Viable management of a renewable resource with a quota market

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May 29, 2006

Abstract

This paper deals with the sustainable management of a renewable resource based on individual and transferable quotas (ITQs). The aim of that paper is to determine the conditions under which a regulating agency can achieve through a quota market both ecological and economic objectives along time when agents are myopic, heterogeneous and non-compliant. To achieve this, a dynamic bio-economic model is build where the performance of the different quota (TAC) policies is evaluated with respect to the satisfaction at each time of a constraint of guaranteed harvesting. We show that this constraint induces intergenerational and intragenerational equity along with conservation of the stock. The viability kernel provides the analytical tool to handle such a feasibility problem. Thus indicators of maximal guaranteed catches, minimal resource state together with viable quota controls are displayed. Specific policies are analyzed, including conservative, sustainable yield and maximin strategies. An example illustrates the main analytical findings.

Keyword: Renewable resource; Sustainability; TAC; ITQs; Noncompliance; Viability kernel, maximin.

JEL Classification: Q01, Q32, O13, C61

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1 Introduction

Renewable resources are under extreme pressure worldwide despite the endeavors for better regulations in terms of economic instruments and measures of stocks and catches. Without any regulation, numerous stocks will be further depleted or extinct in the future. Hence, to prevent over-exploitation, regulating agencies has to drive the renewable resource management on more sustainable paths which could conciliate both ecological and economic goals in an intergenerational and intragenerational perspective.

Many managed renewable resources are regulated with quantity restriction [30]. The system of individual and transferable quotas (ITQ) offers a decentralized coordination method of management promoting an efficient resource use [8]. Under the ITQ system, a regulating authority is assumed to set at the beginning of each season the total allowable catch (TAC) that cover all mortality to a resource stock caused by human activity. Then the regulator allocates shares of the TAC to a limited set of agents who have the choice of either harvesting their share or selling it, or a portion of it, to another licensed agent. This instrument has been implemented by significant fishing nations all around the world. A great deal of accumulated experiences display interesting performance in terms of stocks or catches. As emphasized by [25] for the New Zealand case, ITQ markets seem to operate reasonably well.

However, not all is rosy. If ITQs are used to achieve the economic efficiency, some issues coping with the social equity and the ecological sustainability induced by the system remain at stake [10]. In particular, the equity issue may come from the implications of potential concentration of quotas within the most efficient users of the resource. Moreover, when setting the TAC's accurate level, the regulating agency may face several problems as high-grading, non selective gear or by-catch related to multi-species management. Non compliance problems occur and ITQs, like all other management systems, need enforcement. But as pointed out in [8, 19], all regulatory instruments are difficult to implement because restrictions are often perceived as constraining economic opportunities. Thus, exploiting firms act as opponents to management authorities, often resulting in illegal landings or mis-reported catches. In fisheries, the share of this illegal landing has been estimated to be close to an average of 20% of total landings [1]. It means that limited budgets and prohibitive monitoring costs make complete enforcement impossible. The frequency of landings inspections and the levels of penalties imposed by courts are insufficient to induce perfect compliance with quota holdings. As shown by [15, 17], violations of quota occur and are mainly driven by economic rationale. It turns out that noncompliance may affect the performance of the ITQ system. An imperfectly enforced system makes more difficult recommendations on the TAC's accurate level to achieve the sustainability of the resource. It implies that regulating agencies, when setting the TAC in a dynamic setting, have to take into account the presence of violation. The theoretical implications of non-compliance on the equilibrium permit price and the level of harvest has been analyzed in [16, 21, 22]. In particular for a given TAC, noncompliance may imply a lower or a higher equilibrium permit price depending on the shape of the penalty function and/or its specification in absolute or relative terms. This will provide misleading information convey by ITQs price on the current and expected state of the market [3, 4].

This aim of this paper is to determine in a dynamic setting the conditions under which a regulating agency can achieve a sustainable management of renewable resources through an ITQ market with a given incomplete enforcement. We extend the stationary and steady state work of [6] in a dynamic bio-economic model based on weak invariance method [9] or viable control [2]. Basically, the viable control method focuses on inter-temporal feasible paths. The idea of this approach is to determine desirable corridors paths for the evolution of resources which are constrained by normative measures representing our knowledge of what should at least be avoided in order to achieve sustainability and prevent catastrophic collapses. The main constraint that we consider here consists in a total guaranteed level of catch to be fulfilled at any time. It turns out that this basic need constraint promotes intergenerational and intragenerational equity along with conservation of the stock. Such outcomes may not be achieved by usual optimality control approach. Indeed, optimal control modeling for the sustainable management of renewable resource can be criticized because it may imply dictator of future or present [18] and favor extinction of stock as shown by [8]. In particular, the optimal solution may require the shutdown of the activity for a while and thus leads to the violation of a basic need constraint. The viability method offers an another way to deal with the sustainability by ensuring to the agents a minimum catch at each period of time and by assigning an equal weight to every time period. We match this approach with the definition of sustainable development as a development "that meets the needs of the present without compromising the ability of future generations to meet their own needs". This is quite close to the maximin approach [18] as proved in [24] in the exhaustible resource context. From the ecological viewpoint, the so-called population viability analysis (PVA) [26] and conservation biology has concerns closed to viable control by focusing on extinction process generally within an stochastic framework. Connections with safe standard of conservation invoked in [27] are worth to be pointed out. The viability approach is proposed as a relevant framework for ecosystem approach to fisheries in [11] and has been used for renewable resources management in [5, 12, 13].

The bio-economic model we develop is a two-stage model. In the first stage, the regulating agency sets the amount of quota in an imperfectly enforced system in which quota violations occur. In the second stage, agents set the amount of harvest. The model is solved by backward induction starting with the static optimization problem of the agents followed by the dynamic program faced by the regulator. At each period the allocation of quotas through a market clearing condition is realized since inter-temporal quotas trade is not allowed. The objective assigned to the regulator is to satisfy economic constraints as a guaranteed catch level together with ecological constraint given by a minimal level of a resource. Hence, it requires to determine the configurations which allow for such sustainable paths to exist. Then, in the favorable case, viable stocks and quotas are displayed. The so-called viability kernel plays a key methodological role to handle such a feasibility problem.

The paper is organized as follows. In section 2, we present the general dynamic bio-economic model operating in an ITQs framework. Section 3 proposes a taxonomy of sustainable and unsustainable configurations through the viability kernel analysis. Viable quota policies are exhibited in the favorable cases. Section 4 presents an example with specific functions of stock recruitment, catch cost and non compliance penalty. Section 5 concludes. To restrict the mathematical content in the core of the text, the proofs of the formal results are exposed within the appendix A.

2 The bio-economic model

2.1 The resource dynamics

A natural resource is described by its population state (biomass, densities,..) $x(t) \in \mathbb{R}$ at time t. Without catches, this resource evolves through a stock recruitment relation f which corresponds to:

$$x(t+1) = f(x(t)),\ t \geq 0. \tag{1}$$

The dynamics f is supposed to be continuous, increasing and to be zero at the origin. Whenever it is regular enough, this means that[1]

$$f(0) = 0,\ f_x(x) \geq 0. \tag{2}$$

When catches $h(t)$ occur at the beginning of the period, the dynamics of the exploited resource $x(.)$ becomes:

$$x(t+1) = f(x(t) - h(t)). \tag{3}$$

Since take-off can not exceed resource stock, a scarcity constraint holds:

$$0 \leq h(t) \leq x(t). \tag{4}$$

2.2 The ITQ market

The renewable resource management system is assumed to be based on individual transferable quota (ITQ). At the beginning of each period t, a regulator is assumed to allocate a total allowable catch (TAC) among the n agents $q(t) = \sum_{i=1}^{n} q_i^-(t)$ where $q_i^-(t)$ stands for the initial amount of quota[2] given to the agent i. Let us denote $q_i(t)$ the amount of quota hold by agent i after trade. The quota market clearing condition is given by:

$$q(t) = \sum_{i=1}^{n} q_i(t).$$

[1]Typical instances of such dynamics are given by the Beverton-Holt relation $f(x) = Rx/(1+Sx)$ that will be used in the example examined in section 4.

[2]We assume that the original allocation of ITQs has been given free to the agents in proportion to their historical catch record. Moreover quotas can be freely traded in a secondary market and inter-temporal trade of quotas is not allowed.

The total catch $h(t)$ is the sum of n amount of harvests $h_i(t)$ with:

$$h(t) = \sum_{i=1}^{n} h_i(t).$$

Agents are assumed to be price takers both in the quota market and in their output market. They face a common quota price m and an exogenous unit price p for the resource. The ith agent's profit is defined as the difference between the income of its harvest ph_i and its production costs $C^i(x, h_i)$, the gain or the loss in the quota market whether he bought or sold quotas $m(q_i - q_i^-)$ and a penalty related to its noncompliance behavior $\Gamma^i(v_i)$ with $v_i = h_i - q_i$ the magnitude of the violation:

$$\pi_i(h_i, q_i) = ph_i - C^i(x, h_i) - m(q_i - q_i^-) - \Gamma^i(h_i - q_i). \tag{5}$$

For any agent i, its cost function $C^i(x, h)$ depends negatively on stock x, positively on harvest h, is assumed to be smooth enough ($\mathcal{C}^2$) on $I\!R^+$, strictly convex with respect to h:

$$C_x^i(x, h) < 0,\ C_h^i(x, h) > 0,\ C_{hh}^i(x, h) > 0,\ \forall x \geq 0,\ h \geq 0. \tag{6}$$

We also assume that the marginal cost of harvest is zero when catch vanishes while the marginal cost of harvest with respect to stock decreases toward zero with the stock[3]:

$$C_h^i(x, 0) = 0,\ \lim_{x \to +\infty} C_x^i(x, h) = 0. \tag{7}$$

An example of such a cost function is used in the example of section 4.

The term $\Gamma^i(h_i - q_i)$ stands for the penalty faced by a non-compliant agent when its amount of harvesting exceed the quota it gets[4] $h_i > q_i$. A compliant agent always operates with $h_i = q_i$. This penalty function consists in the product of the expected probability that the non-compliant agent will be audited and found in violation and a monetary fine [29]. We assume that the penalty $\Gamma^i(v)$ is zero only on negative values:

$$\Gamma^i(v) = 0 \text{ if } v \leq 0, \quad \text{and } \Gamma^i(v) > 0 \text{ if } v > 0. \tag{8}$$

[3]This may be related to zero sunk costs.

[4]As [16] has pointed out, a compliant and a non-compliant agent perform the same catches but by definition the non-compliant firm's quota demand is lower for a given quota price.

We also assume that Γ^i is continuously differentiable and convex. Let us remark that the first part of the assumption (8) induces the marginal relation:

$$\Gamma^i_v(0) = 0. \tag{9}$$

On $(I\!R^+)^*$, Γ^i is smooth enough ($\mathcal{C}^2$) and strictly convex:

$$\Gamma^i_{vv}(v) > 0 \text{ if } v > 0. \tag{10}$$

Every agent i solves the following optimization problem:

$$(h_i^*, q_i^*) \in \arg \max_{0 \le h \le x, 0 \le q} \pi_i(h, q) \tag{11}$$

with $\pi_i(h, q)$ given by (5). The following Proposition provides the optimal catch and quota.

Proposition 1 *The optimal solution of the problem (11) leads to a individual amount of catches h_i^* for a non-compliant agent greater than the corresponding quota q_i^* he/she buy at the given price m:*

$$h_i^* = (C_h^i(x, .))^{-1}(p - m) = q_i^* + (\Gamma^i_v)^{-1}(m)$$

To justify the implement of a regulation through a ITQ market, we also assume that the amount of catches without regulation (business as usual) $h^{\text{bau}}(x)$ lead to overexploitation and is not viable. This level corresponds to:

$$h^{\text{bau}}(x) = \sum_i (C_h^i(x, .))^{-1}(p).$$

Hence we assume that the resource always decrease with such a catch[5]:

$$f(x - h^{\text{bau}}(x)) \quad < \quad x, \ \forall x > 0.$$

From the quota market clearing condition $q = \sum_{i=1}^n q_i^*$, the following Proposition provides the statements on equilibrium price, optimal catches and quotas.

Proposition 2 *If the quota supply does not exceed the amount of catches without regulation $0 \le q \le h^{\text{bau}}(x)$, then:*

[5] This sequence is decreasing and bounded from below. Hence it converges toward the solution: $x^* = f(x^* - h^{\text{bau}}(x^*))$ which is negative. Thus $h^{\text{bau}}(x)$ is not a viable strategy.

- *there exists a unique quota price* $m^*(q,x)$ *(*$0 \leq m^* \leq p$*) such that the quota market clearing condition* $q = \sum_{i=1}^n q_i^*$ *holds.*

- *The equilibrium price function* $m^*(q,x)$ *is continuously differentiable (*$\mathcal{C}^1$*) with* $m_q^*(q,x) < 0$ *and* $m_x^*(q,x) > 0$*.*

- *The total harvesting* $h^*(x,q) = \sum_i h_i^*(x,q)$ *is given by the following general equation:*

$$h^*(x,q) \quad = \quad q + \sum_i (\Gamma_v^i)^{-1}(m^*(q,x)).$$

Note that a rise in the quota supply implies a fall in the quota price ($m_q^*(q,x) < 0$). An increase in the stock implies a decrease in the production cost and a rise in the amount of harvest creating a incentive for agents to buy more quotas. This yields to an increase in the quota price ($m_x^*(q,x) > 0$) as pointed out by [6].

2.3 The objectives of the regulating agency

The sustainability of the system is measured from the regulating agency view-point in terms of equity including inter and intra generational perspective. Such requirements are captured by effectiveness constraints at each period. Thus it turns out as a feasibility problem.

A direct use value requirement: The eco-system provides services or consumption through harvesting. The regulating agency requires at each time a guaranteed service:

$$U(h(t)) = U\Big(h^*\big(x(t), q(t)\big)\Big) \geq U_\flat \tag{12}$$

where $U_\flat > 0$ stands for some guaranteed utility level. We assume that the function U is increasing. This implies the equivalence with a guaranteed harvesting namely:

$$h(t) = h^*\big(x(t), q(t)\big) \geq h_\flat. \tag{13}$$

An implicit intragenerational equity requirement: Equity between agents can be derived from the guaranteed total catch constraint (13) or global utility requirement (12). An additional technical assumption on marginal costs is needed[6]:

$$0 < \gamma_i = \inf_{x \geq 0, y \geq 0} \frac{C_h^i(x,.)^{-1}(y)}{\sum_j C_h^j(x,.)^{-1}(y)} \tag{14}$$

Under the previous assumption, we derive the individual guaranteed catch constraint[7]:

$$h_i(t) = h_i^*\big(x(t), q(t)\big) \geq \gamma_i h_\flat.$$

An implicit conservation requirement: Combining the scarcity constraint (4), the increasing shape of U along with guaranteed utility requirement (12), we derive a conservation constraint for the exploited resource:

$$U(x(t)) \geq U\left(h(t)\right) \geq U_\flat. \tag{15}$$

Again, whenever U is an increasing and continuous function, this induces that the population of concern in the guaranteed utility goal is not extinct in the sense that a conservation level $h_\flat$ is:

$$x(t) \geq h_\flat. \tag{16}$$

Thus one can see this guaranteed catch constraint as a way to reconcile the ecological and economic points of view.

3 A viability analysis of the ITQ management system

We use the mathematical concept of viability kernel to characterize the sustainability of the system. This kernel is the set of initial resource for which

[6]For instance, any cost function of the type $C_i(x,h) = c_i\alpha(x)h^\beta$ with $\beta > 1$ is a bijection on $\boldsymbol{R}^+$ that meets the desired condition (14). An example is displayed in section 4.

[7]The coefficient γ_i might rely on historical catches record as detailed in the example 4,

$$\gamma_i = \frac{h_i^{\mathrm{bau}}(x)}{h^{\mathrm{bau}}(x)} = \frac{(C_h^i(x,.))^{-1}(p)}{\sum_j (C_h^j(x,.))^{-1}(p)}$$

This initial allocation of quota might also be related to the social equity constraint $q_i^-(t) = \gamma_i q(t)$.

an acceptable regime of quotas exists. Therefore, the viability kernel refers to some *ex post* viability features and provides the "true" constraints of the system. We start by determining the configurations which allow for sustainable catch paths. Then, for a given sustainable configuration, the initial resource conditions for which such policies exist are defined. Lastly, we analyze some specific TAC policies associated with these sustainable states, namely the conservative, greedy, sustainable yield and maximin strategies. In this dynamic analysis, we assume that the enforcement policy is insufficient to induce perfect compliance, so that violations of the quota occur. We also assume that the regulator has perfect knowledge of the penalty function and is able to estimate the aggregate violation when setting the TAC[8]. Our framework allows us to consider a mix of compliant agents and non-compliant agents.

3.1 The viability kernel

The dynamic of the whole system controlled by total quotas $q(t)$ becomes:

$$x(t+1) = f\Big(x(t) - h^*\big(x(t), q(t)\big)\Big) \tag{17}$$

where the catches derived from quota $q(t)$ and stock $x(t)$ according to Proposition (2) are given by:

$$h^*\big(x(t), q(t)\big) \quad = \quad q(t) + \sum_i (\Gamma_v^i)^{-1}(m^*(q(t), x(t))) \tag{18}$$

with $m^*(q(t), x(t))$ the equilibrium quota price. The sequential decision corresponds to the recommendation signal $q(t)$ provided by the regulating agency. The question that arises now is to determine whether the evolution (17) is compatible with the set of constraints defined by (12). In other words, we aim at revealing levels of resource $x(t)$ and recommendations $q(t)$ that are associated with a sustainable trajectory.

The set of such initial states is called the viability kernel associated with the dynamics and the constraints:

$$\mathrm{Viab}(f, h_\flat) = \left\{ x_0 \;\middle|\; \begin{array}{l} \text{there exists quotas } q(t) \text{ and resource states } x(t) \\ \text{starting from } x_0 \text{ satisfying conditions} \\ (12),(17) \text{ for any time } t \in \mathbf{R}^+ \end{array} \right\}.$$

[8]This assumption could be relax by assuming that the regulator holds a probability distribution over possible penalty rate values.

In the general mathematical framework, this set can alternatively be empty, or be the whole state constraint set or even a strict part of the initial state constraint domain. Moreover, the viability kernel captures an irreversibility mechanism. Indeed, from the very definition of this kernel, every state outside the viability kernel will violate the constraints in finite time whatever decisions applied. This means that the crisis is unavoidable. For instance, the extreme case where the viability kernel is empty corresponds to a hopeless configuration[9].

3.2 A ceiling guaranteed harvest

An upper bound on total guaranteed harvesting $h_\flat$ appears as depicted by the Proposition 3 below. This means that some sustainable harvesting are not possible. Not surprisingly, the maximal sustainable catch corresponds to the so-called maximum sustainable yield h_{MSY} defined by:

$$h_{\mathrm{MSY}} = \sup_{x \geq 0,\, x = f(x-h)} h \tag{19}$$

Proposition 3 *No sustainable resource management is possible if the guaranteed harvest is too large namely:*

$$\mathrm{Viab}(f, h_\flat) = \emptyset \ \textit{if } h_\flat > h_{\mathrm{MSY}}.$$

Thus the maximal viable guaranteed catch h_{MSY} only depends on the biological parameters.

3.3 A floor stock

Whenever the total required guaranteed catch $h_\flat$ is not too high according to the previous Proposition 3, it turns out that a viable resource management is possible as claimed by Proposition 4 below. In this case, a minimal stock level $x_\flat(h_\flat)$ is required. Note that this level is greater that the initial stock threshold $h_\flat$ of (16) which means that viability or safety margins are required.

Another restrictive condition is needed to take into account the impact of noncompliance. More specifically, a positive quota is required to achieve

[9] In particular, if the kernel is empty, this allows to eliminate "red herrings", objectives functions for optimal control problems that could never have a feasible solution.

the total guaranteed catches whatever resource state x is in the sense that:

$$\sup_{x\geq 0} h^*(0,x) \leq h_\flat.$$

From the definition of effective catches $h^*(0,x)$ in Proposition (2) and calculus explained in Lemma (2), we thus assume that:

$$h_{\min} = \sum_i (\Gamma_v^i)^{-1}(p) \leq h_\flat.$$

Proposition 4 *Assume that the guaranteed catches $h_\flat$ are such that $h_{\min} \leq h_\flat \leq h_{\mathrm{MSY}}$. Then sustainable resource managements are available if the current stock remains sufficiently large namely:*

$$\mathrm{Viab}(f, h_\flat) = [x_\flat(h_\flat), +\infty[$$

where $x_\flat(h_\flat)$ is the solution of

$$x_\flat(h_\flat) = \min\Big(x,\ x \geq h_\flat,\ f(x - h_\flat) = x\Big).$$

Let us note that an implicit viability condition is captured by the relation:

$$\sum_i (\Gamma_v^i)^{-1}(p) \leq h_{\mathrm{MSY}}$$

mixing economic and biological characteristics. Moreover, the floor stock $x_\flat(h_\flat)$ has connections with safe standards of conservation invoked in [27] and the safe stock level in [20]. Note that $x_\flat(h_\flat)$ is well-defined and is greater than initial stock threshold $h_\flat$ from the definition[10]:

$$x_\flat(h_\flat) > h_\flat.$$

Hence, a viability margin can be deduced from the difference between $x_\flat(h_\flat)$ and $h_\flat$:

$$\mathrm{ViabilityMargin} = x_\flat(h_\flat) - h_\flat.$$

It corresponds to a security margin to maintain in order to guarantee the sustainability of required harvesting $h_\flat$.

[10] If not, $x_\flat = h_\flat$ which implies $f(0) = 0 = x_\flat$ and a contradiction occurs.

3.4 The viable quota corridor

Whenever they are possible, the relevant policies consist in maintaining the state within the viability kernel, avoiding the crisis mentioned before. Thus, given a viable current state $x \in \mathrm{Viab}$, the relevant viable recommendations $q(x)$ (that exist) ensure that the state remains inside the viability kernel. Consequently the set of viable feedback is described by:

$$Q_{\mathrm{Viab}}(x) = \Big\{ q \geq 0 \mid h^*(q,x) \geq h_\flat,\ f(x - h^*(q,x)) \in \mathrm{Viab} \Big\}.$$

The characterization of the viability kernel Viab in Proposition (4) leads to the following assertion.

Proposition 5 *Given any initial viable state $x \in \mathrm{Viab}$, the set of viable quotas is not empty and corresponds to the interval:*

$$Q_{\mathrm{Viab}}(x) = [q_\flat(x), q_\sharp(x)]$$

where feedbacks $q_\flat(.)$ and $q_\sharp(.)$ are respectively defined by:

$$\begin{cases} h^*(q_\flat(x), x) & = h_\flat, \\ h^*(q_\sharp(x), x) & = x - x_\flat(h_\flat) + h_\flat. \end{cases}$$

Consequently, in the interior of the viability kernel where $x > x_\flat(h_\flat)$, the sustainable policies and thus paths are not reduced to one solution since $q_\sharp(x) > q_\flat(x)$. However, on the boundary of the viability kernel, the specific quota $q_\flat(x)$ is needed. Therefore there is not uniqueness of the sustainable policies in the viability kernel except on its boundary[11]. In this sense, this method does not provide a unique policy but the set of all viable policies. At this stage, let us point out that the multiplicity of sustainable policies allows for distinct viable strategies and room for manoeuvre. This includes policies based on resource conservation goals or optimal strategies such as the maximin approach.

3.5 Some viable quota strategies

Assume that $x \in \mathrm{Viab}$, the following strategies can be implemented:

[11]For the boundary $x = x_\flat(h_\flat)$, we get uniqueness of viable decisions since $q_\sharp(x) = q_\flat(x)$.

- The **minimal viable strategy** defined by $q_\flat(x)$ is viable since $q_\flat(x) \in Q_{\mathrm{Viab}}(x)$. Such a quota policy refers to an ecological and conservation viewpoint in the sense that it favors the resource.

- The **maximal viable strategy** defined by $q_\sharp(x)$ is viable since $q_\sharp(x) \in Q_{\mathrm{Viab}}(x)$. Such a policy promotes the catches and the economics. However it might be termed a greedy policy since it can only be implemented once a time:
$$f(x - h^*(q_\sharp, x)) = f(x_\flat - h_\flat) = x_\flat.$$

- Any **mixed viable quota strategies** defined by:
$$q_\alpha(t, x) = \alpha(t) q_\flat(x) + (1 - \alpha(t)) q_\sharp(x),\ \forall \alpha(t) \in [0, 1]$$
are also viable. The term α measures the trade-off between conservation and economics viewpoints which can depend on time t.

- **More efficient rules** (in the sense of inter-temporal optimality) might be considered among the viable quotas. The case of maximin is examined later on. Some net present value (utility, rent) criteria under the viability constraints can also be considered. Such a problem consists in optimizing the inter-temporal utility under the guaranteed catch constraint:
$$\max_{q(t)} \sum_{t=0}^{\infty} \rho^t U(h^*(q(t), x(t)))$$
under the constraint $h^*(q(t), x(t)) \geq h_\flat$ with ρ the discount rate. It can also be rewritten as:
$$\max_{q(t) \in \left[q_\flat(x), q_\sharp(x)\right]} \sum_{t=0}^{\infty} \rho^t U\left(h^*(q(t), x(t))\right)$$
for $x_0 \in \mathrm{Viab}$, namely $x_0 \geq x_\flat$.

3.6 Sustainable Yield is not a viable quota policy

The so-called Sustainable Yield refers to a situation where both harvest and resource are at equilibrium. Such an itchyocentrism viewpoint related to the sustainable yield quota $q_{\mathrm{SY}}(x)$ is the solution of:
$$f(x - q) = x.$$

Thus under assumptions (2) on the dynamics f, it corresponds to:

$$q_{\mathrm{SY}}(x) = x - f^{-1}(x).$$

It turns out that Sustainable Yield is not a viable quota policy as claimed by the Proposition below. This stems from non compliance of agents since catches are strictly larger than quotas. Hence harvests associated with a Sustainable Yield quota $q_{\mathrm{SY}}(x)$ induce a declining dynamics of the stock $x(t)$.

Proposition 6 $q_{\mathrm{SY}}(x)$ *is not viable feedback.*

In particular, the maximum sustainable yield MSY is not sustainable quota policy. The intuition for such a result is that the sequence:

$$x(t+1) = f(x(t) - h^*(q_{\mathrm{SY}}(x(t)), x(t))) < f(x(t) - q_{\mathrm{SY}}(x(t))) = x(t)$$

is strictly decreasing and bounded from below on $\mathbb{R}^+$. Hence it converges toward zero resource stock. Therefore it is not a viable strategy.

3.7 A maximin quota policy

A characterization in terms of maximin can be derived as follows. This enhances the viability results in terms of intergenerational equity.

Proposition 7 *Given initial state x_0, the maximin solution is obtained by:*

$$\begin{aligned} \max_{q(.)} \min_{t \geq 0} U(h^*(q(t), x(t))) &= \max_{x_0 \in \mathrm{Viab}(f, h_\flat)} U(h_\flat) \\ &= \max_{x_0 \geq x_\flat(h_\flat)} U(h_\flat) \end{aligned}$$

Let us stress the fact that the use of the viability kernel is a straightforward way of dealing with this kind of maximin concern: maximizing the guaranteed catch so that the current state lies on the boundary of the viability kernel.

4 An example

The present section aims at illustrating the previous general analytical results through an example where population dynamics, cost and penalties functions are specified. Our example allows us to consider a mix of compliant agents and non-compliant agents according the possible compliance rate values.

The resource dynamics: It is now assuming that the population dynamics is given by the Beverton-Holt relation:

$$f(x) = \frac{Rx}{1 + Sx}.$$

which satisfies required assumption (2). We set the conditions $R > 1$ and $S > 0$ to warrant a positive carrying capacity k defined as $k = (R-1)/S$. The sustainable yield function SY corresponds to

$$h \quad = \quad SY(x) = x\left(1 - \frac{1}{S(k-x)+1}\right).$$

For $0 < x < k$, the term into brackets is positive and lower than unity. The maximum sustainable stock x_{MSY} and harvest h_{MSY} are derived from (19). We obtain:

$$x_{\text{MSY}} = \frac{R - \sqrt{R}}{S}, \; h_{\text{MSY}} = \frac{(R - \sqrt{R})^2}{RS}.$$

The ITQ system: We consider the following specifications for the cost function:

$$C^i(x, h_i) = \frac{c_i}{2x} h_i^2$$

and the expected penalty function:

$$\Gamma_i(v_i) = \frac{\tau_i}{2}(v_i^+)^2 = \begin{cases} 0 & \text{if } v_i \leq 0 \\ (\tau_i/2)v_i^2 & \text{otherwise.} \end{cases}$$

The required assumptions (6), (7), (8), (10) and (14) for both cost and penalty functions hold true.

The profit of the ith agent can be written as:

$$\pi_i(h_i, q_i) \quad = \quad ph_i - \frac{c_i}{2x}h_i^2 - m(q_i - q_i^-) - \frac{\tau_i}{2}(h_i - q_i)_+^2.$$

The first-order conditions with respect to h_i and q_i leads to the optimal individual amount of catches h_i^* that exceeds the corresponding quota q_i^* he/she buy at the given price m:

$$h_i^* \quad = \quad \frac{(p-m)x}{c_i} = q_i^* + \frac{m}{\tau_i} > q_i^*.$$

The quota market clearing condition $q = \sum_{i=1}^n q_i^*$ yields the quota equilibrium price $m^*(q, x)$ with $\widetilde{c}^{-1} = \sum_{i=1}^n c_i^{-1}$ and $\widetilde{\tau}^{-1} = \sum_{i=1}^n (\Gamma_v^i)^{-1}$:

$$m^*(q,x) = \widetilde{\tau}\left(\frac{px - \widetilde{c}q}{\widetilde{\tau}x + \widetilde{c}}\right)$$

where $m_q^* < 0$ and $m_x^* > 0$. The individual and total harvests are given by the following general equations:

$$\begin{aligned} h_i^*(q,x) &= a_i(x)q + b_i(x) \\ h^*(q,x) &= a(x)q + b(x) \end{aligned}$$

with $a(x)$, $b(x)$, $a_i(x)$, $b_i(x)$ given by:

$$a(x) = \frac{\widetilde{\tau}x}{\widetilde{\tau}x + \widetilde{c}}, \quad b(x) = \frac{px}{\widetilde{\tau}x + \widetilde{c}}, \quad \frac{a_i(x)}{a(x)} = \frac{b_i(x)}{b(x)} = \frac{\widetilde{c}}{c_i}.$$

Note that that $a_x > 0$ and $b_x > 0$.

Without regulation, the amount of catches $h^{\text{bau}}(x)$ corresponds to:

$$h^{\text{bau}}(x) \quad = \quad \frac{px}{\widetilde{c}}.$$

Such a case refers to a situation where quota price is set to zero, optimal quotas and catches coincides so to imply:

$$h^*(h^{\text{bau}}(x), x) \quad = \quad h^{\text{bau}}(x).$$

To justify the implement of a regulation through a quota market, we assume that the amount of catches $h^{\text{bau}}(x)$ lead to overexploitation and is not viable. From the value of $h^{\text{bau}}(x(t))$, we get:

$$x(t+1) \quad = \quad f\left(x(t)\left(1 - \frac{p}{\widetilde{c}}\right)\right).$$

Hence, an over-exploitation configuration occurs for a large price-cost ratio $(p/\widetilde{c})$ greater than unity [8].

The viable resource states and the viability kernel: To exhibit some viable state, a ceiling guaranteed catch threshold is $h_{\text{MSY}} = ((R - \sqrt{R})^2/RS)$

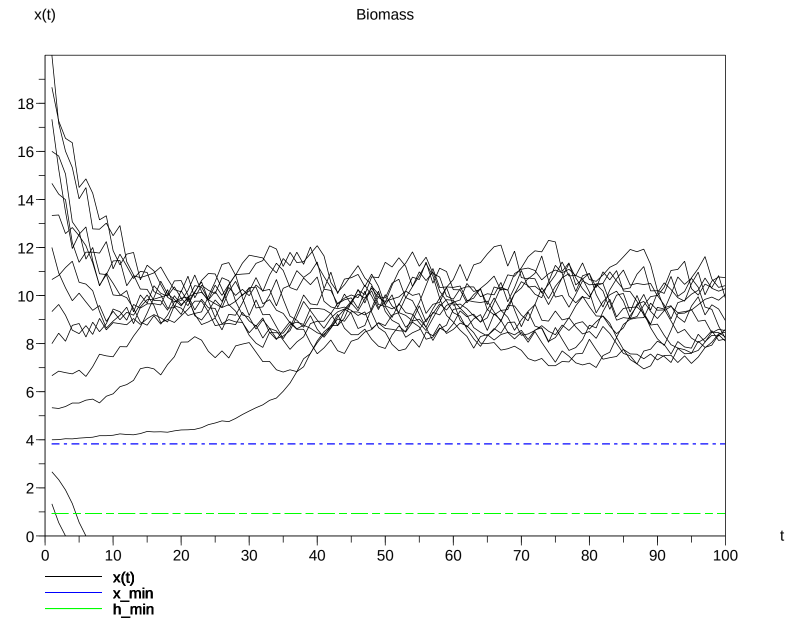


Figure 1: Paths of stock $x(t)$ over a period $[0, 100]$ with population parameters are productivity $R = 1.4$, carrying capacity $K = 20$. Viability requirement corresponds to total guaranteed catches along time of $h_\flat \approx 0.93$ (lower dashed line). Viable states lie over the threshold $x_\flat \approx 3.82$ (higher dashed line); the viability kernel corresponds to $\mathrm{Viab} = [x_\flat, +\infty[$.

from Proposition 3. Moreover, a floor threshold $h_{\min}$ for the total guaranteed harvesting is needed for Proposition 4. We obtain:

$$h_{\min} = \sup_{x \geq 0} h^*(0, x) = \frac{p}{\widetilde{\tau}}.$$

Let us remark that this minimal threshold tends to zero when agents get more compliant. Applying Proposition 4, we claim that condition $(p/\widetilde{\tau}) \leq h_\flat \leq ((R - \sqrt{R})^2/RS)$, ensures a nonempty viability kernel equals to $[x_\flat, +\infty[$. The value $x_\flat$ is the smallest equilibrium solution of $\mathrm{SY}(x) = h_\flat$ namely:

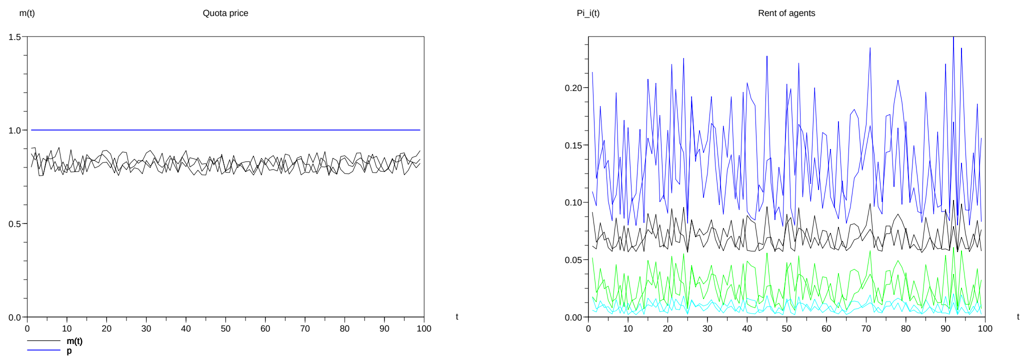
$$x = \frac{R(x - h_\flat)}{1 + S(x - h_\flat)}.$$

This yields to $Sx^2 - (R + Sh_\flat - 1)\, x + Rh_\flat = 0$. Setting $\Delta = (R - 1 + Sh_\flat)^2 - 4RSh_\flat$, it can be easily check that $\Delta > 0$ for $h_\flat < h_{\mathrm{MSY}}$ and $\Delta = 0$ for

$h_\flat = h_{\mathrm{MSY}}$. Hence, the value of $x_\flat$ is:

$$x_\flat = \frac{(k + S h_\flat)}{S} - \frac{\sqrt{\Delta}}{2S}.$$

The figure 1 depicts several evolutions of the stock $x(t)$. Over the viability threshold $x_\flat$, the resource evolves in a sustainable way remaining over the precautionary floor $x_\flat$ and consequently $h_\flat$ while every stock lying below the viability threshold declines and collapses. Population parameters of productivity $R = 1.4$, carrying capacity $K = 20$ together with harmonic mean compliance rate $\widetilde{\tau} = 8.85$, fixed price $p = 1$, and harmonic mean costs $\widetilde{c} = 1$ provide the conditions of sustainability $h_{\min} = \frac{p}{\widetilde{\tau}} < h_{\mathrm{MSY}} \approx 1.67$.



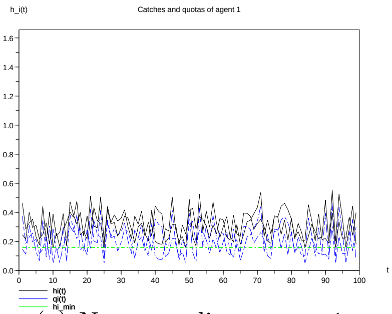
(a) Viable quota prices $m^*(t)$ and price p

(b) Rent of agents $\pi_i(q(t), h(t))$

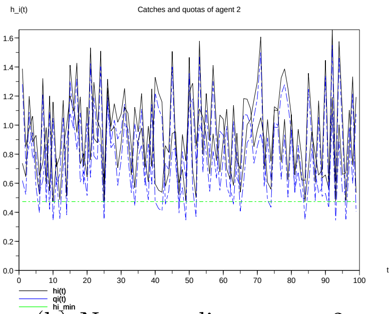
Figure 2: Viable economics paths over a period $[0, 100]$ in terms of (a) quota prices $m^*(t)$, (b) rent of four agents $\pi_i(q(t), h(t))$. Marginal costs are $c \approx (5.19, 1.73, 5.80, 17.42)$ while expected penalties are $\tau \approx (7.56; 3.30; +\infty; +\infty)$. Note that the quota price $m^*(t)$ remains below the resource price $p = 1$ (straight line).

The viable quotas: Now from Proposition (5), we derive the viable regulations:

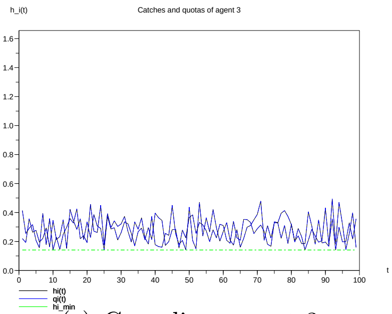
$$Q_{\mathrm{Viab}}(x) = [q_\flat(x), q_\sharp(x)]$$



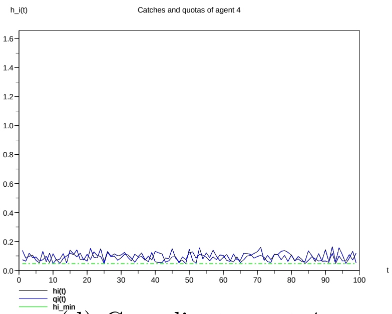
(a) Non compliant agent 1



(b) Non compliant agent 2



(c) Compliant agent 3



(d) Compliant agent 4

Figure 3: Viable catches $h_i(t)$ (plain) and quotas $q_i(t)$ (dash) for four distinct agents. Agents 1 and 2 are non compliant $h_i(t) > q_i(t)$ while agents 3 and 4 are compliant since expected penalties are $\tau \approx (7.56; 3.30; +\infty; +\infty)$. For every agent, an individual guaranteed catch $\gamma_i h_\flat$ (straight line) is displayed with $\gamma_i = \widetilde{c}/c_i$.

where feedbacks $q_\flat(.)$ and $q_\sharp(.)$ are respectively defined by:

$$
\begin{aligned}
q_\flat(x) &= \frac{h_\flat - b(x)}{a(x)} \\
q_\sharp(x) &= \frac{h_\flat - b(x)}{a(x)} + \frac{x - x_\flat}{a(x)}.
\end{aligned}
$$

Let us remark that only the means of costs $\widetilde{c}$ and penalty $\widetilde{\tau}$ are needed by the regulator to set adequate viable quota policy.

From Proposition 7, the maximin solution is:

$$\begin{aligned}\max_{q(.)}\min_{t\geq 0} U(h^*(q(t),x(t))) &= \max_{x_0\geq x_\flat(h_\flat)} U(h_\flat)\\ &= \begin{cases} U\left(x_0\left(1+\frac{1}{Sx_0-R}\right)\right) & \text{if} \quad x_0 \leq x_{\mathrm{MSY}}\\ U\left(h_{\mathrm{MSY}}\right) & \text{if} \quad x_0 > x_{\mathrm{MSY}}\end{cases}\end{aligned}$$

since $x_\flat(h)$ increases with h on $[0, h_{\mathrm{MSY}}]$ and $x_\flat(h_{\mathrm{MSY}}) = x_{\mathrm{MSY}}$.

For viable paths, the evolution of economic variables such as quota price $m^*(t)$ and rents of agents $\pi(t)$ are plotted in figure 2. Note that a guaranteed rent for every agent appears as an interesting consequence of guaranteed catches.

Figure 3 illustrates the equity for each agent through individual guaranteed catches $\gamma_i h_\flat$ as explained[12] in paragraph 2.3. Here the allocation $\gamma_i h_\flat$ depends on costs since $\gamma_i = \widetilde{c}/c_i$.

5 Conclusion

This paper addresses the problem of the sustainable management of a renewable resource in an ITQs system. A bio-economic model is built with general and abstract assumptions on dynamics, cost and penalty functions. Through a quota policy, the planner aims at ensuring catches along time to myopic, optimizing and non compliant agents that coordinate on a quota market at each time. Using the viable control approach and especially the viability kernel, we identify feasible resource state and possible TAC policies under a constraint of guaranteed catch. It turns out this constraint induces intergenerational and intragenerational equity along with conservation of the stock. Thus, both ecological and economics objectives are achieved by the regulating agency.

Although illustrative in nature, the study highlights some important considerations for decision making. Our results show how the ITQ management

[12] If we assume that the original allocation of ITQs has been given free to the agents in proportion to their historical catch record then we can show that an agent i buys (or sales) permits $q_i^* - q_i^- > 0$ (or $q_i^* - q_i^- < 0$) when $(\widetilde{c}/c_i) > (\widetilde{\tau}/\tau_i)$ (or $(\widetilde{c}/c_i) > (\widetilde{\tau}/\tau_i)$) where q_i^- stands for the number of permits given to agent i and q_i^* the number of permits hold by agent i after trade.

system is viable under accurate conditions. Firstly, noncompliance has to be weak with respect to the productivity of the resource. A high amount of noncompliance harvest reduces the regulator's room for manoeuvre to guarantee an amount of catch for the agents through a quota policy. Secondly, our analysis also puts forward the necessity for the regulator to preserve the resource stock above a safety level. If the resource decreases below this critical level, the regulator is not able to ensure utility to the agents. Lastly, the regulator have to care about the quota level that is implemented. A desirable corridor of quotas is required. It includes a ceiling quota allowing the guaranteed catch and taking into account non compliance of agents. It also includes a upper precautionary threshold which provides the respect of the previous catch constraint along time.

But important questions remain for future research. A modeling of multi-species in an ITQs system might shed further light on the relationship between the by-catch problem and the resource sustainability. The extension of the work to deal with uncertainties is also a challenging task. In our framework, it will concern ecological uncertainties on the stock recruitment function and economic uncertainties about the relation between the enforcement policy and the level of noncompliance. Lastly, assuming imperfect competition on the resource market [28] and on the quota market [23, 29] will raise several questions on the impact of the initial allocation of permits on the level of compliance and on the trade-off between efficiency and equity.

A The proofs

Proof of Proposition 1 — The necessary and sufficient first-order conditions of the problem (11) with respect to h_i and q_i are:

$$\left\{\begin{array}{lcl} 0 & = & p - C_h^i(x, h_i) - \Gamma_v^i(h_i - q_i) \\ 0 & = & -m + \Gamma_v^i(h_i - q_i) \end{array}\right. \tag{20}$$

Since $h \to C^i(x, h)$ is regular and convex, the function $h \to C_h^i(x, h)$ is increasing, continuous and thus invertible. Consequently $C_h^i(x, .)^{-1}$ makes sense. Combining the two conditions (20) yields the optimal harvesting:

$$h_i^* = (C_h^i(x, .))^{-1}(p - m).$$

Since $h \to \Gamma^i(h)$ is $\mathcal{C}^1$ and convex, the function $h \to \Gamma^i(h)$ is increasing, continuous and thus invertible. Consequently, the optimal demand of quota for an agent is characterized by:

$$q_i^* = h_i^* - (\Gamma_v^i)^{-1}(m)$$

$\square$

Proof of Proposition 2 — For sake of clarity, let us now introduce the following notations:

$$\begin{aligned} Z^i(x,y) &= (C_h^i(x,.))^{-1}(y), \\ W^i(y) &= (\Gamma_v^i)^{-1}(y). \end{aligned}$$

Summing to the n agents the demand of quotas leads to:

$$D(m) = \sum_{i=1}^{n} q_i^* = \sum_i \big(Z^i(x,p-m) - W^i(m)\big)$$

while the supply of the regulating agency is given by $S(m) = q$. Thus, the quota market clearing condition $S(m) = D(m)$ yields:

$$q = \sum_i \Big(Z^i(x,p-m) - W^i(m)\Big).$$

Now consider any q such that $q \leq \sum_i Z^i(x,p)$. We prove in that case that:

- $S(p) - D(p) > 0$
- $S(0) - D(0) \leq 0$
- $m \to S(m) - D(m)$ continuous monotonic

Indeed, using the condition on marginal cost function and positivity of marginal penalty function, we claim that:

$$\begin{aligned} S(p) - D(p) &= q - \sum_i \big(Z^i(x,0) + W^i(p)\big) \\ &= q + \sum_i W^i(p) \\ &> 0. \end{aligned}$$

Moreover, since $q \leq \sum_i Z^i(x,p)$ and condition (9) $W^i(0) = 0$, we get:

$$\begin{array}{rcl} S(0) - D(0) & = & q - \sum_i \left(Z^i(x,p) + W^i(0)\right) \\ & = & q - \sum_i Z^i(x,p) \\ & \leq & 0. \end{array}$$

Furthermore, since $C_h^i(x,.)$ is increasing and continuous, so is the inverse function $Z^i(x,.)$. Then the function $m \to \sum_i Z^i(x, p-m)$ is decreasing and continuous. Similarly, since $\Gamma_v^i(.)$ is increasing and continuous, $m \to \sum_i W^i(m)$ is increasing and continuous. Thus $D(m)$ is decreasing and $S(m) - D(m)$ is increasing.

Therefore, from some intermediary theorem, there exists a unique positive price $m^*(q,x) \in [0,p[$ such that the equilibrium occurs:

$$0 = S(m^*(q,x)) - D(m^*(q,x)).$$

Furthermore the equilibrium price function $m^*(q,x)$ is continuously differentiable ($\mathcal{C}^1$) from an implicit theorem together with the regularity of demand $D(.)$ and supply $S(.)$ on $\mathbb{R}^+$.

Since all the partial derivatives Z_x^i,Z_y^i, W_y^i are positive, the qualitative marginal properties reads as follows:

$$\begin{array}{rcrcl} 0 & < & m_x^*(q,x) & = & \left(\sum_{i=1}^n Z_h^i + \sum_{i=1}^n W_h^i\right)^{-1} \sum_{i=1}^n Z_x^i \\ 0 & > & m_q^*(q,x) & = & -\left(\sum_{i=1}^n Z_h^i + \sum_{i=1}^n W_v^i\right)^{-1} \end{array}$$

□

Proof of Proposition 3 — Assume that $h_\flat > h_{\mathrm{MSY}}$ and suppose for a while that Viab $\neq \emptyset$. Pick up any $x_0 \in$ Viab. From the very definition of the viability kernel, there exists a path $(x(t), q(t))$ with $x(0) = x_0$ that remains in Viab in the sense that:

$$h_\flat \leq h^*(q(t), x(t)) \leq x(t).$$

Since $h_\flat > h_{\mathrm{MSY}}$, Lemma (3) yields $x_\flat(h_\flat) = +\infty$. Then, combining the lemmas (4) and (5) in this case, we deduce the existence of a time T such that $x(T) \leq x_\flat(T) < h_\flat$. Consequently, a contradiction occurs. Therefore $x_0 \notin \text{Viab}$ which ends the proof. $\square$

Proof of Proposition 4 — Now we assume that $h_\flat \leq h_{\text{MSY}}$. We used the three lemmas (3), (4) and (5) proved later on to obtain the desired assertion. Following [2], we proceed in two steps:

1. We first prove that the set $V = [x_\flat(h_\flat), +\infty[$ is weakly invariant and is a subset of Viab.
2. Second we prove that $[h_\flat, x_\flat(h_\flat)[\subset \mathbb{R}_+ \backslash \text{Viab}$.

1) From lemma (3), we know that $x_\flat(h_\flat) < +\infty$ and the set V makes sense. Now, consider any $x \in V$ namely $x \geq x_\flat(h_\flat)$. Pick the lowest admissible quota $0 \leq q_\flat(x)$ defined by $h^*(q_\flat(x), x) = h_\flat$ which exists from lemma (1). Since f is non decreasing, we can write:

$$f(x - h^*(q_\flat(x)), x) \geq f(x_\flat(h_\flat) - h_\flat) = x_\flat(h_\flat).$$

Thus for any $x \in V$, there exists $q = q_\flat(x)$ such that $f(x - h^*(q, x)) \in V$. Consequently, V is weakly invariant. Since $x_\flat(h_\flat) > h_\flat$ we also have $V \subset [h_\flat, +\infty[$. Thus we conclude that $V \subset \text{Viab}$.

2) Now consider any x_0 such that $h_\flat \leq x_0 < x_\flat(h_\flat)$ and assume for a while that $x_0 \in \text{Viab}$. Then there exists a solution $x(t)$ starting from x_0 that remains in Viab. Combining lemmas (5) and (4), we deduce the existence of a time T such that $x(T) \leq x_\flat(T) < h_\flat$. We derive a contradiction with $x_0 \in \text{Viab}$. Therefore $x_0 \notin \text{Viab}$ which ends the proof. $\square$

Proof of Proposition 5 — The set of viable quotas corresponds to:

$$Q_{\text{Viab}}(x) = \left\{ q \geq 0,\ h^*(q, x) \geq h_\flat,\ f(x - h^*(q, x)) \in \text{Viab} \right\}.$$

Given any initial state $x \in \text{Viab}$, this set $Q_{\text{Viab}}(x)$ is not empty. Using the Proposition (4), any $q \in Q_{\text{Viab}}(x)$ satisfies the relations:

$$f(x - h^*(q, x)) \geq x_\flat(h_\flat),\ h^*(q, x) \geq h_\flat.$$

Since $x_\flat(h_\flat) = f(x_\flat(h_\flat) - h_\flat)$, we obtain equivalently:

$$f(x - h^*(q,x)) \geq f(x_\flat(h_\flat) - h_\flat),\ h^*(q,x) \geq h_\flat.$$

Since f is non decreasing, this means that:

$$x - h^*(q,x) \geq x_\flat(h_\flat) - h_\flat,\ h^*(q,x) \geq h_\flat$$

or

$$h_\flat \leq h^*(q,x) \leq x - x_\flat(h_\flat) + h_\flat.$$

Since $q \to h^*(q,x)$ is not decreasing and continuous, we deduce the assertion: $Q_{\mathrm{Viab}}(x) = [q_\flat(x), q_\sharp(x)]$. □

Proof of Proposition 7 — Given any initial state x_0, consider $(q^*(t), x^*(t))$ an optimal solution of the maximin problem:

$$V(x_0) = \min_{t\geq 0} U(h^*(q^*(t), x^*(t))) = \max_{q(.)} \min_{t\geq 0} U(h^*(q(t), x(t))).$$

Then $x_0 \in \mathrm{Viab}(f, U^{-1}(V(x_0)))$ since $(q^*(t), x^*(t))$ is a viable path with:

$$U(h^*(q^*(t), x^*(t))) \geq V(x_0),\ \forall t \geq 0.$$

Thus:

$$V(x_0) \leq \max_{x_0 \in \mathrm{Viab}(f,h_\flat)} U(h_\flat).$$

Conversely, for any $h_\flat$ and any $x_0 \in \mathrm{Viab}(f, h_\flat)$, there exists $(x(t), q(t))$ such that:

$$h^*(q(t), x(t))) \geq h_\flat,\ \forall t \geq 0.$$

Consequently:

$$\min_{t\geq 0} U(h^*(q(t), x(t))) \geq U(h_\flat)$$

and

$$\max_{q(.)} \min_{t\geq 0} U(h^*(q(t), x(t))) \geq U(h_\flat).$$

Therefore:

$$V(x_0) \geq \max_{x_0 \in \mathrm{Viab}(f,h_\flat)} U(h_\flat)$$

and the equality holds true. □

Lemma 1 *Assume that* $h_{\min} = \sup_{x\geq 0} h^*(0,x) \leq h_\flat$. *Then*

$$\min_{q\geq 0,\ h^*(q,x)\geq h_\flat} h^*(q,x) = h_\flat.$$

Lemma 2 *We get* $\sup_{x\geq 0} h^*(0,x) = \sum_i (\Gamma_v^i)^{-1}(p)$.

Lemma 3 *We get:*

$$\inf\left(x,\ x \geq h_\flat,\ f(x-h_\flat) \geq x\right) = \begin{cases} +\infty & \text{if } h_\flat > h_{\mathrm{MSY}} \\ x_\flat(h_\flat) < +\infty & \text{if } h_\flat \leq h_{\mathrm{MSY}} \end{cases}$$

Lemma 4 *Consider any* x_0 *such that* $h_\flat \leq x_0 < x_\flat(h_\flat)$ *and*

$$M = \max_{x,\ h_\flat\leq x\leq x_0} \left(f(x-h_\flat) - x\right)$$

then $M < 0$.

Lemma 5 *Consider* $x_\flat(t)$ *the solution of* $x(t+1) = f(x(t) - h_\flat)$ *starting from* x_0 *with* $h_\flat \leq x_0 < x_\flat(h_\flat)$. *Then, for any* $t \geq 0$, *and any admissible solution* $x(.)$ *of (17), we have:*

$$x(t) \leq x_\flat(t) \leq x_0 + tM \leq x_0.$$

Proof of Lemma 1 — First, it is clear that:

$$\min_{q\geq 0,\ h^*(q,x)\geq h_\flat} h^*(q,x) \geq h_\flat.$$

Second, let us assume for a while that the inequality is strict for some $x \geq 0$. Then:

$$\forall q \geq 0,\ h^*(q,x) > h_\flat.$$

In particular, we derive that $h^*(0,x) > h_\flat$. This is in contradiction with the assumption that $h_{\min} = \sup_{x\geq 0} h^*(0,x) \leq h_\flat$. Consequently, the equality holds true. □

Proof of Lemma 2 — The function $x \to h^*(0,x)$ is not decreasing. Thus:

$$\sup_{x\geq 0} h^*(0,x) = h^*(0,+\infty).$$

Letting x converge to the $+\infty$ in the optimality condition:

$$0 = p - C_h^i(x, .)h_i - m^*(0, x).$$

We derive from the condition (7) $C_x^i(+\infty, h) = 0$ so that:

$$p = m^*(0, +\infty),\ h^*(0, +\infty) = \sum_i W^i(p)$$

which is the desired statement. □

Proof of Lemma 3 — Assume that $h_\flat \le h_{\mathrm{MSY}}$. Consider the set:

$$\mathcal{A} = \{x,\ x \ge h_\flat,\ f(x - h_\flat) \ge x\} \tag{21}$$

This set $\mathcal{A}$ is not empty because x_{MSY} belongs to it. Indeed, $x_{\mathrm{MSY}} \ge h_{\mathrm{MSY}} \ge h_\flat$. Moreover since $h_\flat \le h_{\mathrm{MSY}}$ and f non decreasing, we can write that:

$$f(x_{\mathrm{MSY}} - h_\flat) \ge f(x_{\mathrm{MSY}} - h_{\mathrm{MSY}}) = x_{\mathrm{MSY}}.$$

Furthermore $\mathcal{A}$ is bounded from below by $h_\flat$. This implies the existence of the minimum for this set. Such a minimum lies on the boundary of the set and thus satisfies the equality $f(x - h_\flat) = x$. Thus $x_\flat(h_\flat)$ exists. □

Proof of Lemma 4 — Assume for a while that $M \ge 0$. Then there exists $x^* \in [h_\flat, x_0]$ such that:

$$f(x^* - h_\flat) - x^* \ge 0.$$

This implies that $x^* \in \mathcal{A}$ where $\mathcal{A}$ is the set defined in (21). Thus:

$$x^* \ge \min_{x \in \mathcal{A}} x = x_\flat(h_\flat).$$

We derive a contradiction since $x^* \le x_0 < x_\flat(h_\flat)$. □

Proof of Lemma 5 — Recursive proof. At time $t = 0$, we get:

$$x_0 = x(0) =\le x_\flat(0) \le x_0 + tM \le x_0.$$

Assume the relation holds true at time t. Then:

$$x_\flat(t+1) = x(t) + f(x(t) - h_\flat) - x(t) \le x(t) + M \le x_0 + (t+1)M \le x_0.$$

Moreover, since f is non decreasing, for any admissible $q(t)$ and $x(t)$, we claim:

$$x(t+1) = f(x(t) - h^*(q(t), x(t)) \le f(x_\flat(t) - h_\flat) = x_\flat(t+1).$$

This concludes the proof. □

References

[1] D.J. Agnew, The illegal and unregulated fishery for toothfish in the southern ocean and the CCAMLR catch documentation scheme, *Marine Policy.* 24, 361-374(2000)

[2] J.P. Aubin, *Viability Theory.* Birkhäuser, Springer Verlag (1991).

[3] R. Arnason, Minimum Information Management in Fisheries, *Canadian J. of Economics.* 3, 630-653 (1990).

[4] C.J. Batsone, B.M.H. Sharp, Minimum information management systems and ITQ fisheries management, *J. of Environ. Econ. Manage.* 45, 492-504 (2003).

[5] C. Béné, L. Doyen, D. Gabay, A viability analysis for a bio-economic model, *Ecological Economics.* 36, 385-396 (2001).

[6] C. Chavez, H. Salgado, Individual Transferable Quota Markets under Illegal Fishing, *Environmental and Resource Economics.* 31, 303-324 (2005).

[7] V. Christensen, D. Pauly, ECOPATH II : a software for balancing steady-state models and calculating network characteristics, *Ecological Modelling.* 61, 169-185 (1992).

[8] C.W. Clark, *Mathematical Bio-economics: The Optimal Management of Renewable Resources, Second Edition.* J.Wiley & Sons, New York (1990).

[9] F.H. Clarke, Y.S. Ledyaev, R.J. Stern, P.R. Wolenski, Qualitative properties of trajectories of control systems: a survey, *J. of Dynamical Control Systems.* 1, 1-48 (1995).

[10] J.M. Conrad, *Resource economics*, Cambridge University Press (1999).

[11] P. Cury, C. Mullon, S. Garcia and L. J. Shannon, Viability theory for an ecosystem approach to fisheries, *ICES Journal of Marine Science.* 62(3), 577-584 (2005).

[12] L. Doyen, C. Bene, Sustainability of fisheries through marine reserves: a robust modeling analysis, *J. of Environmental Management.* 69, 1-13 (2003).

[13] K. Eisenack, J. Sheffran and J. Kropp, The Viability Analysis of Management Frameworks for fisheries, *Environmental Modeling and Assessment*. 11(1), 69-79 (2006).

[14] T. Lauck, C.W. Clark, M. Mangel, G.R. Munro, Implementing the Precautionary Principle in Fisheries Management through Marine Reserves. *Ecological Applications*. 81, Special Issue S72-78 (1998).

[15] A. Hatcher, S. Jaffry, O. Thebaud, E. Bennet, Normative and Social Influences Affecting Compliance with Fishery Regulations, *Land Econonmics*. 76(3), 448-461 (2000).

[16] A. Hatcher, Non-compliance and the quota price in a ITQ fishery. *J. of Environ. Econ.Manage.* 49, 427-436 (2005).

[17] A. Hatcher, D. Gordon, Further Investigations into the factors Affecting Compliance with U.K. Fishing Quotas. *Land Econonmics.* 81(1), 71-86 (2005).

[18] G. Heal, *Valuing the Future, Economic Theory and Sustainability*, Columbia University Press, New York (1998).

[19] J.M. Hartwick, N.D. Olewiler, *The Economics of Natural Resource Use*, second edition. Harper and Row, New York (1998).

[20] F.R. Homans, J.E. Wilen, A Model of Regulated Open Acess Resource Use, *J. of Environ. Econ.Manage.* 32, 1-21 (1997).

[21] A. Keeler, Noncompliant Firms in Transferable Discharge Permit markets: Some Extensions, *J. of Environ. Econ.Manage.* 21, 180-189 (1991).

[22] A. Malik, Markets for pollution control when firms are noncompliant, *J. of Environ. Econ.Manage.* 18, 97-106 (1990).

[23] A. Malik, Further Results on permit Markets with Market Power and Cheating, *J. of Environ. Econ.Manage.* 44, 371-390 (2002).

[24] V. Martinet, L. Doyen, Sustainable management of an exhaustible resource: a viable control approach, *Resource and Energy Economics*, forthcoming.

[25] R.G. Newell, J.N. Sanchirico, S. Kerr, Fishing quota markets, *J. of Environ. Econ. Manage.* 49,(2005).

[26] W.F. Morris, D.F. Doak, *Quantitative Conservation Biology: Theory and Practice of Population Viability Analysis*, Sinauer Associates (2003).

[27] L.J. Olson, S. Roy, Dynamic Efficiency of Conservation of Renewable resources under Uncertainty, *J. of Economic Theory.* 95, 186-214 (2000).

[28] L.K. Sandal, S.I. Steinshamn, Dynamic Cournot-competitive harvesting of a common pool resource,*J. of Econ. Dynam and Control.* 28, 1781-1799 (2004).

[29] H. van Egteren, C. Weber, Marketable Permits, Market Power, and Cheating, *J. of Environ. Econ.Manage.* 30, 161-173 (1996).

[30] J.E. Wilen, Renewable Resource Economists and Policy: What Difference have We Made, *J. of Environ. Econ. Manage.* 39, 306-327 (2000).