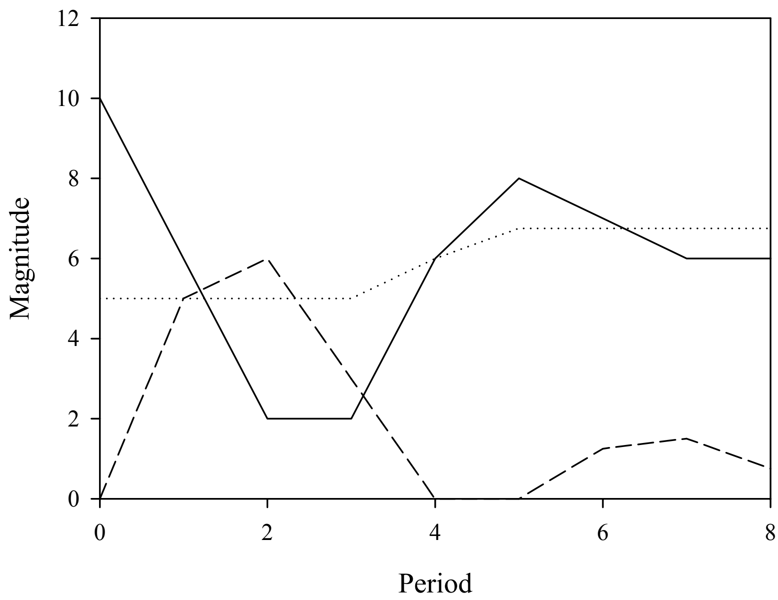
stating that (starting from period T-k) the average of the grants of the first n+1 periods must exceed the average of the grants of all l+1 periods. Equation (30) then says that l+1 is the maximum possible number of consecutive periods such that picking any number $0 \le n \le l$-1 we find that the average grant in the first n+1 periods is higher than that considering all l+1 periods. The reason for this result is that according to Eqs. (27) and (28) it is optimal to re-allocate money from periods with high payments to periods with lower payments such that in the end all periods receive the same amount. Of course, this reallocation is only possible forwards in time, i.e. only from periods T-k+n to periods T-k+n+j with $j \ge 1$. An even allocation of the sum of all l+1 grants, Σh_{T-k+n}, is therefore only possible if the first periods receive higher grants on average than the following ones, and this is guaranteed by Eq. (29).

To obtain a clearer understanding of the arguments, consider the following two cases with T+1 periods, t=0…T (without loss of generality, F_0=0). The simplest case is a constant negative trend in the grants: h_t=h_0-δt>0 for all T+1 periods. As an even allocation of the payments is optimal (Eq. 28), it makes sense to save money in the first half of all periods (when the grants are above average) and to spend this money in the second half (when the grants are below average). The money saved is accumulated in the fund, which increases in the first periods and decreases in the last ones. The average of the grants the first *n* periods receive is always higher than the average of the grants of all periods and *l* assumes its maximum possible value: $l=T$. The optimal payment for the first period is

$$p_0{*} = p_0{*}(l=T) = \frac{1}{T+1}\left((T+1)h_0 - \sum_{t=0}^{T} t\delta\right) = h_0 - \frac{T}{2}\delta \tag{32}$$

and the same optimal payment is obtained for all the following periods, $t=1\ldots T$ (note that a chain of length *l* means that constant payments can be achieved over $l+1$ periods).

Figure 1: *Optimal payments (dotted line) when grants (solid line) first decrease, increase, and then decrease again. The evolution of the fund is shown by the dashed line.*



In the second case (Fig. 1) the grants decrease in the first periods and then increase before decreasing again. In the first period a chain of length $l=3$ exists which contains periods $t=0,1,2,3$. In periods 0 and 1, money is saved which is spent in periods 3 and 4. The payment can be maintained at the level $p^*=5$ during these four periods. Between periods 3 and 5 the grant increases, such that in periods 3 and 4 the chain length is 0 and constant payments cannot be maintained. In period 5, again a chain of length l=3 exists and constant payments can be maintained over periods 5–8 by saving money in period 5 and 6 and spending the saved amount in periods 7 and 8. The analysis demonstrates that the whole process may contain several chains. Each chain can be identified by a plateau in the payment (leading to the required even allocation of the money; (cf. Eqs. (27) and (28)). In times of increasing grants, all the money is spent (expressed in mathematical terms, we are confronted with a corner solution (cf Eq. (20)) and saving money is pointless if an increase in the grants can be expected (in particular, consider the periods $t=3$–5 between the two chains in Fig. 1). Another observation is that the dotted payments curve looks a bit like a gliding average of the solid grants curve, levelling out some of the fluctuations in the grants, and the general rule for selecting the optimal investment then reads: Have as little variation in the payments as feasible within the budget constraints. Equal payments in all $T+1$ periods may be infeasible as we demand that $F_t>0$ for all $t=0\ldots T$. As demonstrated in Fig. 1, equal payments may be achievable only *within* sections (“chains”) of the entirety of all periods. Payments *between* chains differ.

Example 2: Stochastic grants without any deterministic changes, F_0=0, h_t=h and $\varepsilon_t \neq 0$

In this example we investigate the effect of the uncertainty in the grants: $\varepsilon_t \neq 0$ on the optimal payment with the grants having the same magnitude in all periods, t=0…T: F_0=0, and h_t=h_0=h. For the first period, t=0, Eq. (25) becomes

$$p_0*(l) = \frac{1}{l+1}\left(F_0 + g_0 + \sum_{n=1}^{l} h_n\right) - \frac{(\alpha+2)}{6}\cdot\frac{\sigma^2}{\frac{1}{l+1}\left(F_0 + g_0 + \sum_{n=1}^{l} h_n\right) + C}\cdot\frac{\sum_{n=1}^{l} 1/n}{l+1} \tag{33}$$

$$= h - \frac{\sigma^2}{(h+C)}\frac{(\alpha+2)}{6}\frac{1}{l+1}\sum_{n=1}^{l}\frac{1}{n}$$

As the chain length l is not known, we have to take the average over all payments with chain length probabilities $P(l)$, and with Eq. (26) the optimal payment becomes

$$\langle p_0*\rangle = h - \frac{\sigma^2}{(h+C)}\frac{(\alpha+2)}{6}\sum_{l=1}^{k}\frac{P(l)}{l+1}\sum_{n=1}^{l}\frac{1}{n} \tag{34}$$

(note that the first sum now starts from l=1 because for l=0 the second sum is zero). In Appendix B we show that in the absence of deterministic changes in the grants, h_t=h, the probabilities $P(l)$ can be approximated by $P(l)$=1/T for l>0 and $P(0)$=0., i.e. the chain length l may range from 1 to the maximum possible value T, and each l value has the same probability. With this, the optimal payment in period t=0 becomes[5]

$$\langle p_0*\rangle \approx h - \frac{\sigma^2}{(h+C)}\frac{(\alpha+2)}{6}\frac{1}{T}\sum_{l=1}^{T}\frac{1}{l+1}\sum_{n=1}^{l}\frac{1}{n} \approx h - \frac{\sigma^2}{(h+C)}\frac{(\alpha+2)}{6}\frac{(\ln(T)+1)^2}{2T(T+1)}. \tag{35}$$

It can be seen that the uncertainty in the grants reduces the optimal payment by an amount

$$\Delta p_0* \approx \frac{\sigma^2}{h+C}\cdot\frac{(\alpha+2)}{6}\cdot\frac{(\ln(T)+1)^2}{2T(T+1)} \tag{36}$$

which is the product of three factors:

I. The variance of the grants, σ^2, divided by the deterministic value of the grants, h, plus C, which is the initial habitat capacity divided by the scaling factor b (cf. Eq. 7).

II. The term (α+2)/6, where α is species-specific and characterises the relationship between extinction risk and habitat capacity (Eq. 4).

III. A term that depends only on the number of periods, T, and decreases with increasing T.

Altogether, the larger the relative variation in the payments (I), the stronger the non-linearity in the benefit function (II), and the smaller the number of periods (III), the more money is saved for the future.[6]

[5] The optimal decision in the following periods, t>0 can be obtained in an analogous manner (the only difference from the above calculations being that the amount of money available in period t=1 is not h but F_1+h+ε_1=2h-p_0*+ε_1; the analogue applies to all other periods, t>1).

[6] These results could already be derived from the structure of Eq. (25) if we relate the chain length l to the number of periods T.

The reason why stochasticity in the grants leads to reduced payments is due to the fact that the third derivative of the benefit function (Eqs. 8 and 12), d^3Z/dp^3, is positive. This means that the higher the payment, the less the degree of concavity at this point of the benefit function and the less the effect of variation in the payments on the expected benefit. This tallies with results of Leland (1968), who showed in a two-period consumption model with a concave benefit function that if the third derivative of this benefit function is positive, consumption in the first period is reduced in favour of higher consumption in the second – an effect he dubbed "precautionary saving". It also explains the role of the parameter α. The third derivative of the benefit function is proportional to $\alpha(\alpha+1)(\alpha+2)$ and increases with increasing α. Thus, increasing α leads to more precautionary saving. In addition to Leland's result, Eq. (33) shows the effect of the number of periods if this exceeds 2. As stated above, it turns out that the smaller the number of periods, the more money is saved. The reason for this is that the more periods exist, the more likely the stochastic variations in the grants are to cancel each other out, and the less the requirement for precautionary saving.

Example 3: Stochastic grants with a negative expected trend: $\varepsilon_t \neq 0$, $F_0=0$ and $h_t=h_0-t\delta$ with $\delta>\sigma/2$

We now turn to the example that captures the possibility of a political rollback in species conservation. Grants are expected to decline in the medium term but may temporarily increase. This requires the consideration of at least three periods, $t=0,1,2$ such that grants are most likely to decline between the first and the second periods and may further decrease, or change trend and increase, between the second and the third period.[7] The expected decline in grants is described by $h_t=h_0-t\delta$ and possible deviations are described by stochastic variation of magnitude σ. We distinguish between three cases:

1. $\sigma < 2\delta$
2. $\sigma > 4(h_0+1)/(\alpha+2)-2\delta$
3. $2\delta \leq \sigma \leq 4(h_0+1)/(\alpha+2)-2\delta$

In the first two cases, according to Appendix B we find $P(2)=1$ and $P(1)=P(0)=0$ (note that capital P denotes a probability and lower case p a payment). The chain length is exactly $l=2$ and the optimal payment is given by Eq. (25) for $l=2$:

$$p_0^*(l=2) = h_0 - \delta - \frac{\sigma^2(\alpha+2)/12}{h_0+C} \tag{37}$$

We can see that both the trend (δ) and the uncertainty in the grants (σ) reduce the optimal payment p_0*, as expected from the previous examples. In the third case the situation is less clear, because here the probability of obtaining a chain of length $l=2$ is less then one and reads

$$P(l=2) = \frac{1}{2} + \frac{\delta}{\sigma} + \frac{\sigma(\alpha+2)/8}{h_0-\delta+C} \tag{38}$$

For the optimal payment we obtain [with $P(l=1)=1-P(l=2)$]

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[7] A larger number of periods would complicate the analysis significantly because the complexity of the nested integral (Eq. (C12) in Appendix C determining the probability of observing a particular chain length) would increase considerably.

$$p_0{*} = \sum_{l=0}^{2} P(l) p_0{*}(l) = h_0 - \frac{\sigma^2 (\alpha+2)/12}{h_0 + C} - \delta\left(\frac{3}{4} + \frac{\delta}{2\sigma}\right) + O(\sigma^3) \tag{39}$$

Whether $p_0{*}$ increases or decreases with increasing σ can be seen from the first derivative, $\mathrm{d}p_0{*}/\mathrm{d}\sigma$. Astonishingly, it turns out that the first derivative is not always negative, i.e., increasing uncertainty σ does not necessarily lead to a reduction in the optimal payment. Instead, a decrease, i.e. a positive first derivative, is found if

$$\sigma^3 < \frac{3\delta^2}{\alpha+2}(h_0 - \delta) \tag{40}$$

which means that uncertainty calls for an increased payment. The reason for this is that the expected chain length is decreased from $l=2$ to a value $l<2$ (cf. Eq. 38), which has to mutually opposing consequences. First, according to Eq. (32), if there is no uncertainty and a constant negative trend in the grants ($\delta>0$), a shorter chain length leads to *higher* payments and the effect is proportional to δ. Second, according to Eq. (35), if there is no trend but uncertainty in the grants ($\sigma>0$), a shorter chain length leads to *lower* payments and the effect is proportional to σ^2. In Eq. (39) both effects are present. If σ is sufficiently small (Eq. 40), the former dominates and a shorter chain length altogether increases the optimal payment.

The fact that there exist situations that require precautionary saving as well as situations that require precautionary spending is worrying if the aim is to develop guidelines for decision-making. Therefore we investigate how critical this ambiguity is and what possible errors may arise if it is ignored. We hypothesise that even in the difficult third case the optimal payment $p_0{*}$ which is correctly described by Eq. (39) can well be approximated by $p_0{*}(l=2)$ of Eq. (37). This would mean that the relative deviation between the two quantities,

$$\frac{p_0{*} - p_0{*}(l=2)}{p_0{*}(l=2)} = \frac{\frac{\delta}{4}\left(1 - 2\frac{\delta}{\sigma}\right)}{h_0 - \delta - \frac{\sigma^2(\alpha+2)}{12(h_0-\delta)}} \tag{41}$$

is sufficiently small. As grants must be nonnegative, we have $\sigma \le h_0$, and due to $2\delta \le \sigma$ (case 3), the numerator in Eq. (41) is not larger than $\delta(1-2\delta/h_0)/4$. Furthermore, the denominator is not smaller than h_0. Then altogether,

$$\frac{p_0{*} - p_0{*}(l=2)}{p_0{*}(l=2)} \le \frac{\delta}{4h_0}\left(1 - 2\frac{\delta}{h_0}\right) \le \frac{1}{32} \tag{42}$$

i.e. in the third case, the correct payment $p_0{*}$ (Eq. 39) can be approximated by $p_0{*}(l=2)$ of Eq. (37) with a relative error of approximately 3%. As $p_0{*}(l=2)$ clearly decreases with increasing σ, the same holds for $p_0{*}$ with an error less than 3%. Thus any possible increases in the payment are negligible. We conclude that in the three-period case with negative trend and uncertainty in the grants, uncertainty decreases the optimal payment – with possible exceptions of negligible magnitude.

5. Discussion

The starting point of this paper was the problem of political uncertainty, i.e. doubts about political commitment to the protection of natural resources and the environment continuing. Within this general political framework we focus on species conservation, specifically on the important situation where the survival of an endangered species depends on regularly carrying out certain types of conservation measures. It is assumed that future budgets for conservation are expected to decline. The relevance of the paper goes beyond the problem of political uncertainty as such an assumption also encompasses situations where an economic downturn is likely, probably entailing conservation budget cuts, as well as circumstances in which the donations to an NGO that finances conservation measures can be expected to decline. Against this background a conceptual model is developed to determine the efficient allocation of the available financial resources over time to maximise the survival probability of endangered species. The model suggests that the population survival probability is maximised if the payments inducing the biodiversity-enhancing land-use measures are allocated as evenly as possible over time.

The requirement of even payments stems from the structure of the ecological benefit function, Eq. (6), which is additive over the periods with each summand being concave. The additivity of the benefit function reflects the fact that the probability of surviving several periods is the product of the probabilities of surviving the individual periods (Eq. 2). This multiplicative characteristic makes ecological sense: if we acknowledge that memory effects in the population dynamics can be ignored with respect to all the other factors influencing the dynamics, the probability of surviving one period of time is independent of the probability of having survived the previous period of time.

Due to the restriction that the fund must not be negative and the fact that grants may temporarily increase, evenly allocating the payments over all periods may be impossible. In such a case the entirety of all periods has to be divided into appropriate sections called 'chains', such that the periods belonging to the same chain receive identical payments.

To reflect the possibility of a decreasing commitment to conservation, we assumed that a negative trend in the grants can be expected, but that the exact temporal development of the grants is subject to uncertainty. The model results show that in the presence of uncertainty we find cases where the optimal payments in the first periods are smaller compared to a situation where future grants are known, but we also find cases where the optimal payments are larger! The reduction of the optimal payments in the former cases is known as precautionary saving and is explained by the positivity of the third derivative of the benefit function. This argumentation, however, ignores the fact that uncertainty affects the optimal payment via not only the shape of the benefit function but also the chain length. As demonstrated in Example 3 of Section 4, uncertainty may decrease the expected chain length and increase the optimal payment in the first period. Clearly this ambiguity could considerably complicate the management of funds for species protection. However, our results suggest that the chain length effect that may lead to increased payments is relatively small.

The magnitude of precautionary saving also depends on characteristics of the species to be conserved. The variable α (Eq. 3) indicates the variation in a species' population growth rate, and as a general rule a large α corresponds to species with small variation in the population growth rate such as large mammals and a small α to species with large variations such as various insects. The model shows that the larger α, the higher the level of precautionary saving.

In the present paper we have assumed zero interest rates, because the analysis of non-zero interest rates would be extremely laborious. The reason is two-fold. Firstly, as known from the standard microeconomic analysis of an optimal intertemporal consumption decision in a two-period case, there are various ways in which the interest rate affects the allocation of resources over time and there is no single answer. If there are $T>2$ periods, the situation is much more complex and therefore general guidelines are difficult to derive. Secondly, including uncertainty into the analysis turns the allocation problem from a linear into a quadratic one: in the two-period case, the equation that determines the optimal payment[8] becomes quadratic in p_0 if uncertainty ($\sigma^2>0$) is added. Again, this problem multiplies if there are $T>2$ periods. We leave it to future research to analyse in detail the effect of non-zero interest rates on the problem addressed in this paper.

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[8] In two periods the optimal payment p_0* is given by
$\left((C+p_0)(1+r)^{1/(\alpha+1)}\right)\left(C+(g_0-p_0)(1+r)+h_1\right)^{-1} \approx 1-\sigma^2(\alpha+2)(C+g_0+h_1-p_0)^{-2}/6$ (cf. Eq. A9).

Appendix A: Optimal payments p_t* for periods t=T and t=T-1

The optimal payment in period t=T is determined by

$$J(p_T,T)=\max_{p_T}\langle Z_T\rangle=\max_{p_T}\left\langle -\frac{1}{(C+p_T)^\alpha}\right\rangle \tag{A1}$$

under the budget constraint

$$0\le p_T\le \bar{p}_T=F_T+g_T \tag{A2}$$

In this terminal period the agency is faced with a decision under certainty, as both the fund F_T and the grant g_T are known and no further decision will have to be made in the future. Therefore the argument to be maximised in Eq. (A1) is simply $-(1+p_T)^{-\alpha}$. With $\alpha>0$ (Eq. 4) the solution is

$$p_T{}^*=\bar{p}_T=F_T+g_T \tag{A3}$$

with the corresponding value

$$J(p_T,T)=-\frac{1}{(C+F_T+g_T)^\alpha} \tag{A4}$$

In the preceding period t=T-1, the grant in this period, $g_{T\text{-}1}$, is known but the grant in the following period, g_T=h_T+ε_T, is not yet known to the agency, and thus a decision has to be made under uncertainty. The optimal payment is the payment that maximises the SDP recursion equation (also called the Bellman equation) for period t=T-1, which reads (cf. Clark 1990, p. 345)

$$J(p_{T-1},T-1)=\max_{p_{T-1}}\{J(p_{T-1},T)+Z_{T-1}\}=\max_{p_{T-1}}\left\{-\left\langle\frac{1}{(C+F_T(p_{T-1})+g_T)^\alpha}\right\rangle-\frac{1}{(C+p_{T-1})^\alpha}\right\} \tag{A5}$$

Note that the term in brackets $\langle ...\rangle$ contains $J(p_T,T)$ of Eq. (A4) where F_T is evaluated as a function F_T=$F_T(p_{T\text{-}1})$ of the payment $p_{T\text{-}1}$.[9] Due to Eq. (13), F_T depends on $p_{T\text{-}1}$ via F_T=$F_{T\text{-}1}$+$g_{T\text{-}1}$-$p_{T\text{-}1}$. All these quantities are known with certainty in the current period t=T-1. The uncertain element in Eq. (A5) is the grant in the terminal period t=T: g_T=h_T+ε_T with $\varepsilon_T\in[-\sigma,\sigma]$ (Eqs. (9) and (10)).

The exact evaluation of the term in brackets, $\langle ...\rangle$, is not possible. Therefore we proceed as is customary in stochastic dynamic programming and assume that the magnitude of stochastic variation, σ, in the grant g_t is sufficiently small compared to the deterministic value h_t. In particular, if σ/h_T is small then the quantity

$$x=\frac{\varepsilon_T}{C+F_{T-1}+g_{T-1}-p_{T-1}+h_T}<\frac{\varepsilon_T}{h_T}\le\frac{\sigma}{h_T} \tag{A6}$$

is small, too. With x sufficiently small, a mathematical standard technique, Taylor expansion (see e.g. Bronstein and Semendyayew 1985), can be employed to expand the first fraction in Eq. (A5), including terms up to the order of x^3:

[9] This leads to the typical recursive nature of the Bellman equation (cf. Clark 1990).

$$\frac{1}{(C+F_T(p_{T-1})+g_T)^\alpha}=\frac{1}{(C+F_{T-1}+g_{T-1}-p_{T-1}+h_T+\varepsilon_T)^\alpha}$$
$$=\frac{1}{(C+F_{T-1}+g_{T-1}-p_{T-1}+h_T)^\alpha}\left[1-\alpha x+\frac{\alpha(\alpha+1)}{2}x^2+\frac{\alpha(\alpha+1)(\alpha+2)}{6}x^3+O(x^4)\right] \tag{A7}$$

The term $O(x^4)$ contains terms of the order of x^4 or higher which can be neglected if x is not too large (see below).

With Eq. (10) we have

$$\left\langle \varepsilon_T{}^n \right\rangle=\int_{-\infty}^{\infty} f(\varepsilon_T)\varepsilon_T{}^n d\varepsilon_T=(2\sigma)^{-1}\int_{-\sigma}^{\sigma}\varepsilon_T{}^n d\varepsilon_T=\begin{cases}\sigma^n/(n+1) & \text{if } n \text{ even}\\ 0 & \text{if } n \text{ odd}\end{cases}$$

and the expected value on the right-hand side of Eq. (A5) becomes

$$\left\langle \frac{1}{(C+F_T+g_T)^\alpha}\right\rangle \approx \frac{1}{(C+F_{T-1}+g_{T-1}+h_T-p_{T-1})^\alpha}\left[1+\frac{\sigma^2\alpha(\alpha+1)/6}{(C+F_{T-1}+g_{T-1}+h_T-p_{T-1})^2}\right] \tag{A8}$$

with a relative error of the order of x^4 which can be neglected.[10]

We search for the optimal payment p_{T-1}* that solves Eq. (A5) and maximises its right-hand side. To maximise the right-hand side of Eq. (A5), we take the derivative with respect to c_{T-1} and set it to zero: d{…}/dc_{T-1}=0 (the second derivative can easily be shown to be negative (Appendix B), indicating that we indeed obtain a maximum). The solution of this equation is denoted as $\hat{p}_{T-1}$. Using Eq. (A8), with some algebra the equation can be reformulated as

$$\frac{C-\hat{p}_{T-1}}{C+F_{T-1}+g_{T-1}+h_T-\hat{p}_{T-1}}\approx\left[1+\frac{\sigma^2(\alpha+1)(\alpha+2)/6}{(C+F_{T-1}+g_{T-1}+h_T-\hat{p}_{T-1})^2}\right]^{-1/(\alpha+1)}$$
$$\approx 1-\frac{\sigma^2(\alpha+2)/6}{(C+F_{T-1}+g_{T-1}+h_T-\hat{p}_{T-1})^2} \tag{A9}$$

which is solved by

$$\hat{p}_{T-1}=\frac{F_{T-1}+g_{T-1}+h_T}{2}-\frac{\sigma^2(\alpha+2)/6}{F_{T-1}+g_{T-1}+h_T+2C} \tag{A10}$$

Whether Eq. (A10) is the solution of Eq. (A5) depends on whether it fulfils the budget constraint

$p_{T-1}\le\bar{p}_{T-1}=F_{T-1}+g_{T-1}$ (Eq. 14). Three cases are possible:

a. $\hat{p}_{T-1}<\bar{p}_{T-1}$: $\hat{p}_{T-1}$ fulfils the budget constraint and thus the optimal payment is $p_{T-1}{*}=\hat{p}_{T-1}<\bar{p}_{T-1}$

[10] Mathematical textbooks like that by Bronstein and Semendyayev (1985) provide estimations of the remainder terms of Taylor expansions, such as that in Eq. (A7), and it can be shown that the relative error in Eq. (A8) is less than $\alpha(\alpha+1)(\alpha+2)(\alpha+3)[\sigma/(1+F_{T-1}+g_{T-1}+h_T)]^4/96$. If we assume a plausible value α=2, the relative error can be shown to be less than 10%, as long as $\sigma<(h_{T-1}+h_T)/3$. In other words, if we accept a relative error of 10%, the stochastic variation σ in the grants may be up to $(h_{T-1}+h_T)/3$, which is about two-thirds of the deterministic values of the grants! For instance, if $h_{T-1}=h_T$=€9,000, then the stochastic variation σ must not exceed €6,000 to keep the model error below 10%. This is an acceptable constraint and does mean a significant loss of generality.

b1. $\hat{p}_{T-1} = \overline{p}_{T-1}$: $\hat{p}_{T-1}$ just fulfils the budget constraint and thus the optimal payment is $p_{T-1}{*} = \hat{p}_{T-1} = \overline{p}_{T-1}$

b2. $\hat{p}_{T-1} > \overline{p}_{T-1}$: Eq. (A10) violates the budget constraint. As the second derivative of the right-hand side of Eq. (A5) is positive, this right-hand side is a strictly monotonically increasing function of p_{T-1} on the interval [0, $\overline{p}_{T-1}$] and is thus maximised by $p_{T-1} = \overline{p}_{T-1}$. Therefore, again, the optimal payment is $p_{T-1}{*} = \overline{p}_{T-1}$.

Appendix B: Proof of Eq. (25)

The proof is by backward induction. Let l be the chain length. We claim that

- the optimal payment $p_{T-k}{*}(l)$ is given by Eq. (25), and
- the value function has the form

$$J(p_{T-k}, T-k) = -\frac{1}{\left(C + u/(l+1)\right)^{\alpha}}\left[(l+1) + \frac{\beta^2 \sum_{j=1}^{l} 1/j}{\left(C + u/(l+1)\right)^2}\right] + \varphi(h_{T-k+l+1}, h_{T-k+l+2}, \ldots, h_T) \quad \text{(B1)}$$

  where $u = F_{T-k} + g_{T-k} + \sum_{j=1}^{l} h_{T-k+j}$ and $\beta^2 = \sigma^2\alpha(\alpha+1)/6$. The last term in Eq. (B1), φ, is a function that depends only on the deterministic grants of the periods $t=T-k+l+1$ to $t=T$. It is zero if $l=k$. Below we are only interested in the first and second derivatives of $J(p_{T-k},T-k)$. As φ is not dependent on p_{T-k-1} and thus drops out in the derivative, its particular structure is not of interest.

We perform the induction from k to $k'=k+1$ and maximise the value function for $t=T-k-1$:

$$J(p_{T-2}, T-k-1) = \max_{p_{T-k-1}}\left\{Z_{T-k-1} + J(p_{T-k-1}, T-1)\right\}$$

$$= \max_{p_{T-k-1}}\left\{-\left\langle \begin{array}{l} -\dfrac{1}{\left(C + \dfrac{u' + \varepsilon_{T-k} - p_{T-k-1}}{l+1}\right)^{\alpha}}\left[(l+1) + \dfrac{\beta^2\sum_{j=1}^{l} 1/j}{\left(C + \dfrac{u' + \varepsilon_{T-k} - p_{T-k-1}}{l+1}\right)^2}\right] \\ + \varphi(h_{T-k+l+1}, h_{T-k+l+1}, \ldots, h_T) \end{array}\right\rangle - \frac{1}{(C - p_{T-k-1})^{\alpha}}\right\}$$

$$= \max_{p_{T-k-1}}\left\{-\frac{1}{\left(C + \dfrac{u' - p_{T-k-1}}{l+1}\right)^{\alpha}}\left[(l+1) + \frac{\beta^2\sum_{j=1}^{l+1} 1/j}{\left(C + \dfrac{u' - p_{T-k-1}}{l+1}\right)^2}\right] + \varphi(h_{T-k+l+1}, h_{T-k+l+2}, \ldots, h_T) - \frac{1}{(C + p_{T-k-1})^{\alpha}}\right\}$$

(B2)

where $u' = F_{T-k-1} + g_{T-k-1} + \sum_{j=0}^{l} h_{T-k+j}$ and the mean $\langle\ \rangle$ is taken over ε_{T-k} which is uncertain in period $t=T-k-1$.

First we discuss the second derivative of the term in braces {...} of Eq. (B2) with respect to the payment p_{T-k-1}:

$$\frac{d\{...\}}{\alpha(\alpha+1)dp_{T-k-1}} = -\frac{1/(l+1)}{\left(C+\frac{u'-p_{T-k-1}}{l+1}\right)^{\alpha+2}}\left[1+\frac{\sigma^2(\alpha+2)(\alpha+3)/6\sum_{j=1}^{l+1}1/j}{(l+1)^2\left(C+\frac{u'-p_{T-k-1}}{l+1}\right)^2}\right]-\frac{1}{\left(C+p_{T-k-1}\right)^{\alpha}} \quad \text{(B3)}$$

The last term of Eq. (B3) is always negative. If we restrict $p_{T\text{-}k\text{-}1}$ to the feasible interval $[0, \overline{p}_{T-k-1}]$ where $\overline{p}_{T-k-1}=F_{T\text{-}k\text{-}1}+g_{T\text{-}k\text{-}1}$ is the budget constraint in period T-k-1 (cf. Eq. 14), we have $p_{T\text{-}k\text{-}1} \le u'$ and the first term in Eq. (B3) is negative, too. Altogether, the second derivative is negative for all payments $p_{T\text{-}k\text{-}1}$ that fulfil the budget constraint $p_{T\text{-}k\text{-}1} \le \overline{p}_{T-k-1}$.

The optimal payment is now obtained by taking the first derivative of the term in braces {…} of Eq. (B2) with respect to $p_{T\text{-}k\text{-}1}$. This derivative is zero if $p_{T\text{-}k\text{-}1}$ is given by the unique solution $\hat{p}_{T-k-1}$, which is identical to Eq. (25) with k being replaced by $k'=k+1$ and l being replaced by $l'=l+1$. The following two conclusions directly follow from the negativity shown above of the second derivative: (1) If $\hat{p}_{T-k-1} < \overline{p}_{T-k-1}$ (interior solution: cf. case a in Eq. (20)) it maximises the term in braces, {…}, of Eq. (B2) and thus uniquely solves Eq. (B2). (2) If alternatively, $\hat{p}_{T-k-1} \ge \overline{p}_{T-k-1}$ (corner solution: case b in Eq. (20)), then due to the uniqueness of the solution $\hat{p}_{T-k-1}$, the term in braces increases strictly monotonically with $p_{T\text{-}k\text{-}1}$ on the interval $[0, \overline{p}_{T-k-1}]$ and $p_{T\text{-}k\text{-}1} = \overline{p}_{T-k-1}$ uniquely solves Eq. (B2). Altogether, we either have a chain of length $l'=l+1$ (case a) or $l'=0$ (case b) with the optimal payment $p_{T\text{-}k\text{-}1}$* being given by Eq. (25) with $l'=l+1$ or $l'=0$, respectively.

Having completed the induction for the optimal payment and proved Eq. (25), we still have to prove Eq. (B1). For this we insert the two alternative solutions, $\hat{p}_{T-k-1}$ (in case a) and $\overline{p}_{T-k-1}$ (in case b) into Eq. (B2). We start with case a and find

$$J(p_{T-k-1},T-k-1) = -\frac{1}{\left[C+u'/(l+2)\right]^{\alpha}}\left[(l+2)+\frac{\beta^2\sum_{j=1}^{l+1}1/j}{\left[C+u'/(l+2)\right]^2}\right]+\varphi(h_{T-k+l+1},h_{T-k+l+1},...,h_T) \quad \text{(B4)}$$

which is identical to Eq. (B1) if we replace k by $k'=k+1$ and l by $l'=l+1$ in Eq. (B1). For case b we find

$$J(p_{T-k},T-k) = -\frac{1}{\left(C+F_{T-k-1}+g_{T-k-1}\right)^{\alpha}}+\varphi'(h_{T-k},h_{T-k+1},...,h_T) \text{ with}$$

$$\varphi'(h_{T-k},h_{T-k+1},...,h_T) = \frac{l+1}{\left(C+\frac{1}{l+1}\sum_{j=0}^{l}h_{T-k+j}\right)^{\alpha}}+\varphi(h_{T-k+l+1},h_{T-k+l+1},...,h_T) \quad \text{(B5)}$$

The first term in Eq. (B4) is identical to that of Eq. (B1) if we replace k by $k'=k+1$ and set $l'=0$ in Eq. (B1). Like in Eq. (B1), the second term, φ' (Eq. B5) is a function of the deterministic grants of the periods $t=T-k'+l'+1$ to $t=T$ (note that due to $l'=0$ and $k'=k+1$ we have T-k+l+1=T-k), whose particular structure is not of interest. This completes the induction from k to $k'=k+1$ and the proof of the two introductory claims.

Appendix C: The chain length distribution *P*(*l*)

Assume $\sigma \geq 0$ and we are in period $t=T-k$. We are interested in the chain length l, i.e. the *maximum* number of periods such that $\hat{p}_{T-k+n} < \overline{p}_{T-k+n}$ for all $n=1 \ldots l$. The probability that l is the chain length then is the probability $\pi(l)$ that $\hat{p}_{T-k+n} < \overline{p}_{T-k+n}$ for all $n=1 \ldots l$ multiplied by the probability that no larger $l' > l$ can be found where the condition $\hat{p}_{T-k+n} < \overline{p}_{T-k+n}$ is fulfilled for all $n=1 \ldots l'$. Expressed in mathematical terms, the probability that l is the chain length is

$$P(l) = \pi(l) \prod_{m=l+1}^{k} (1 - \pi(m)) \tag{C1}$$

where

$$\pi(l) = \Pr\left(\hat{p}_{T-k+n} < \overline{p}_{T-k+n} \mid n \in \{0,1,2,...l-1)\right) \tag{C2}$$

(note that Eqs. (C1) and (C2) are equivalent to Eq. (24) but explicitly consider that the chain length l is not known with certainty. It follows that

1. The minimum possible chain length is 0, i.e. $\pi(0)=1$;
2. The maximum possible chain length in period $t=T-k$ is k;
3. Long chains are not necessarily less likely than short chains (i.e., $\pi(l)$ may increase with l), as can be seen below and in Example 3 of Section 4).

To determine the probability $\pi(l)$ we start with period $t=T-k$ and determine the likelihood that an interior solution exists, i.e. that $\hat{p}_{T-k} < \overline{p}_{T-k}$ (set $n=0$ in Eq. C2). Assuming that an interior solution exists we project into the uncertain future (the ε_{T-k+n}, $n=1 \ldots k$ are uncertain), proceed to the next period, $t=T-k+1$ and determine the (conditional) probability that $\hat{p}_{T-k+1} < \overline{p}_{T-k+1}$ ($n=1$ in Eq. C2). We proceed in this manner until we have reached period $t=T-k+l-1$ ($n=l-1$ in Eq. C2).

The first condition for an interior solution, $\hat{p}_{T-k}(\text{l}) < \overline{p}_{T-k}$, reads

$$\Delta_{T-k}(l) \equiv \overline{p}_{T-k} - \hat{p}_{T-k}(l) = F_{T-k} + g_{T-k} - \frac{d_l}{l+1} + \frac{s_l}{d_l + (l+1)C} > 0 \tag{C3}$$

where

$$d_l = F_{T-k} + g_{T-k} + \sum_{j=1}^{l} h_{T-k+j} \quad \text{and} \quad s_l = \sum_{j=1}^{l} \sigma^2 (\alpha + 2)/(6j) \tag{C4}$$

(cf. Eqs. 14 and 25).

If Eq. (C3) is valid, the fund in the following period will be $F_{T-k+1} = \Delta_{T-k}$. The money that can be spent in the next period $t=T-k+1$ will be

$$\overline{p}_{T-k+1} = F_{T-k+1} + h_{T-k+1} + \varepsilon_{T-k+1} \tag{C5}$$

where ε_{T-k+1} is uncertain. With this the optimal decision in period $t=T-k+1$ will be $p_{T-k+1}{}^*(l-1) = \min(\overline{p}_{T-k+1}, \hat{p}_{T-k+1}(l-1))$ where

$$\hat{p}_{T-k+1}*(l-1)=$$

$$=\frac{1}{l}\left(F_{T-k+1}+h_{T-k+1}+\varepsilon_{T-k+1}+\sum_{n=2}^{l}h_{T-k+n}\right)-\frac{\sum_{n=1}^{l-1}\sigma^2(\alpha+2)/(6n)}{F_{T-k+1}+h_{T-k+1}+\varepsilon_{T-k+1}+\sum_{n=2}^{l}h_{T-k+n}+lC} \tag{C6}$$

With Eqs. (C5) and (C6), the condition for an interior solution, $\hat{p}_{T-k+1}$(l-1)< $\overline{p}_{T-k+1}$, reads

$$\Delta_{T-k+1}(l)=$$

$$=F_{T-k}+g_{T-k}+h_{T-k+1}-2\frac{d_l}{l+1}+\frac{l-1}{l}\varepsilon_{T-k+1}+\frac{l-1}{l}\frac{s_l}{d_l+(l+1)C}+\frac{l+1}{l}\frac{s_{l-1}}{d_l+(l+1)C}>0 \tag{C7}$$

Similar to above, if Eq. (C7) is valid, the fund in the following period will be $F_{T-k+2}=\Delta_{T-k+1}$. We proceed in the same way as from T-k to T-k+1 and find for arbitrary $n<l$:

$$\begin{aligned}\Delta_{T-k+n}(l)=&F_{T-k}+g_{T-k}+\sum_{i=1}^{n}h_{T-k+i}-\frac{n+1}{l+1}d_l\\&+\sum_{i=1}^{n}\frac{l-n}{l+1-i}\varepsilon_{T-k+i}+\frac{1}{d_l+(l+1)C}\left(\frac{l-n}{l}\sum_{i=1}^{n}s_{l-i+1}\prod_{j=1}^{i-1}\frac{l+2-j}{l-j}+\frac{l+1}{l+1-n}s_{l-n}\right)\end{aligned} \tag{C8}$$

and Eq. (C2) is rewritten

$$\pi(l)=\Pr\left(\Delta_{T-k+n}(l)>0\,|\,n\in\{0,1,2,...l-1)\right) \tag{C9}$$

with Δ_{T-k+n} given by Eq. (C8).

Now consider a relatively general case: a positive initial fund, $F_0>0$, a positive first period grant, $h_0>0$, and a constant non-positive deterministic trend in the grants: $h_t=h_0-t\delta$ ($\delta\geq 0$). Note that all grants have to be positive, $h_t>0$, which requires $h_0>k\delta>l\delta$. We consider the first period, t=0, of k+1 periods. Equation (C8) becomes

$$\begin{aligned}\Delta_n(l)=&(l-n)\left(\frac{F_0}{l+1}+\frac{(n+1)\delta}{2}\right)+\sum_{i=1}^{n}\frac{l-n}{l+1-i}\varepsilon_{T-k+i}\\&+\frac{1}{F_0-\delta(l+1)l/2+(l+1)(h_0+C)}\left(\frac{l-n}{l}\sum_{i=1}^{n}s_{l-i+1}\prod_{j=1}^{i-1}\frac{l+2-j}{l-j}+\frac{l+1}{l+1-n}s_{l-n}\right)\end{aligned} \tag{C10}$$

For arbitrary l,

$$\Delta_0(l)=\frac{lF_0}{l+1}+\frac{l\delta}{2}+\frac{s_l}{F_0-\delta(l+1)l/2+(l+1)(h_0+C)}>0 \tag{C11}$$

which is always positive, given $h_0>l\delta$. An immediate consequence of Eqs. (C9) and (C11) is that the chain length is at least 1: $\pi(1)=1$. For $\sigma>0$ the probability $\pi(l)$ of obtaining a chain of greater length, $l>1$, is the nested integral

$$\pi(l)=\frac{1}{(2\sigma)^{l-1}}\int_{-a_1^0(l)}^{\sigma}d\varepsilon_1\int_{-(a_2^0(l)+a_2^1(l)\varepsilon_1)}^{\sigma}d\varepsilon_2\;\Lambda\int_{-(a_n^0(l)+\sum_{i=1}^{n-1}a_n^i(l)\varepsilon_i)}^{\sigma}d\varepsilon_n\;\Lambda\int_{-(a_{l-1}^0(l)+\sum_{i=1}^{l-2}a_{l-1}^i(l)\varepsilon_i}^{\sigma}d\varepsilon_{l-1} \tag{C12}$$

with coefficients (for all n=1…l-1)

$$a_n^0(l) = \frac{l-n+1}{l-n}\left[\begin{array}{l}(l-n)\left(\dfrac{F_0}{l+1}+\dfrac{(n+1)\delta}{2}\right) \\ +\dfrac{1}{F_0-\delta(l+1)l/2+(l+1)(h_0+C)}\left(\dfrac{l-n}{l}\sum_{i=1}^{n} s_{l-i+1}\prod_{j=1}^{i-1}\dfrac{l+2-j}{l-j}+\dfrac{l+1}{l+1-n}s_{l-n}\right)\end{array}\right]$$

$$a_n^i(l) = \frac{l+1-n}{l+1-i} \quad (i = 1 \ldots n-1) \tag{C13}$$

An analytical solution can easily be obtained for l=2 where we have to consider only the first integral in Eq. (C12) whose lower bound is

$$-a_1^0(2) = -\frac{2}{3}F_0 - 2\delta - \frac{\sigma^2(\alpha+2)/4}{F_0/3+h_0+C-\delta} \tag{C14}$$

If $a_1^0 < \sigma$, the lower bound of the integral, $-a_1^0$, falls within the range of the ε_1 (Eq. 10) and the first integral becomes

$$\pi(2) = \frac{1}{2} + \frac{F_0/3+\delta}{\sigma} + \frac{\sigma(\alpha+2)/8}{F_0/3+h_0+C-\delta} \tag{C15}$$

Otherwise, i.e. if $a_1^0 > \sigma$ (which may result, for instance, from F_0=0 and $\delta > \sigma/2$), the first integral in Eq. (C12) assumes its maximum value, 2σ, and consequently, $\pi(2)=2\sigma(2\sigma)^{-1}=1$, leading to $P(0)=P(1)=0$ and $P(2)=1$. For larger chain lengths, $l>2$, a tractable analytical solution requires simplifying assumptions: we have to set the initial fund and the deterministic trend in the grants to zero ($F_0=\delta=0$) and we have to ignore the terms of the order σ^2 in Eq. (C13). Under these assumptions we find $\pi(1)=1$, $\pi(2)=1/2$, $\pi(3)=1/3$, $\pi(4)=1/4-\tau$ with $\tau=5/1728<<1$. If there are altogether 5 periods (k=4), the corresponding chain length probabilities $P(l)$ are $P(1)=P(2)=P(3)=1/4+\tau/3$, $P(4)=1/4-\tau$, and $P(0)=0$. Altogether, with small error the probabilities $P(l)$ for $l>0$ are almost identical, i.e. the chain length distribution is almost uniform. The shape of the distribution is not expected to change drastically if there are more periods, $k>4$, and we conclude that $P(l)=1/k$ for $l=1\ldots k$ is a reasonable approximation of the chain length distribution.