

# A Tourism Strategy Optimally Balancing Recreation and the Conservation of the Golden Eagle

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## Abstract

We set up an optimal control model to identify a social welfare maximizing tourism strategy for national park areas which guarantees the survival of a threatened species, such as the Golden Eagle in the Alps, by ensuring the existence of the supporting habitat. The associated ecosystem is described by a predator–prey–type system. We compare this ‘traditional approach to conservation’ with one solely targeting the survival of the endangered species native to the national park of concern. In both cases the social welfare maximizing level of tourism is inherently nonstatic and guarantees the existence of the ecosystem, i.e. a national park. Cycling occurs rather due to changes in the public valuation of the ecosystem than due to biological changes in the ecosystem itself.

Keywords: *Optimal control theory, predator–prey systems, charismatic species, tourism, conservation*

## 1 Introduction

The foundation of a national park, or more precisely, the conversion of private land into an area protected for the sake of conservation of a unique ecosystem, asks for two prerequisites: (a) an area recognized to be highly valuable for society due to its richness in species (i.e. its biodiversity) and/or its uniqueness as a habitat for rare and endangered species, and

(b) a strong public interest in protecting this area from human intrusion, such as logging, hunting, or farming.

A straightforward way to raise public interest in conservational issues is to allow the public to experience pristine nature and rare animals living in their natural habitats (as opposed to zoos). For instance, in the ‘Hohe Tauern’ National Park (situated in the Eastern Alps and including the well-known Grossglockner) the release of a pair of Bearded Vultures attracted some 6,000 visitors to the National Park.

It is without doubt that enthusiastic national park visitors contribute positively to a park’s reputation as a jewel of countryside and that they are more likely to raise financial support for environmental protection campaigns. This way of fund raising is, however, not without drawbacks. Racks of visitors disturb sensitive parts of natural systems and, therefore, contribute negatively to a park’s state and reputation. Consequently, an upper limit of human access to national parks (=tourism) must exist where the existence of unique ecosystems is supported to the highest possible extent without severely harming the habitat or rare species living therein. If we agree to refer to such a threshold level as *optimal* if it additionally maximizes the *welfare of the entire society*, our first task is to figure out what is beneficial to all of us. This issue is addressed in the second section of the current paper. Moreover, in search for ‘guidelines for (eco)tourism’ one has to answer a long list of questions: E.g., is the optimal policy nearly static and does not change over time? Is social welfare maximizing tourism inherently nonstatic and requires to be permanently adjusted to the current state of the ecosystem? And above all—does the resulting strategy differ, if we assume that society acquires benefits rather from the existence of a single endangered species (the national park’s ‘flagship’ or ‘icon’ as it is the Golden Eagle for the Eastern Alps) than from the existence of the national park *per se* (without emphasizing a species in particular)? Our tool for answering these questions (in Section 3 and 4 of the current paper) is an optimal control model aiming at addressing the trade-off between a recreational value of a national park and the protection of an unique ecosystem. In Section 5, the model is calibrated for the case of the Golden Eagle, who is viewed as the symbol for the unspoiled and wild natural beauty of the Alps. Although its hunting is prohibited throughout the Alps, it is listed as a potentially threatened species in the Appendix 1 to the EU Birds Directive.

## 2 Model Formulation

Some authors frame the task of conservation from the perspective of a national park administration or private owners of land (to be) included in the park rather than from the viewpoint of an entire society (see, e.g., Conrad and Salas, 1993; Dubey, 1997). Conrad and Salas (1993), for instance, assume that the land owners try to maximize their income from economic activities in the *buffer zone*.<sup>1</sup> Other approaches to conservation omit the explicit influence of a decision-maker and include the hypothetical development (and influence) of control in the framework of nonlinear dynamical systems. E.g. Shukla and Dubey (1996) analyze a plant growing so rapidly that it drives other plants into extinction. To overcome this undesired (natural) process, they successfully introduce *buffalo grazing* as a regulatory mechanism to guarantee biodiversity in an Indian national park. — The number of contributions to topics like these is remarkable. Maximizing something like society’s welfare derived from several services of a national park is, however, not addressed. We argue that this measure is most likely to serve as basis for decision-making — paying tribute to it in our modelling approach.

Recreation (due to tourism) increases social welfare. Furthermore, society derives benefits from protecting an entire ecosystem including rare species. Loomis and White (1996), for instance, state that society benefits from the protection of a certain endangered species (a) by deriving a *use value* by, e.g., viewing it, (b) by securing genetic material that might be useful, e.g., in medicine (*option value*), (c) by knowing that the species of concern continues to live in its natural habitat on a sustainable level (*existence value*), and (d) by preserving the species for further generations (*bequest value*). Thus, one way of setting up a social welfare function is to include at least (the weighted sum of) tourism and conservation benefits (see Figure 1):<sup>2</sup>

$$U(u, N) := \eta \ln(1 + u(t)) + \nu_1 \ln(1 + N(t)) \geq 0, \quad (1)$$

where the scaling factors  $\eta$  and  $\nu_1$  are positive constants, and the variables  $u(t)$  and  $N(t)$  represent the number of tourists and the extent of pristine nature (as a proxy for the value of the national park), respectively, both at time  $t$ . The relative size of  $\eta$  to  $\nu$  determines the society’s preference for tourism relative to conservation. I.e. the higher the societal valuation of conservation (for a given value  $\eta$ ) the higher will be the value of the parameter  $\nu$ . To put it in a nutshell, welfare function (1) refers to what is often called the *traditional approach* to conservation.

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<sup>1</sup>This is the outer zone of a national park where harvesting activities are allowed within limits.

<sup>2</sup>Both of these effects are confirmed empirically by Kontoleon and Swanson (2003).

In recent years conservationist NGOs such as the World Wildlife Fund for Nature (WWF) increasingly targeted their protection campaigns at specific *charismatic species*, e.g. the Giant Panda, the Monk Seal, or, for the case of Austria, the Bearded Vulture. This approach partially replaced the traditional one because these endangered species have the ‘advantage’ of being easily recognized by picture and name as opposed to a vast number of less known (endangered) animals. Furthermore, their existence crucially depends on the conservation of the supporting ecosystem in sufficient quantity and quality. Therefore, this approach implicitly includes the more general way of conserving entire ecosystems. To find out whether these two approaches are truly equivalent (see Section 4) we compare welfare function (1) with a modelling approach where society’s conservational benefits are derived from the existence of the charismatic species. This is way of phrasing benefits is expressed by the function:<sup>3</sup>

$$\tilde{U}(u, R) := \eta \ln(1 + u(t)) + \nu_2 \ln(1 + R(t)), \quad (2)$$

where the scaling factors  $\eta$  and  $\nu_2$  are positive constants, and the variables  $u(t)$  and  $R(t)$  represent the number of tourists and the population size of the endangered species, respectively. Since a charismatic species native to a particular ecosystem is likely to be regarded as ‘icon’ or ‘flagship’ of a particular region, the associated protection campaign is called the *flagship approach* to conservation (compare Leader–Williams and Dublin 2000, cited in Kontoleon and Swanson, 2003). The parameter  $\tilde{\nu}$  corresponds to the existence of a species on a higher level in the food chain. We assume that these species are valued more highly by society than biomass. Thus,  $\tilde{\nu}$  exceeds the parameter  $\nu$  considerably ( $\nu \ll \tilde{\nu}$ ).

While it is obvious that tourism contributes positively to social welfare (see Eqs. 1 and 2, respectively), any tourism — even ecotourism — has a negative impact on the ecosystem. We integrate this negative feedback effect by altering the natural growth rate of the charismatic species, called *flagship* from now on (see Figure 1). More specifically, for  $u(t)$  representing the number of tourists visiting the national park at time  $t$ , e.g. hikers, the function  $T(t) = T(u(t))$  captures the percentage reduction in the natural growth rate of the flagship species due to tourism. Any additional tourist leads to a reduction in the flagship’s fertility rate — but at a decreasing rate ( $0 \leq T(u(t)) \leq 1$ , and  $T'(u(t)) < 0$ ).

To optimally counterbalance recreational benefits of park visitors and the ecosystem’s needs for conservation (see Figure 1), we have to specify what we want to conserve: To simplify matters we assume that our modelled ecosystem consists of a single flagship

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<sup>3</sup>To distinguish the traditional from the flagship approach, we mark the later by tildes.

species, and its supporting biomass, i.e. trees, grass, watersheds, and other animals (see Figure 1). The associated interdependencies are modelled by a predator–prey–type system (e.g. Rosenzweig and MacArthur, 1963), which is discussed in detail by a rich stream of literature (e.g. Cheng *et al.*, 1981; Myerscough *et al.*, 1996).<sup>4</sup> Since Myerscough *et al.* (1996) confirmed that the stability behavior of (even uncontrolled) predator–prey systems is sensitive to the choice of the functional form for growth and decay (for either of the populations) we perform the analysis of the ecosystem’s dynamics without specifying the associated functional forms.

Let  $N(t)$  represent the volume of biomass in the national park area and  $R(t)$  the population size of the flagship species, e.g. the Golden Eagle. Then, the function  $g = g(N(t))$  denotes the specific growth rate of the biomass, and  $f = f(N(t))$  is the so-called *predator response function*<sup>5</sup>. Subsequently, the entire ecosystem’s dynamics is described by

$$\dot{N}(t) = g(N(t))N(t) - f(N(t))R(t), \quad N(0) > 0, \quad (3a)$$

$$\dot{R}(t) = [T(u(t))c f(N(t)) - d]R(t), \quad R(0) > 0, \quad (3b)$$

where  $cf(N(t))$  denotes the birth rate of the flagship species, and  $d$  is their death rate.

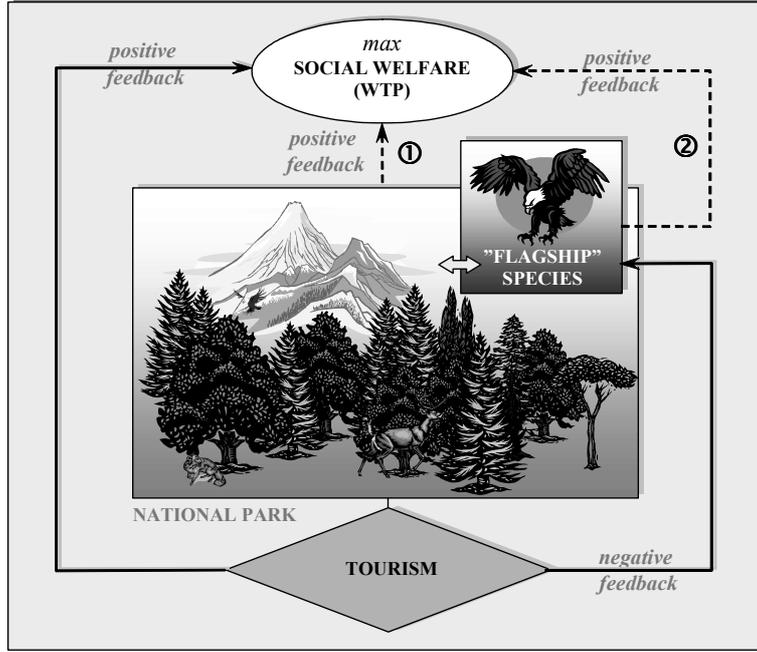
**Assumption 1 (Properties of  $g$  and  $f$ )** : *The general assumptions on growth function  $g$  and the predator response function  $f$  are:*

- *$g$  is continuously differentiable,  $g \geq 0$ ,  $g'(0) > 0$ , and there exists a carrying capacity  $\omega > 0$  such that  $g(\omega) = 0$ . Thus, the growth of biomass  $g$  is limited by its carrying capacity  $\omega$  and the ‘pressure’ from the flagship species. According to Cheng et al. (1981), functional forms fulfilling these requirements are, e.g.,  $g(N) = a(1 - N/\omega)$  or  $g(N) = a(\omega - N)/(\omega + \epsilon N)$ .*
- *$f$  is continuously differentiable,  $f(0) = 0$ ,  $f(N) > 0 \forall N > 0$ ,  $f' > 0 \forall N \geq 0$ . The predator response function  $f$  measures the endangered species’ difficulty of finding suitable food. Appropriate functional forms are, e.g.,  $f(X) = mN^n/(x + N^n)$ ,  $n \geq 1$  or  $f(N) = mN^n$ ,  $1 \geq n > 0$  (Cheng et al., 1981).*

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<sup>4</sup>For examples of optimal control models constrained by predator–prey–type system with other objective functions than ours, see e.g. Hartl *et al.* (1992), Wirl (1996), or Hrinca (1997).

<sup>5</sup>For our purposes,  $R(t)$  does not necessarily represent a predator or raptor but an animal in one way or another exploiting the biomass, e.g. the supporting ecosystem in the national park. For the case of the Golden Eagle who is an territorial animal and needs large areas for hunting, biomass can be interpreted as the natural area necessary as an habitat for the eagle.



**Figure 1:** The structure of the national park model. Feedbacks associated with the dotted lines are not regarded within the same optimal control model, i.e. arrow 1 corresponds to the traditional approach and arrow 2 to the flagship approach to conservation.

For the purpose of determining a degree of tourism maximizing the intertemporal social welfare (following the traditional approach), we consider a nonlinear, autonomous, infinite-horizon optimal control problem with a single control variable,  $u(t)$  (number of tourists) and two state variables,  $N(t)$  (the extent of pristine nature or biomass), and  $R(t)$  (the population size of the endangered (flagship) species):<sup>6</sup>

$$J^* = \max_{u(t) \geq 0} \int_0^{\infty} e^{-rt} U(u, N) dt, \quad (4)$$

subject to Eqs. 3a and 3b,

where  $U(tu, N)$  is defined by Eq. 1, and  $r$  denotes a positive discount rate.<sup>7</sup> Tourism is chosen as control variable  $u(t)$  for either approach since it is easily observable and possible to control, i.e. it can be reduced by limiting entry to the park or raised by advertisement or positive word of mouth. Thus, the resulting optimal time paths of  $u$  can be interpreted as upper bound on tourism. As long as the actual amount of tourism does not surpass this

<sup>6</sup>The only restriction for tourism is a standard nonnegativity assumption. Furthermore, the limitations on the states ( $0 < N(t) \leq \omega$ ,  $0 \leq R(t)$ ) are always satisfied as long as  $u(t) \geq 0$ .

<sup>7</sup>The formulation of the optimal control problem for the flagship approach can be derived analogously by substituting  $U(u, N)$  by  $U(u, R)$ .

threshold, the ecosystem is not severely in danger. However, only if the current number of visitors has the optimal size of  $u$ , social welfare will be maximal as well.

### 3 The Optimal Tourism Policy for the Traditional Approach

#### 3.1 The Optimal Control Policy

For the traditional approach to conservation, the Hamiltonian function  $\mathcal{H}$  is defined by

$$\mathcal{H} = \pi_0 U(u, N) + \pi_1 \dot{N} + \pi_2 \dot{R}, \quad (5)$$

where  $\pi_0 \geq 0$  is a nonnegative constant multiplier associated with the integrand of the objective function,  $\pi_1$  and  $\pi_2$  are the costate variables,  $U(\cdot)$  represents the dynamic utility function as defined by Eq. 1 and  $\dot{N}$  and  $\dot{R}$  are the predator-prey dynamics as determined by Eq. (3).<sup>8</sup>

Thus, applying the maximum principle (Pontryagin *et al.*, 1962) for an inactive constraint ( $u > 0$ ,  $\sigma = 0$ ) results in a necessary condition for an optimal interior control, i.e.  $\mathcal{H}_u^* = 0$ ,

$$\frac{\eta_1}{(1 + u^*)} = \pi_2^* c T'(u^*) f(N^*) R^*. \quad (6)$$

Note that (6) states that marginal utility of tourism (L.H.S.) should just equal the opportunity costs of tourism at any instance of time along the optimal path. The opportunity costs of tourism are measured by the damage to the rate of reproduction of the endangered species caused by an incremental extra unit of tourism, evaluated by the optimal imputed value, or shadow price, of the endangered species ( $\pi_2^*$ ). Therefore, the optimal degree of tourism crucially depends on the current state of the ecosystem ( $N^*$ ,  $R^*$ ).

#### 3.2 Equilibrium Properties

The above subsection showed that the extinction of the endangered species and/or its supporting biomass can never be optimal. — But since the conservation of pristine nature and the conservation of a rare species native to this area do not necessarily go hand in hand in reality, it seems possible that our flagships dies out, leaving the ecosystem injured but capable to recover to some extent. Then, after a while the biomass would grow and

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<sup>8</sup>Since the constraint,  $u \geq 0$ , satisfies the so-called rank condition (Leonard and Long, 1992, p.38) and model (4) is an infinite time horizon problem, it follows from e.g. (Feichtinger and Hartl, 1986, pp.161) that, without loss of generality,  $\pi_0 = 1$ .

reach its carrying capacity (because there are no more flagship animals ‘exploiting’ it).  
— Obeying the guidelines for optimal tourism this (natural) process can never happen (because  $N = K$  implies that  $R = 0$ , which is not optimal).<sup>9</sup>

**PROPOSITION 1 (Equilibrium for Traditional Approach)** *Given that Assumption 1 and Assumption ?? on the properties of the functional forms for growth and decay,  $g$  and  $f$ , hold, the ecologically desirable steady state is defined by  $\hat{E} = (\hat{X}, \hat{Y}, \hat{\pi}_1, \hat{\pi}_2)$ ,  $0 < \hat{N} < K$ ,  $\hat{R} > 0$ , follows from the simultaneous solution of the following set of equations:*

$$f(\hat{N}) \left( \frac{r\eta}{d\hat{N}g(\hat{N})} - \frac{\nu_1}{(1 + \hat{N})(r - G(\hat{N}))} \right) + \frac{\eta f'(\hat{N})}{r - G(\hat{N})} = 0 \quad (7a)$$

$$\hat{R}(\hat{N}) = \frac{\hat{N}g(\hat{N})}{f(\hat{N})} > 0 \quad (7b)$$

$$\hat{\pi}_1(\hat{N}) = \frac{1}{f(\hat{N})(r - G(\hat{N}))} \left( \frac{\nu_1 f(\hat{N})}{1 + \hat{N}} + \eta f'(\hat{N}) \right) > 0 \quad \text{if } r > G(\hat{N}) \quad (7c)$$

$$\hat{\pi}_2(\hat{N}) = \frac{\eta f(\hat{N})}{d\hat{N}g(\hat{N})} > 0 \quad (7d)$$

where

$$G(\hat{N}) := g(\hat{N}) - \frac{\hat{N}}{f(\hat{N})} \left( g(\hat{N})f'(\hat{N}) - g'(\hat{N})f(\hat{N}) \right). \quad (8)$$

The associated equilibrium level of optimal tourism is determined by

$$\frac{\eta_1}{(1 + \hat{u})} = \hat{\pi}_2 c T'(\hat{u}) f(\hat{N}) \hat{N} \hat{R} \quad (9)$$

**PROPOSITION 2 (Equilibrium Stability Properties)** *For any parameter set satisfying simultaneously*

$$K := (r - G(\hat{N})) G(\hat{N}) - dr < 0, \quad (10)$$

$$D := \frac{1}{\hat{\pi}_2^2} \left( \hat{\pi}_2^2 G(\hat{N}) (-r + G(\hat{N})) \alpha_2 (r + \alpha_2) + \eta_1 f(\hat{N})^2 \left( -(\hat{\pi}_1 F(\hat{N})) - \frac{\nu_1}{(1 + \hat{N})^2} + \frac{\eta_1 (r - G(\hat{N})) G(\hat{N})}{\hat{N}^2 g(\hat{N})^2} \right) - \eta_1^2 f'(\hat{N})^2 + \eta_1 f(\hat{N}) \left( \hat{\pi}_1 (r - 2G(\hat{N})) f'(\hat{N}) + \eta_1 f''(\hat{N}) \right) \right) > 0, \quad (11)$$

we observe a monotonous or an oscillating approach of the equilibrium point  $\hat{E}$  along the stable manifold. A parameter set satisfying  $K > 0$ ,  $D > 0$ , and  $D = (K/2)^2 + r^2(K/2)$

<sup>9</sup>We might, however, numerically approach the states  $(N_1, R_1) = (0, 0)$  and  $(N_2, R_2) = (K, 0)$  arbitrarily close!

leads to persistent oscillation as observed for the ecological system in absence of tourism (see e.g. Cheng et al., 1981; Myerscough et al., 1996, pp.36–40).

## 4 The Traditional Versus the Flagship Approach

This section is devoted to the comparison of the two concepts of approaching conservational issues outlined in Section 2. The traditional approach targets the entire (unique) ecosystem via conserving the supporting biomass — the flagship approach pegs the conservation of pristine nature and all animals native to this area to the existence/survival of a single endangered species (= the flagship). To investigate under which circumstances these two approaches are equivalent, let us have a look at the respective optimal tourism strategies.

The associated functional forms  $u^*(t)$  and  $\tilde{u}^*(t)$ , respectively, are identical for both approaches as shown in Appendix A.2. Thus all properties of the optimal tourism policy described in Section 3 also apply for the flagship approach. Recall that among these features are that the extinction of the endangered species can never be optimal for society. If the flagship species dies out, not even fully exploiting the national park ‘today’ can offset ‘tomorrow’s’ loss for society. One has to keep in mind, however, that the functions used to determine the optimal control path (see Eq. ??) differ for the traditional and the flagship approach.

Therefore, we have to get further insight into these ‘differences’. Thus, let us regard the differential equations for states and costates and start with the one for the imputed value of an endangered animal:

$$\dot{\tilde{\pi}}_2(t) = \dot{\pi}_2(\tilde{X}(t), \tilde{Y}(t), \tilde{\pi}_1(t), \tilde{\pi}_2(t)) - \frac{\tilde{\nu}}{1 + \tilde{Y}(t)}, \quad (12)$$

where  $\pi_2(t)$  denotes the second costate variable for the traditional approach and  $\tilde{\pi}_2(t)$  for the flagship approach. Thus, the change in the endangered animal’s imputed value is smaller for the flagship approach than for the traditional one, ceteris paribus. Since

$$\dot{\tilde{\pi}}_1(t) = \dot{\pi}_1(\tilde{X}(t), \tilde{Y}(t), \tilde{\pi}_1(t)) + \frac{\nu}{1 + \tilde{X}(t)}, \quad (13)$$

the opposite is true for the change in the imputed value of an unit of biomass. If the protection campaign is pegged to the existence (and maximization) of the volume of biomass instead of to the existence of the national park’s flagship,  $\tilde{\pi}_1(t)$  varies more than  $\pi_1(t)$ , ceteris paribus. As shown in the Appendix,  $\tilde{\pi}_1(t)$  contributes to the development of  $\tilde{\pi}_2(t)$ . Thus it depends on the current social per unit valuation of biomass ( $\frac{\nu}{1 + \tilde{X}(t)}$ ) and the current

social per capita valuation of the flagship population ( $\frac{\bar{v}}{1+Y(t)}$ ), whether these stabilizing and destabilizing effects will more or less cancel out.

## 5 Illustrating the System Behavior

### 5.1 Specification of Functional Forms and Parameter Values

As outlined in the previous sections, optimal tourism management is nonstatic and its evolution depends explicitly on the optimal values of  $N(t)$ ,  $R(t)$ , and  $\pi_2(t)$ . To further illustrate the rather technical results and the associated interdependencies derived in the last two sections, we shall refer to the protection of the Golden Eagle in the Eastern Alps.

The Golden Eagle lives in open or partly open landscapes and can be found in the higher regions of the European and Asian mountain ranges as well as in the tundra landscapes in Northern Asia. His main characteristic is its territorial behavior – a breeding pair has a territory in the size of 30 to up to 100 square kilometers. Therefore, they require huge nature reserves and an internationally coordinated protection strategy. The project AQUILALP.NET is targeted at this aim. Severely diminished by hunting at the end of the 19th century, the eagle is nowadays not under direct threat in the Alps and populations are rather stable. However, especially due to shrinking natural areas, the habitat for the eagle is under threat and therefore the Golden Eagle is listed as a threatened species according to the EU Birds Directive (AQUILALP, 2003).

The Austrian–Italian part of the Eastern Alps is one of the most important habitats of the Golden Eagle. Three national parks, namely the Hohe Tauern National Park (Austria), the Nature Park Rieserferner–Ahrn (Autonomous Province of Bozen, Italy), and the Zillertal Alps Natural Park (Austria) constitute the largest spatially integrated nature reserve of the Alps (2500 km<sup>2</sup>). The area includes a range of alpine habitats from montane woodlands to the highest peaks of the Eastern Alps (Grossglockner 3,798 m). In the National Park Hohe Tauern, the density of the eagle is significantly higher than in the other areas of the Eastern Alps: 33–35 breeding pairs were counted in the park in 2003, this is 10% of the Austrian eagle population (for comparison: the size of the natural park is approximately 1/37 of the area of Austria). Also the breeding success is higher than in other areas (AQUILALP, 2003). These facts indicate that the establishment of nature reserves contributes to the survival of the Golden Eagle.

The development of the Golden Eagle and its supporting ecosystem over time is described by a predator–prey system as described in general in Section 3. Therein, growth

and decay (satisfying Assumption 1) will be specified by the logistic growth function, for the supporting biomass (natural habitat),

$$g(N(t)) = aN(t) \left( 1 - \frac{N(t)}{\omega} \right), \quad (14)$$

where  $N(t)$  represents the volume of biomass and  $\omega$  denotes its carrying capacity. A Holling (1959) predator response function is used to describe the growth of the eagle in dependency from the biomass

$$f(N(t)) = \frac{bN(t)}{1 + \gamma N(t)}, \quad (15)$$

where  $b$  is the consumption rate of the flagship species and parameter  $\gamma$  captures the effort necessary to convert the supporting ecosystem into a source of food. Thus, the rate of consumption is limited by the predator's 'handling time' for each unit of food (Myerscough *et al.*, 1996, p.33). In the case of the Golden Eagle,  $\gamma$  describes the difficulty of hunting and preparing the prey for eating, while  $b$  would describe the amount of biomass (or territorial size) necessary per eagle.

A simple choice for the tourism function meeting the requirements specified above is given by

$$T(u(t)) = \frac{\varphi}{1 + u(t)}, \quad u(t) \geq \varphi \geq 0. \quad (16)$$

Thus, for  $u(t) = 0$ , the tourism function is equal to parameter  $\varphi$ . On the other hand, the larger the number of visitors the lower is the value of  $T(u)$  and the larger is the decrease in the rate of reproduction of the endangered species,  $R(t)$ .

We perform all calculations described in this paper by using Mathematica 4.2 (Wolfram Research Ltd.). The intertemporal paths for states, costates and control are derived by backward integration. The parameter values are given in Table 1. These values are chosen for purely illustrative reasons and do not (yet) refer to any real world process.<sup>10</sup>

## 5.2 Equilibrium Behavior

For the base case parameter values (see Table 1) and growth and decay functions as specified by (14) and (15), we observe two interior solutions of which one comes fairly close to the extinction of the entire ecosystem.<sup>11</sup> The second interior solution, denoted by

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<sup>10</sup>This part of the paper is currently revised on the basis of actual data on the development of the Golden Eagle in the Eastern Alps. Results will be available soon.

<sup>11</sup>Note that this solution is unstable.

**Table 1:** Base case parameter values

Parameters	Value	Description
$b$	20	decay rate of biomass
$d$	0.001	decay rate of flagship species
$a$	1.5	growth rate of biomass
$c$	0.5	fertility rate of flagship species
$\gamma$	0.01	rate of conversion of prey
$\eta_1$	500	weighting factor for tourism in social welfare function
$\nu$	50	weighting factor for biomass in social welfare function 1
$\tilde{\nu}$	500	weighting factor for flagship species in social welfare function 2
$\varphi$	1	tourism impact factor
$\omega$	1,000,000	carrying capacity of biomass
$r$	0.04	rate of discount

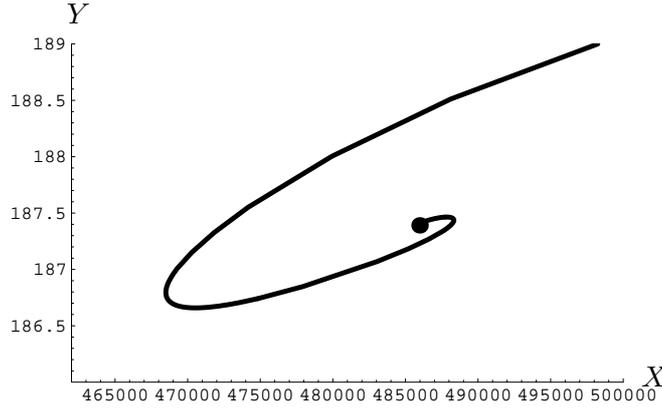
$\hat{E}$  for the traditional approach and by  $\hat{\hat{E}}$  for the flagship approach, is located further right in the  $(X, Y)$ -plane:

$$\hat{E} := \begin{pmatrix} \hat{N} \\ \hat{R} \\ \hat{\pi}_1 \\ \hat{\pi}_2 \\ \hat{u} \end{pmatrix} = \begin{pmatrix} 485,970 \\ 187 \\ -0.053 \\ 2,668 \\ 999,793 \end{pmatrix}, \quad \hat{\hat{E}} := \begin{pmatrix} \hat{\hat{X}} \\ \hat{\hat{Y}} \\ \hat{\hat{\pi}}_1 \\ \hat{\hat{\pi}}_2 \\ \hat{\hat{u}} \end{pmatrix} = \begin{pmatrix} 486,613 \\ 187 \\ -0.052 \\ 2,688 \\ 999,794 \end{pmatrix}. \quad (17)$$

The steady state values for the traditional approach differ only marginally from the values for the flagship approach. This can be seen as a sign for the equivalence of the two approaches (at least in equilibrium terms). Thus, the following equilibrium analysis will be restricted to the traditional approach. (Note however, as discussed in Section 4, that while the steady states are virtually the same, the equilibrium approaches to these states differ.)

For the parameter set specified in Table 1, the Jacobian evaluated at the equilibrium point  $\hat{E}$  (Eq. 17) exhibits four conjugate complex eigenvalues of which two have negative real parts. Thus, the steady state has the properties of a saddle focus (see Figure 2) and the ecosystem (and optimal tourism) undergo damped oscillation approaching  $\hat{E}$ .

The qualitative properties of the equilibrium are very insensitive to changes in the



**Figure 2:** Traditional conservationist approach: Phaseplot in the vicinity of the steady state  $\hat{E}$  for the base case set of parameters (see Tab. 2).

base case parameter values as becomes obvious from calculating the parameter elasticities (outcomes not shown here). This results from the strong stabilizing effect of the optimal tourism strategy that counterbalances changes in the size of biomass and the population size of the flagship species.

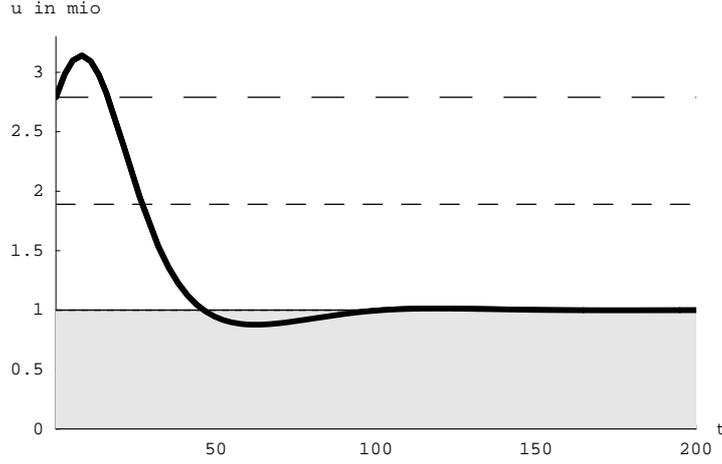
### 5.3 Transient Behavior — The Shape of the Optimal Tourism Strategy

To highlight the intertemporal state dependence of the welfare maximizing tourism strategy, Figure 3 compares three static and the optimal dynamic policy.<sup>12</sup> The static strategy  $\bar{u}_1 = \bar{u}_0^*$  is fixed at a level of visitors which is identical to the ‘initial endowment’ for the optimal dynamic policy (i.e. 2.79 M visitors). While the optimal policy changes over time, a static policy leads to the extinction of the endangered species (and the biomass approaches its carrying capacity  $\omega$ ). Another strategy would be to restrict tourism to the equilibrium level displayed in Eq. 17. If the number of visitors is permanently restricted to  $\bar{u}_4 = \bar{u}_\infty^* = 999,973$ , the flagship species will survive. Clearly, this stringent policy is positive from a conservationist perspective since the flagship’s population size converges to an equilibrium value of 187 — at the cost of having the number of visitors cut by roughly 65%. Thus, the social loss is remarkable, since the dynamically adjustable level of tourists is almost always higher than or equal to  $\hat{u} = 999,793$ . The intermediate level of visitors,  $\bar{u}_2 = 1.89 * 10^6$ , exceeds the level of tourism which guarantees the survival of the endangered species and thus leads to extinction.

To simplify insight, the shaded area in Figure 3 indicates levels for static tourism strategies which are not severely thinning the flagship population but cause a reduction in

<sup>12</sup>A tourism strategy is called ‘static’ if a specific level of visitors is pertained from present till infinity. Any static strategy will be denoted by a bar.

the intertemporal social welfare relative to the optimal dynamic policy. In fact, exceeding the threshold level  $\bar{u}_3 = 999,810$  by an extra visitor will cause the flagship species to disappear from the national park. If the tourism strategy is, however, adjusted permanently to the current state of the ecosystem (see  $u^*(t)$  in Eq. ??), tourism can exceed the static threshold level  $\bar{u}_3$  considerably for longer periods of time. Clearly, intertemporal social welfare is higher for the optimal dynamic tourism strategy than for any static strategy that does not lead to the extinction of the endangered species.



**Figure 3:** Comparison of the optimal dynamic (thick line) and different static tourism strategies (thin lines) in the case of traditional conservation, for initial values  $X_0 = 590,374$  and  $Y_0 = 191$  and base case parameter values.  $u$  is measured in millions.

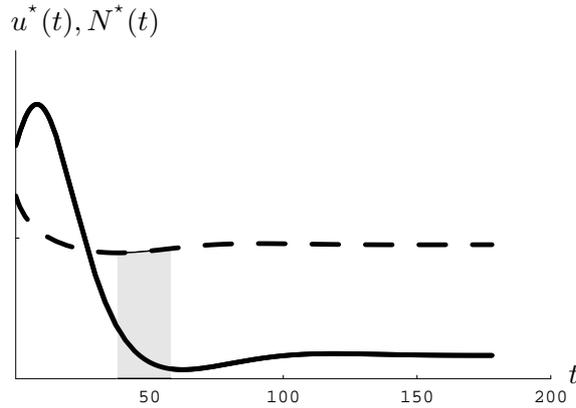
**Table 2:** Comparison of different tourism strategies for the traditional approach to conservation and for the base case parameter values

number of visitors	$\hat{N}$	$\hat{R}$	$J^*$
$\bar{u}_1 = \bar{u}_0^* = 2.79 * 10^6$	$\omega$	0	(201, 952)
$\bar{u}_2 = \frac{\bar{u}_0^* + \bar{u}_\infty^*}{2} = 1.89 * 10^6$	$\omega$	0	(192, 721)
$\bar{u}_3 = 999,810$	529,001	187	182,930
$\bar{u}_4 = \bar{u}_\infty^* = \hat{u}^* = 999,793$	485,970	187	182,929
$u^*(t) = \frac{\varphi}{\eta_1} \pi_2(t) cf(N(t)) R(t) - 1$	485,970	187	197,860

When interpreting the objective functional values for the static strategies leading to extinction, i.e.  $\bar{u} > \bar{u}_3$ , one has to be careful. Parameter  $\nu$  values the benefit per unit of biomass *including* the existence of the flagship species. Thus, if our species of concern is severely diminished one can imagine that the value of  $\nu$  changes considerably. Accordingly, it is very unlikely that the number of visitors will continue to be high if the attracting

flagship species dies out. Neither of these effects is captured in the present version of this model. Thus, the social welfare,  $J^*$ , associated for such policies, i.e.  $\bar{u}_1 = \bar{u}_0^*$  and  $\bar{u}_2$  are only mentioned in parentheses and should not be compared with the remaining cases.

We have seen in Section 3 that optimal tourism contributes positively to conversational aims. Figure 4 helps to understand this process. For the traditional approach and initial values  $X_0 = 590,374$  and  $Y_0 = 191$ , the optimal time path of tourism,  $u(t)$ , is depicted by the solid black line, while the dashed line depicts the optimal time path for the volume of biomass,  $N(t)$ . In the initial phase, the number of visitors increases because the marginal benefit derived from tourism exceeds the marginal benefit from conservation. — But there is no tourism without supporting biomass and a surviving ‘flagship’. Thus, after the upswing in  $u(t)$ , the valuation of  $N(t)$  in the objective function ‘takes the lead’ and  $u(t)$  diminishes delayed to the decay in  $N(t)$ . The trajectories do not increase and decay simultaneously, however, but with a delay of approximately 25 periods (see the shaded area in Figure 4). Note, that while  $N(t)$  takes the lead,  $u(t)$  also exerts a positive influence on the extent of  $N(t)$ , as does  $R(t)$ .



**Figure 4:** Comparison of optimal tourism (—) and optimal size of biomass (---) in the case of traditional conservation for initial values  $X_0 = 590,374$  and  $Y_0 = 191$  and the base case parameters.

Finally, a word about the transients for the traditional and the flagship approach for the base case parameter values (see Table 1). We assume for our modelling approach that  $\nu \ll \tilde{\nu}$ , since it is reasonable to assume that society values an endangered species higher than plain ‘biomass’. In this case, a protection campaign pegged to the existence (and maximization) of an endangered species (= the national park’s flagship), leads to less variability in the optimal time paths of the entire ecosystem. E.g. in the case of transient oscillations,  $\tilde{\pi}_2(t)$  swings less than  $\pi_2(t)$ . This has a stabilizing effect on the trajectory for  $\tilde{Y}(t)$  and, consequently on the orbit for  $\tilde{X}(t)$ . If this is not the case and  $\nu \approx \tilde{\nu}$ , however,

the opposite is true and the flagship approach exhibits more variability in the state of the ecosystem. *Thus, given that society wants to maximize its welfare, variation in the ecosystem is not (primarily) caused by interior biological forces, but by society's conception of what is 'worth saving (to a certain extent)' and what is not.*

## 6 Conclusions

The insights gained from the model presented in this paper are as follows. A static tourism strategy (i.e. one that requires a constant level of visitors over time) has to be very restrictive to guarantee the survival of an endangered species. The optimal dynamic tourism strategy, however, allows for higher levels of tourism for quite a long period of time, if tourism is reduced at later stages of time. The reason why tourism does not lead to extinction in this latter case is that the optimal tourism strategy depends on the current state of the ecosystem and is therefore adjusted permanently to the needs of the endangered species and its supporting biomass. Put differently, a clever tourism strategy is required to be inherently nonstatic. One interesting observation of the current model is the positive (delayed) relationship between the optimal time paths of tourism and the volume of biomass. For policies guaranteeing the survival of the endangered species of concern, intertemporal social welfare is highest for the optimal dynamic tourism strategy.

A major focus of this paper is the comparison between the traditional approach to conservation, where the protection of the complete ecosystem is the aim, and the so-called flagship approach to conservation, concentrating on one popular species. Kontoleon and Swanson (2003) have shown empirically that the willingness to pay for conservation is positively influenced when conservation is linked to a charismatic species (this is also the strategy followed recently by environmental NGOs such as the WWF). However, van Kooten and Bulte (2000) warn that a too strong focus on one species could endanger the ecosystem's integrity and by this harm its habitat. We try to answer this question by comparing the solution to two optimal control problems where in the first case recreational values and the underlying ecosystem (habitat) go into the welfare function, and in the second case we replace the ecosystem by the population size of the endangered species, i.e. the golden eagle. Our main result is that, irrelevant of the specification of the conservation part in the welfare function, optimal (dynamic) tourism guarantees that neither the biomass nor the endangered species can be driven to extinction (even in the long-run). Therefore, we conclude that a high interest in national parks for recreational reasons can

go hand in hand with a high devotion to conservational values when tourism is chosen at its state-dependent welfare maximizing level. The differences between the approaches are apparent mainly in the optimal approach to the steady state, where for the traditional approach the population size fluctuates differently from the flagship approach according to the social valuation of pristine nature and flagship species, respectively.

While we can determine the optimal path of tourism over time (the maximum level for tourism in accordance with conservation), the demand for tourism is not modelled in this approach. However, the optimal level of tourism depends on the variables one would expect to be the relevant determinants for recreation demand, namely, the population size of the endangered species and the relative importance of recreation to conservation. This result is valid for both the traditional and the flagship approach to conservation.

Finally, from a conservationist's point of view, one could question the sufficiency of guaranteeing that the endangered species does not die out. Clearly, we cannot make sure that the population size does not fall below its 'survival size'. A way of overcoming this problem is to put a high weight on conservation relative to tourism in the welfare function. This would increase, however, the variability of the ecosystem. A second approach (for further research), is to introduce a constraint in the optimal control problem which obeys some measure of, e.g., biodiversity (as, e.g., in Conrad and Salas, 1993). This would refer to a kind of *safe minimum standard*, a concept developed by Ciriacy-Wartrup (1968), for a discussion see e.g. van Kooten and Bulte (2000, pp.293–307).

## A Appendix

### A.1 Equilibrium Properties for the Traditional Approach

The costates are defined by the following set of equations<sup>13</sup>

$$\dot{\pi}_1 = r\pi_1 - \mathcal{H}_N = \pi_1 (r - g(N) - Rf'(N) - Ng'(N)) - \pi_2 cT(u)f'(N)R - \frac{\nu - 1}{1 + N} \quad (\text{A.1})$$

$$\dot{\pi}_2 = r\pi_2 - \mathcal{H}_R = \pi_1 f(N) + \pi_2 (r + d - cT(u)f(N)) \quad (\text{A.2})$$

The limiting transversality conditions for the costates hold along the optimal path since (nonnegative and bounded) states and (bounded) costates approach a stable state (defined by the simultaneous solution of Eqs. 7a–7d below):

$$\lim_{t \rightarrow \infty} e^{-rt} \pi_1(t)N(t) = 0 \quad (\text{A.3})$$

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<sup>13</sup>Note that from now on we omit the notion of the asterisks as well.

$$\lim_{t \rightarrow \infty} e^{-rt} \pi_2(t) R(t) = 0 \quad (\text{A.4})$$

If we specify the impact of tourism on the birth rate of the endangered animal as described by Eq. 16, the canonical system for an optimal interior control  $u$ ,  $\pi_2 \neq 0$  (and positive stocks of both species) is defined by

$$\dot{N} = g(N)R - f(N), R \quad (\text{A.5})$$

$$\dot{R} = \frac{\eta_1}{\pi_2} - dR, \quad (\text{A.6})$$

$$\dot{\pi}_1 = \pi_1 \left( r - g(N) + Rf'(N) - Ng'(N) \right) - \frac{\eta_1 f'(N)}{N} - \frac{\nu_1}{1+N}, \quad (\text{A.7})$$

$$\dot{\pi}_2 = \pi_1 f(N) + \pi_2 (r + d) - \frac{\eta_1}{R}. \quad (\text{A.8})$$

The second order derivative of the Hamiltonian  $\mathcal{H}$  with respect to  $u$  simplifies to

$$\mathcal{H}_{uu} = - \underbrace{\frac{\eta_1}{(1+u)^2}}_{>0} + \underbrace{\pi_2 c T''(u) f(N) R}_{>0}. \quad (\text{A.9})$$

According to Feichtinger and Hartl (1986, p.84) the optimal interior control,  $u$ , is therefore uniquely determined at each instant of time if and only if

$$\eta_1^3 > \varphi^2 T''(u) (\pi_2 c f(N) R)^3. \quad (\text{A.10})$$

Unfortunately we cannot prove that  $\mathcal{H}$  is jointly concave with respect to states and control and, consequently, we cannot generally state that the necessary conditions are sufficient.

But since the motion towards the equilibrium state  $\hat{E}$  happens on a two-dimensional stable manifold, each pair of initial values,  $(N_0, R_0)$ , uniquely defines a trajectory (on this manifold) being a candidate for the optimal solution and satisfies,

$$J = \frac{1}{r} \mathcal{H}(N(0), R(0), \pi_1(0), \pi_2(0)), \quad (\text{A.11})$$

according to Michel (1982), for the choice of the corresponding optimal initial conditions of the costates,  $(\pi_{10}, \pi_{20})$ , from this stable manifold. The choice of any other combination of initial conditions for the costates leads to divergence of the trajectories. Therefore, each pair of initial values  $(N_0, R_0)$  defines a unique trajectory on the stable manifold, which is the optimal solution of the well-defined model 5.

## A.2 Optimal Tourism Strategy for the Flagship Approach

The derivation of the necessary conditions for the flagship approach is analogous to the one for the traditional approach. For the sake of comparison, we derive the conditions in

terms of the traditional approach, where all expressions with a tilde refer to the flagship approach.

The Hamiltonian for the flagship approach is determined as

$$\mathcal{H} = \tilde{U}(\tilde{u}, \tilde{R}) + \tilde{\pi}_1 \dot{\tilde{N}} + \tilde{\pi}_2 \dot{\tilde{R}} \quad (\text{A.12})$$

For an inactive constraint the necessary condition for an optimal interior control is given by

$$\begin{aligned} \eta_1(1 + \tilde{u}^*) &= \varphi \tilde{\pi}_2^* c f(\tilde{N}^*) \tilde{R}^* = \\ &= \eta_1(1 + u^*(\tilde{N}^*, \tilde{R}^*, \tilde{\pi}_2^*)) > 0. \end{aligned} \quad (\text{A.13})$$

If we omit the asterisks from now on, the canonical system for the interior optimal control is defined by the following set of equations for  $\tilde{N} \neq 0$ ,  $\tilde{R} \neq r$ ,  $G(\tilde{N}) \neq 0$

$$\dot{\tilde{N}} = \dot{N}(\tilde{N}, \tilde{R}), \quad (\text{A.14})$$

$$\dot{\tilde{R}} = \dot{R}(\tilde{R}, \tilde{\pi}_2), \quad (\text{A.15})$$

$$\dot{\tilde{\pi}}_1 = \dot{\pi}_1(\tilde{N}, \tilde{R}, \tilde{\pi}_1) + \frac{\nu}{1 + \tilde{N}}, \quad (\text{A.16})$$

$$\dot{\tilde{\pi}}_2 = \dot{\pi}_2(\tilde{N}, \tilde{R}, \tilde{\pi}_1, \tilde{\pi}_2) - \frac{\tilde{\nu}}{1 + \tilde{R}}. \quad (\text{A.17})$$

The equilibrium for the flagship approach is then defined by the simultaneous solution of the following set of equations

$$F(\hat{N}) + \frac{\nu}{(1 + \hat{N})(r - G(\hat{N}))} - \frac{\tilde{\nu}}{f(\hat{N})} + \hat{N}g(\hat{N}) = 0 \quad (\text{A.18a})$$

$$\hat{R}(\hat{N}) = \hat{R}(\hat{N}) \quad (\text{A.18b})$$

$$\hat{\pi}_1(\hat{N}) = \hat{\pi}_1(\hat{N}) - \frac{\nu f(\hat{N})}{f(\hat{N})(r - G(\hat{N}))(1 + \hat{N})} \quad (\text{A.18c})$$

$$\hat{\pi}_2(\hat{N}) = \hat{\pi}_2(\hat{N}) \quad (\text{A.18d})$$

where

$$F(\hat{N}) := f(\hat{N}) \left( \frac{r\eta_1}{d\hat{N}g(\hat{N})} - \frac{\tilde{\nu}}{(1 + \hat{N})(r - G(\hat{N}))} \right) + \frac{\eta f'(\hat{N})}{(r - G(\hat{N}))} \quad (\text{A.19})$$

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