Natural vs. financial insurance in the management of public-good ecosystems

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Abstract

In the face of uncertainty, ecosystems can provide insurance to risk averse users of these systems. We employ a conceptual ecological-economic model to analyze the allocation of (endogenous) risk and environmental quality by risk averse decision makers, who can decide upon ecosystem management and who have access to financial insurance, and study the implications for optimal ecosystem management and policy design. We show that while an improved access to financial insurance leads to lower ecosystem quality, the effect on the free-rider problem and on welfare is determined by ecosystem properties. If financial insurance becomes more accessible, (i) the extent of optimal regulation may decrease or increase, depending on the relative size of private and external effects of management effort on biodiversity; (ii) the welfare loss due to free-riding may decrease or increase, depending on how biodiversity influences ecosystem service provision; and (iii) in the absence of environmental regulation, welfare may decrease or increase, depending on both the relative size of private and external effects of management effort on biodiversity and on how biodiversity influences ecosystem service provision; it decreases, if the external effect is large and if higher biodiversity greatly decreases the variance of ecosystem services.

1 Introduction

Human well-being depends in manifold ways on ecosystem services, which are understood as the various benefits provided by natural or managed ecosystems (Daily 1997, Assessment 2005). Examples include goods such as food, fuel or fibre; and services such as pollination or the regulation of local climate, pests, diseases or water runoff from a watershed. In a world of uncertainty, human well-being depends not only on the mean level at which such services are being provided, but also on their statistical distribution. Biodiversity can reduce the variance at which desired ecosystem services are provided. This means, biodiversity can provide insurance to risk averse users of these systems, e.g. crop, orchard or livestock farmers, or water utility managers. Since increasing biodiversity generates this *insurance value* for ecosystem managers, they tend to employ more conservative management strategies in the face of uncertainty (Baumgärtner forthcoming, Baumgärtner and Quaas 2005). Hence, uncertainty may be socially beneficial, because the overuse of public natural resources is reduced (Bramoullé and Treich 2005, Sandler and Sterbenz 1990, Sandler et al. 1987).

On the other hand, rather than making use of natural insurance, ecosystem users can also use other possibilities to insure against income fluctuations. In the USA for over one hundred years crop insurances are offered in agriculture to manage risk. Since traditional crop insurances are particularly vulnerable to classical insurance problems such as moral hazard and adverse selection (e.g., Luo et al. 1994), recently considerable effort is spent to develop alternative possibilities of financial insurance for farmers, e.g. index-based insurances (Miranda and Vedenov 2001, Skees et al. 2002, World Bank 2004).

While this effort to develop instruments of financial insurance is motivated by the idea that reducing income risk is beneficial for ecosystem users, some studies have shown that financial insurances tend to have ecologically negative effects. Horowitz and Lichtenberg (1993) show that financially insured farmers are likely to undertake riskier production - with higher nitrogen and pesticide use - than uninsured farmers do. A similar result is pointed out in Mahul (2001), assuming a weather-based insurance. Wu (1999) estimates in an empirical study the impact of insurances on the crop mix and its negative results on soil erosion in Nebraska (USA).

In this paper, we analyze, how risk-averse ecosystem managers make use of financial insurance and of the natural insurance function of biodiversity. We address the question, how the availability of financial insurance affects the overuse of natural resources and social welfare when management measures generate both a private benefit and, via ecosystem processes at higher hierarchical levels, positive externalities on other ecosystem users.

The analysis is based on a conceptual ecological-economic model. Ecosystem services (e.g. pollination of orchards by insects) are random because of exogenous sources of risk (e.g. winter temperature); their distribution (mean and variance) is determined by ecosystem quality (biodiversity). Ecosystem quality, in turn, can be influenced by management action (e.g. setting aside land for wetlands and hedges as habitat for insects) that affects ecosystem processes at different scales. Ecosystem users are risk-averse and choose a management action such as to maximize utility from ecosystem services (e.g. income from orchard farming). Our modeling of biodiversity and the provision of ecosystem services captures important insights about ecosystem functioning that emerged from recent theoretical, experimental and observational research in ecology (Hooper et al. 2005, Kinzig et al. 2002, Loreau et al. 2001, 2002, Holling 2001, Levin 2000, Peterson et al. 1998, Tilman 1994, O'Neill 1986).[1] Among other insights three 'stylized facts' about biodiversity and ecosystem functioning emerged which are of crucial importance for the issue studied here:

1. *Biodiversity may enhance the mean level of ecosystem services.* In many instances, an increase in the level of biodiversity monotonically increases the mean absolute level at which certain ecosystem services are provided. This effect decreases in magnitude with the level of biodiversity.

2. *Biodiversity may reduce the variance of ecosystem services.* In many instances, an increase in the level of biodiversity monotonically decreases the temporal and spatial variability of the level at which these ecosystem services are provided under changing environmental conditions. This effect decreases in magnitude with the level of biodiversity.

3. *Local biodiversity is affected by ecosystem processes at different hierarchical scales.* Ecosystems are hierarchically structured, with processes operating at different spatial and temporal scales and interacting across scales. Species diversity is typically influenced differently by processes at different scales. Accordingly, biodiversity management measures at different scales have different impact on local biodiversity.

[1] The article by Hooper et al. (2005) is a committee report commissioned by the Governing Board of the Ecological Society of America. Some of its authors have previously been on opposite sides of the debate. This report surveys the relevant literature, identifies a consensus of current knowledge as well as open questions, and can be taken to represent the best currently available ecological knowledge about biodiversity and ecosystem functioning.

These stylized ecological facts are of economic relevance.[2] Biodiversity increasing management creates benefits in terms of a higher mean level and a reduced variance of ecosystem services. In particular, an individual manager's action affects biodiversity via ecosystem processes at different scales. At a lower scale, benefits accrue exclusively to him. At a higher scale, his action contributes to increasing local biodiversity for other users, thereby generating a positive externality. For example, by setting aside land on his farm as habitat for insects, an individual farmer increases the local level of biodiversity on his farm and also contributes – via metapopulation dynamics – to biodiversity on other farms.

Our analysis of environmental risk, ecosystem management and purchase of financial insurance brings together two separate strands in the literature: (i) In the environmental economics literature, Crocker and Shogren (1999, 2001, 2003) and Shogren and Crocker (1999) have developed the idea that environmental risk is endogenous, that is, economic decision makers bearing environmental risk may influence their risk through their actions. They have formalized decision making under uncertainty in this context by conceptualizing ecosystems as lotteries. (ii) In the insurance economics literature, the analysis of the trade-off between 'self insurance' (by acting such as to reduce a potential income loss) or 'self protection' (by acting such as to reduce the probability of an income loss) on the one hand, and 'market insurance' on the other hand goes back to Ehrlich and Becker (1972) . One standard result is that self insurance and market insurance are substitutes, with the result that market insurance, as it becomes cheaper, my drive out self insurance. In this paper, we combine these two lines of argument. The methodological innovation of our analysis is the concept of *insurance value* of some action, which allows us to conceptualize ecosystem management as a form of natural insurance and study its interaction with financial insurance.

We study an economy, where individual ecosystem managers face the trade-off between obtaining natural insurance from ecosystem management and hedging income risk with financial insurance. We show that natural insurance by conservative ecosystem management and financial insurance coverage are substitutes. Hence, availability of financial insurance reduces the demand for natural insurance from ecosystems and, thus, leads to a less conservative management action which results in lower ecosystem quality. In particular, the lower the transaction costs of financial insurance are (i.e. the more actuarially fair the risk premium of financial insurance is), the less conservative are the individually optimal management actions and the

[2] For a more detailed and encompassing discussion of these findings, and references to the literature, see Baumgärtner and Quaas (2005) and Hooper et al. (2005).

lower is the resulting ecosystem quality.

Yet, the effect of an improved access to financial insurance on the free-rider problem is ambiguous. We show that this relationship crucially depends on the ecosystem's properties. The extent of the optimal regulatory intervention may decrease or increase depending on the relative effects of management measures on biodiversity via the lower, i.e. individual, and the higher, i.e. public, scale. Additional information about ecosystem functioning is required in order to assess how the availability of financial insurance influences the welfare loss due to free-riding, which also may decrease or increase with uncertainty. If biodiversity reduces the variance of ecosystem services very strongly, the welfare loss increases, if financial insurance becomes more easily accessible. Moreover, if additionally the external effect of individual management actions has a very strong influence, welfare in the laissez-faire equilibrium actually decreases, if financial insurance is introduced. If, on the other hand, biodiversity hardly reduces the variance of ecosystem services, the welfare loss decreases, if the costs of financial insurance decline. In other words, for poorly manageable ecosystems, the free-rider problem decreases when financial insurance becomes available.

The paper is organized as follows. In Section 2 we introduce the concept of insurance value in general. In Section 3 we then specify an ecological-economic model of an ecosystem which is being managed for the ecosystem services that it provides. The analysis and results are presented in Section 4, with all proofs and formal derivations contained in the Appendix. Section 5 concludes.

2 The concept of insurance value

Consider a world of uncertainty in which a decision maker chooses an n-vector of actions $z = (z_1, \ldots, z_n)$. His income y is random, and the probability distribution of income is uniquely determined by his actions z. The decision maker's preferences over his uncertain income y are represented by a von Neumann-Morgenstern expected utility function

$$U = \mathcal{E}_z[u(y)], \tag{1}$$

where $\mathcal{E}_z$ is the expectancy operator based on the probability distribution of income, which is determined by the action z. The Bernoulli utility function $u(y)$ is increasing ($u' > 0$) and strictly concave ($u'' < 0$), i.e. the decision maker is non-satiated and risk-averse.

By choosing an action z, the decision maker chooses a particular lottery. Therefore, one may speak of 'the lottery z'. One standard method of how to value the riskiness of a lottery to a decision maker is to calculate the *risk premium* $R(z)$ of the lottery z, which is defined by (e.g. Kreps 1990:84)[3]

$$u\left(\mathcal{E}_z[y] - R(z)\right) = \mathcal{E}_z\left[u(y)\right] \ . \tag{2}$$

The risk premium R is the amount of money that leaves the decision maker equally well of, in terms of utility, between the two situations of (i) receiving for sure the expected pay-off from the lottery $\mathcal{E}_z[y]$ minus the risk premium R, and (ii) playing the risky lottery with random pay-off y.[4]

In general, the idea of an *insurance* is that it reduces the (income) risk to which one is exposed. In the extreme, under *full insurance* one does not have any income risk at all. Formally, we conceptualize this notion of insurance by employing the risk premium as a measure of riskiness. The insurance value of the action z can now be defined as follows.

Definition 1

The *insurance value* V_{z_i} of an action z_i (with $i = 1, \ldots, n$) is given by the change of the risk premium $R(z)$ of the lottery z due to a marginal change in the action z_i:

$$V_{z_i}(z) := -\frac{\partial R(z)}{\partial z_i} \ . \tag{3}$$

The minus sign in the defining Equation (3) serves to express the idea that actions which decrease (increase) the risk premium of the lottery have a positive (negative) insurance value. Our notion of insurance deviates from the standard one in the literature in that there is no natural benchmark distribution of income against which the income distribution under insurance is compared. Rather, any marginal change in any action with positive insurance value is understood as an insurance. This is a very general and encompassing notion of risk and insurance. Its strength becomes apparent when analyzing situations of endogenous risk when there is no exogenously given benchmark distribution of income.

The concept of insurance value can be employed to characterize the optimal action.

Proposition 1

Assume that the decision problem can be cast as follows:

$$\max_z U(z) \ , \tag{4}$$

[3] Note that by Equation (2), $\mathcal{E}_z[y] - R(z)$ is the *certainty equivalent* of lottery z, as it yields the expected utility $\mathcal{E}_z\left[u(y)\right]$. If $y \in Y$ with Y as an interval of $\mathbb{R}$, and if u is continuous and strictly increasing, a risk premium R uniquely exists for every lottery z Kreps (1990:84).

[4] In general, if the Bernoulli utility function u characterizes a risk-averse (risk-neutral, risk-loving) decision maker, the risk premium R is positive (zero, negative).

that is, potential constraints can be captured in the objective function. An interior solution $z^\star$ to the decision problem (4), if it exists, is then characterized by

$$\frac{\partial}{\partial z_i}\mathcal{E}_{z^\star}[y] = \frac{\partial}{\partial z_i} R(z^\star) \qquad \textit{for } i = 1, \ldots n\ . \tag{5}$$

Proof: see Appendix A.1.

In words, in the optimum the marginal change due to action z_i in the expected pay-off from the lottery z equals the marginal change in the risk-premium (Equation 5). Equivalently, the marginal change in the expected pay-off and the insurance value of action z_i sum up to zero. This means, in the optimum the insurance value captures all relevant information about the risk and the decision maker's attitude towards risk. It includes the complete information about the income distribution, while the term on the left hand side of Equation (5) only includes the first moment of the income distribution. This result is very general; it does not depend on a specific form of the utility function u or the probability distribution of income.

3 Ecological-economic model

We consider an ecosystem which is managed for some ecosystem service that it provides. Due to stochastic fluctuations in environmental conditions the provision of the ecosystem service is uncertain. Its statistical distribution depends on the state of the ecosystem in terms of biodiversity ('ecosystem quality'), which is influenced by how the system is being managed. As a result, the statistical distribution of ecosystem service and, hence, of income depend on ecosystem management. We capture these relationships in a stylized ecological-economic model as follows.

3.1 Ecosystem management

There are n ecosystem managers, numbered by $i = 1, \ldots, n$. Each ecosystem manager can choose a level x_i of individual effort to improve ecosystem quality. The level of ecosystem quality q_i is specific to user i. It increases with user i's individual effort x_i and the aggregate effort X:

$$\begin{aligned} q_i = q(x_i, X) \quad \text{with} \quad & q_x \geq 0\ , \quad q_{xx} \leq 0\ , \\ & q_X \geq 0\ , \quad q_{XX} \leq 0\ , \quad q_{xX} = q_{Xx} \leq 0\ , \end{aligned} \tag{6}$$

where $X = \sum_{i=1}^{n} x_i$, and subscripts x and X denote partial derivatives with respect to x_i and X respectively. We assume that $q_x > 0$ if $q_X = 0$, and that $q_X > 0$ if

$q_x = 0$ (otherwise results are trivial) and that all individuals face the same type of ecosystem, so that the function $q(\cdot, \cdot)$ has no index i.

Assumption (6) expresses the idea that the level of ecosystem quality relevant to user i is determined by both the individual management action x_i taken by user i and positive externalities from the joint effort X of all ecosystem managers. How the function q_i depends on x_i and X reflects the hierarchical structure of the ecosystem: it captures how the individual effort x_i affects local ecological processes, how the aggregate effort X affects ecological processes at a higher scale, and how these processes interact to determine local ecosystem quality.

In the extreme, $q_x > 0$ and $q_X \equiv 0$ corresponds to a situation where only local ecological processes are relevant and therefore management effort is purely private with no spill-overs to others. The other extreme, $q_x \equiv 0$ and $q_X > 0$, corresponds to a situation where local ecosystem quality is completely determined by higher-scale ecological processes, such that management effort is a pure public good.

Given ecosystem quality q_i, the ecosystem provides user i with the ecosystem service at level s_i which is a random variable that follows a normal distribution. For simplicity we assume that the ecosystem service directly translates into monetary income. Its mean, $\mathcal{E}s_i$, and variance, var s_i, depend on ecosystem quality q_i:

$$\mathcal{E}s_i = \mu(q_i) \quad \text{and} \quad \text{var } s_i = \sigma^2(q_i) \ . \tag{7}$$

Again, since all individuals face the same type of ecosystem, the probability distribution of the ecosystem service is the same for all users who have the same ecosystem quality q. In accordance with ecological evidence (cf. Section 1), the functions μ and σ^2 are assumed to have the following properties:

$$\mu' > 0, \quad \mu'' \leq 0 \qquad \text{and} \qquad {\sigma^2}' < 0, \quad {\sigma^2}'' \geq 0, \tag{8}$$

where the prime denotes a derivative. For each user, the mean level of ecosystem service provision increases, and its variance decreases, with ecosystem quality q. Both effects decrease in magnitude with the level of ecosystem quality.

3.2 Financial insurance

In order to analyze the influence of availability of financial insurance products on the ecosystem managers' choice of activity level x_i, we introduce financial insurance in a simple and stylized way. We assume that manager i has the option of buying financial insurance under the following contract:

- The insurant chooses the fraction $a_i \in [0, 1]$ of insurance coverage.

- He receives (pays)

$$a_i\,(s_i - \mathcal{E}s_i) \tag{9}$$

from (to) the insurance company as an actuarially fair indemnification benefit (insurance premium) if his realized income is below (above) the mean income.[5]

- In addition, he pays the transaction costs of insurance and the insurance company's profit mark-up. The costs of insurance over and above the actuarially fair insurance premium, which are a measure of the 'real' costs of insurance to the insurant,[6] are assumed to follow the cost function

$$\delta\, e(a_i)\,\mathrm{var}\ s_i\ , \tag{10}$$

where $\delta \geq 0$ describes how actuarially unfair is the insurance contract, and

$$e(0) = \ 0\ , \qquad e'(a) > \ 0\ , \qquad e''(a) \geq \ 0\ . \tag{11}$$

This is a highly idealized form of financial insurance, which captures in the most simple way the essence of financial insurance with an actuarially fair insurance premium and some mark-up (e.g., due to transaction costs) on top. The higher the insurance coverage a_i, the lower the risk premium of the resulting income lottery; and the risk premium can be continuously reduced down to zero by increasing a_i to one. This follows the 'Venetian Merchant'-model of insurance: there exists an insurance company (the 'Venetian Merchant') which is ready to (fully or partially) take over the income risk, as measured by variance of income, from the insurant. In order to abstract from any problems related to informational asymmetry, we assume that the normal distribution with mean $\mathcal{E}s_i$ and variance var s_i, as well as the actual level s_i of ecosystem service are observable to both insurant and insurance company.

3.3 Income, preferences and decision

Each ecosystem manager i chooses a vector z_i of actions, which comprises the two components ecosystem management effort x_i and financial insurance coverage a_i,

[5]This benefit/premium-scheme is actuarially fair, because the insurance company has an expected net payment stream of $\mathcal{E}_{z_i}[a_i\,(s_i - \mathcal{E}s_i)] = 0$. This model of insurance is fully equivalent to the traditional model of insurance (e.g. Ehrlich and Becker (1972:627)) where losses compared with the maximum income are insured against and one pays a constant insurance premium irrespective of actual income. In this traditional model, the *net* payment would exactly amount to (9), cf. Appendix A.2.

[6]Since the actuarially fair insurance premium does not cause any expected payoff/costs to the insurant, only the price component over and above the actuarially fair insurance premium constitutes real costs of insurance to the insurant (Ehrlich and Becker 1972626-627).

i.e. $z_i = (x_i, a_i)$. Improving ecosystem quality carries costs, which are purely private and are described by the cost function

$$c(x_i) \quad \text{with} \quad c' > 0, \quad c'' \geq 0\ . \tag{12}$$

Adding up income components, the manager's (random) income y_i is given by

$$y_i = (1 + a_i)\, s_i - c(x_i) - a_i\, \mathcal{E} s_i - \delta\, e(a_i)\, \sigma^2\left(q(x_i, X)\right)\ . \tag{13}$$

Since the ecosystem service s_i is a random variable, net income y_i is a random variable, too. This uncertain part of the income is captured by the first component in Equation (13), while the other components are certain. Obviously, increasing a_i to one allows one to reduce the uncertain income component down to zero. With the normal distribution of ecosystem service s_i, where the mean and variance are given by $\mathcal{E} s_i = \mu\left(q(x_i, X)\right)$ and $\text{var } s_i = \sigma^2\left(q(x_i, X)\right)$ (Equations 7 and 6), the manager's income y_i is normally distributed as well, with mean $\mathcal{E} y_i$ and variance $\text{var } y_i$:

$$\mathcal{E} y_i = \mu(q(x_i, X)) - c(x_i) - \delta\, e(a_i)\, \sigma^2\left(q(x_i, X)\right) \quad \text{and} \tag{14}$$

$$\text{var } y_i = (1 - a_i)^2\, \sigma^2(q(x_i, X))\ . \tag{15}$$

The mean income is given by the mean ecosystem service $\mathcal{E} s_i = \mu(q(x_i, X))$, minus the costs of investment into ecosystem quality $c(x_i)$ and the costs of financial insurance $\delta\, e(a_i)\, \sigma^2(q(x_i, X)$. For an actuarially fair financial insurance contract ($\delta = 0$), the mean income equals mean net income from ecosystem use, $\mathcal{E} s_i - c(x_i)$. The variance of income vanishes for full insurance coverage, $a_i = 1$, and equals the variance of ecosystem service, $\text{var } s_i = \sigma^2(q(x_i, X))$, without any financial insurance coverage, $a_i = 0$.

All ecosystem managers are assumed to have identical preferences over their uncertain income y_i, and to be risk-averse.[7] In order to obtain simple closed-form solutions, we assume that manager i's preferences are given by the constant absolute risk aversion Bernoulli utility function

$$u(y_i) = -e^{-\rho\, y_i}, \tag{16}$$

where $\rho > 0$ is a parameter describing the manager's Arrow-Pratt measure of absolute risk aversion (Arrow 1965, Pratt 1964). Since income follows a normal distribution with mean $\mathcal{E} y_i$ and variance $\text{var } y_i$, the von Neumann-Morgenstern expected

[7]While risk-aversion is a natural and standard assumption for farm *households* (Besley 1995, Dasgupta 1993: Chapter 8), it appears as an induced property in the behavior of (farm) *companies* which are fundamentally risk neutral but act as if they were risk averse when facing e.g. external financing constraints or bankruptcy costs (Caillaud et al. 2000, Mayers and Jr. 1990).

utility function (1) is (see Appendix A.3):

$$U_i = \mathcal{E} y_i - \frac{\rho}{2} \operatorname{var} y_i \ . \tag{17}$$

With this utility function, the risk premium R of a lottery with mean pay-off $\mathcal{E} y_i$ and variance var y_i is simply given by (see Appendix A.4):

$$R_i = \frac{\rho}{2} \operatorname{var} y_i = \mathcal{E} y_i = \frac{\rho}{2} (1 - a_i)^2 \, \sigma^2(q(x_i, X)) \ . \tag{18}$$

Since each ecosystem manager can perform two actions, effort x_i to improve ecosystem quality, and purchasing financial insurance a_i, there are two insurance values, which will play a role in the subsequent analysis:

$$V_{x_i} = -\frac{\partial R_i}{\partial x_i} \quad \text{the insurance value of improving ecosystem quality, and} \tag{19}$$

$$V_{a_i} = -\frac{\partial R_i}{\partial a_i} \quad \text{the insurance value of buying financial insurance.} \tag{20}$$

4 Analysis and results

The analysis proceeds in three steps: First, we discuss the laissez-faire equilibrium, which arises if the n different ecosystem managers optimize their management effort taking the actions of the other managers as given (Section 4.1). Second, we derive the (symmetric) Pareto-efficient allocation (Section 4.2). Finally, we investigate the extent, in welfare terms, of the market failure, and analyze policy measures to internalize the externalities (Section 4.3).

4.1 Laissez-faire equilibrium

As laissez-faire equilibrium, we consider the allocation which results as Nash-equilibrium without regulating intervention. Each ecosystem manager's decision problem is to maximize his expected utility, taking the actions of all other ecosystem managers as given. Formally, manager i's decision problem is

$$\max_{x_i, a_i} \mu(q(x_i, X)) - c(x_i) - \delta \, e(a_i) \, \sigma^2(q(x_i, X)) - \frac{\rho}{2} (1 - a_i)^2 \, \sigma^2(q(x_i, X)) \ , \tag{21}$$

where $X = x_1 + \ldots + x_n$ and all x_j for $j \neq i$ are treated as given. We assume (throughout the remainder of this paper) that an interior solution exists.[8]

[8]Ecosystem properties (6) and (8) and the cost function (12) do not exclude corner solutions. For instance, for very high marginal costs and low marginal benefits of management effort, the Nash equilibrium may be not to make any effort at all. On the other hand, for low marginal costs, the equilibrium could be to make the maximum possible effort, because ecosystem quality has the double benefit of increasing the mean and reducing the variance of ecosystem service provision.

Given the risk-premium R_i of each ecosystem manager, Equation (18), the insurance values of investment into ecosystem quality, $V^\star_{x_i}(x_i, a_i)$, and the fraction a_i of insured income, $V^\star_{a_i}(x_i, a_i)$ are, according to Definition 1,[9]

$$
\begin{aligned}
V^\star_{x_i}(x_i, a_i) &= -\frac{\rho}{2}\,(1-a_i)^2\,\sigma^{2\prime}(q(x_i, X))\;[q_{x_i}(x_i, X) + q_X(x_i, X)] \qquad (22)\\
V^\star_{a_i}(x_i, a_i) &= (1-a_i)\,\rho\,\sigma^2(x_i, q(X))\;. \qquad (23)
\end{aligned}
$$

Obviously, both insurance values, that of ecosystem management effort, as well as the insurance value of financial insurance, are strictly increasing in the degree of risk-aversion ρ.

The insurance value $V^\star_{x_i}$ of ecosystem management effort is a strictly decreasing function of x. (This follows from the assumptions made in Equations (6) and (8).) It is increasing with the elasticity of the variance of the ecosystem service with respect to improved ecosystem quality, $-\frac{q\,\sigma^{2\prime}(q(x,X))}{\sigma^2(q(x,X))}$, with the elasticity of ecosystem quality with respect to individual management effort x, $\frac{x\,q_x(x,X)}{q(x,X)}$, and with the elasticity of ecosystem quality with respect to the aggregate level of management effort X, $\frac{x\,q_x(x,X)}{q(x,X)}$. (This is easily seen by rearranging Equation (22).) This means that the insurance value of ecosystem conservation is higher, the more susceptible the variance of ecosystem services reacts on the change in management effort.

Furthermore, the insurance value of financial insurance increases with the standard deviation σ^2 of ecosystem services, as expected, since higher variance of ecosystem services increases the need for hedging that risk. The mutual dependence of the two insurance values, (22) and (23) is further characterized by the following proposition.

Proposition 2

1. *In the case of full insurance the insurance value $V^\star_{x_i}(x_i, a_i)$ of a marginal change in x_i is zero, i.e. $V^\star_{x_i}(x_i, 1) = 0$.*

2. *For a risk averse ecosystem manager, natural insurance by conservative ecosystem management ($dx_i > 0$) and financial insurance ($da_i > 0$) are substitutes:*

$$
\frac{\partial a_i}{\partial V^\star_{x_i}(x_i, a_i)} = \frac{\partial x_i}{\partial V^\star_{a_i}(x_i, a_i)} < 0\;. \qquad (24)
$$

Proof: see Appendix A.5.

How the equilibrium level of ecosystem management effort depends on the managers' risk aversion and on the transaction costs of financial insurance mainly depends on the properties of the insurance value.

[9]Throughout the paper, we denote the equilibrium values with a subscript $\cdot^\star$, and the values in the social optimum with a hat $(\hat{\cdot})$.

Proposition 3

The laissez-faire equilibrium is unique, and all ecosystem managers choose the same level $x^\star$ of ecosystem management and the same fraction $a^\star$ of financial insurance coverage.

1. *The equilibrium levels $x^\star$ of ecosystem management effort and $q^\star$ of ecosystem quality increase, and the equilibrium level $a^\star$ of financial insurance coverage decreases with the transaction costs of financial insurance:*

$$\frac{dx^\star}{d\delta} > 0, \quad \frac{dq^\star}{d\delta} > 0, \quad \textit{and} \quad \frac{da^\star}{d\delta} < 0 \ . \tag{25}$$

2. *The equilibrium levels $x^\star$ of ecosystem management effort, $q^\star$ of ecosystem quality and $a^\star$ of financial insurance coverage increase with the ecosystem managers' degree ρ of risk aversion:*

$$\frac{dx^\star}{d\rho} > 0, \quad \frac{dq^\star}{d\rho} > 0, \quad \textit{and} \quad \frac{da^\star}{d\rho} > 0 \ . \tag{26}$$

Proof: see Appendix A.6.

The intuition behind the result is as follows. In the absence of transaction costs, i.e. $\delta = 0$, the representative ecosystem manager will choose full insurance, i.e. $a^\star = 1$. If transaction costs are present, i.e. if $\delta > 0$, he chooses partial coverage by financial insurance ($0 < a^\star < 1$) and if transaction costs are prohibitively high, i.e. if $\delta \to \infty$, he will choose no coverage by financial insurance ($a^\star = 0$). This affects the insurance value of ecosystem management effort: the higher these transaction costs are, the higher is the (insurance) value of ecosystem management effort. As a consequence, the equilibrium values of ecosystem management effort and of ecosystem quality are influenced by the transaction costs of financial insurance: the higher the transaction costs of financial insurance are, the higher are ecosystem management effort and ecosystem quality in equilibrium. This result corresponds to Part 2 of Proposition 3; investment into ecosystem quality and financial insurance are substitutes.

The comparative statics of the equilibrium allocation with respect to the degree of risk-aversion ρ shows that an increase in risk-aversion leads to both more conservation and a higher fraction of financial insurance coverage. Both effects are due to the increase in the insurance values: a higher risk-aversion leads to a higher insurance value and, thus, to higher marginal utility of the two measures which reduce income variability. As a consequence, ecosystem quality increases with ρ, too. This, however, is by itself not sufficient to claim that an increase in risk-aversion decreases the market failure due to the public good-externality. The answer to that question will be given in Proposition 6 below.

4.2 Efficient allocation

The next step is to derive the efficient allocation. Since we are interested in comparing the efficient allocation to the laissez-faire equilibrium, we will concentrate on the symmetric Pareto-optimum in which all ecosystem managers make the same effort. To derive this allocation we define social welfare as the sum of the utilities of all n ecosystem managers:

$$W = \sum_{i=1}^{n} \left[\mathcal{E} y_i - \frac{\rho}{2} \operatorname{var} y_i \right] . \tag{27}$$

The efficient allocation is derived by choosing the individual levels of management effort, such as to maximize social welfare (27) subject to Constraints (14) and (15),

$$\max_{x_1,\ldots,x_n;a_1,\ldots,a_n} \sum_{i=1}^{n} \mu\big(q\big(x_i,X\big)\big) - c(x_i) - \delta\, e(a_i)\, \sigma^2\big(q\big(x_i,X\big)\big) - \frac{\rho}{2}\,(1-a_i)^2\, \sigma^2\big(q\big(x_i,X\big)\big) . \tag{28}$$

Given this welfare function, we again can determine two types of insurance values: a representative ecosystem manager's insurance values of ecosystem conservation and of the financial insurance coverage. These are, in the symmetric case $x_i = x$ and $a_i = a$ for all i,

$$\hat{V}_x(x,a) = -\frac{\rho}{2}\,(1-a)^2\, \sigma^{2\prime}(q(x,n\,x))\ [q_x(x,n\,x) + n\, q_X(n,n\,x)] \tag{29}$$

$$\hat{V}_a(x,a) = (1-a)\,\rho\,\sigma^2(q(x,X)) . \tag{30}$$

The insurance value of financial insurance coverage is the same in the individual managers' decision and for determining the efficient allocation, reflecting the fact that the market for financial insurance is undistorted. By contrast, the insurance value of ecosystem management is, which is relevant for determining the socially efficient allocation, takes into account the additional insurance value that each manager's effort has for the $n-1$ other ecosystem users. This is captured by the sum over all ecosystem managers' benefit from a reduced variance of ecosystem service provision. Hence, the qualitative comparative-static results on the insurance values, which hold for the representative ecosystem manager, carry over to the social optimum. In particular, the insurance values of ecosystem conservation and of financial insurance coverage are substitutes.

Proposition 4

The social optimum is unique, and all ecosystem managers make the same management effort $\hat{x}$ and have the same fraction $\hat{a}$ of financial insurance coverage.

1. *The efficient levels $x^\star$ of ecosystem management effort and $q^\star$ of ecosystem quality increase, and the efficient level $a^\star$ of financial insurance coverage decreases with the transaction costs of financial insurance:*

$$\frac{dx^\star}{d\delta} > 0, \quad \frac{dq^\star}{d\delta} > 0, \quad \textit{and} \quad \frac{da^\star}{d\delta} < 0 \ . \tag{31}$$

2. *The efficient levels $x^\star$ of ecosystem management effort, $q^\star$ of ecosystem quality and $a^\star$ of financial insurance coverage increase with the ecosystem managers' degree ρ of risk aversion:*

$$\frac{dx^\star}{d\rho} > 0, \quad \frac{dq^\star}{d\rho} > 0, \quad \textit{and} \quad \frac{da^\star}{d\rho} > 0 \ . \tag{32}$$

Proof: see Appendix A.7

The intuition behind the result is as follows. The difference between the efficient and the equilibrium allocation is that in the efficient allocation, the positive externality, which each ecosystem manager's effort has on the other ecosystem managers due to reduced variance of ecosystem service provision is fully internalized. This changes the effect that an increase in the transaction costs of financial insurance, or an increase in the managers' degree of risk aversion has on the management effort and financial insurance coverage in magnitude, but not in sign. Hence, the same arguments hold, which support Proposition 3.

4.3 Welfare effects of improved access to financial insurance

Due to the external effects of individual ecosystem management effort, the laissez-faire equilibrium is not efficient. In equilibrium, ecosystem managers will spend too little effort to improve ecosystem quality, because they do not take into consideration the positive externality on other ecosystem users. In order to implement the efficient allocation as an equilibrium, a regulator could impose a Pigouvian subsidy on individual management effort. Denoting the subsidy per unit x_i with τ, the optimization problem of ecosystem manager i then reads

$$\max_{x_i,\, a_i} \mu\left(q(x_i, X)\right) - c(x_i) - \delta\, e(a_i)\, \sigma^2\left(q(x_i, X)\right) - \frac{\rho}{2}\left(1 - a_i\right)^2 \sigma^2\left(q(x_i, X)\right) + \tau\, x_i \ . \tag{33}$$

Comparing the first order conditions for the efficient allocation (i.e. the first order condition of maximizing (28) with respect to x_i) and for the regulated equilibrium (i.e. the first order condition of maximizing (33) with respect to x_i), we obtain the optimal subsidy $\hat{\tau}$.

Lemma 1

The efficient allocation is implemented as an equilibrium, if a subsidy $\hat{\tau}$ on individual ecosystem management effort is set with

$$\hat{\tau} = (n-1)\, q_X(\hat{x}, n\,\hat{x}) \left[\mu'(q(\hat{x}, n\,\hat{x})) - \left[\frac{\rho\,(1-\hat{a})^2}{2} + \delta\, e(\hat{a})\right] \sigma^{2\prime}(q(\hat{x}, n\,\hat{x}))\right] \,. \quad (34)$$

Clearly, the optimal subsidy increases with $q'(n\,\hat{x})$, i.e. it is higher, the higher the ecological benefits of ecosystem conservation are. There are three different contributions to the optimal subsidy rate, which are captured by the terms in brackets. In the case of risk-neutrality, $\rho = 0$, we also have $\hat{a} = 0$, and only the first term in brackets remains. Then, the optimal subsidy is $(n-1)\, q'\, \mu_q$, it just internalizes the positive externality that the increase in ecosystem quality has on the expected payoff of the $n-1$ other ecosystem managers.

If $\rho > 0$ and $\delta > 0$, i.e. $\hat{a} < 1$, the second term in brackets captures the positive externality of ecosystem manager $i's$ contribution to ecosystem quality, which is due to the insurance value that the higher ecosystem quality has for the $n-1$ other ecosystem managers. The last term in brackets contains the (pecuniary) externality that the individual ecosystem manager's conservation effort has on the insurance costs of the other ecosystem managers: due to the decrease in variance, the markup on the insurance premiums decrease.

A case of particular interest is the case of actuarially fair ($\delta = 0$) financial insurance. In that case, the last term in brackets of Equation (34) drops out. Also, the term in the middle drops out, since it is optimal to choose full coverage of financial insurance, $\hat{a} = 1$. As a consequence, the optimal subsidy is the same as in the case of risk-neutrality. In other words, the representative ecosystem managers' attitudes towards risk are irrelevant for the market distortion, $d\hat{\tau}/d\rho = 0$.

The optimal subsidy $\hat{\tau}$ can be interpreted as the extent of the regulation necessary in order to solve the public good problem. It has become clear from the discussion so far that the public good problem depends on the degree of uncertainty and of the ecosystem managers' risk-aversion. The question is whether more or less regulation is required if the costs of financial insurance, measured by the parameter δ increase.

Proposition 5

The optimal subsidy decreases / is unchanged / increases with the costs δ per unit of financial insurance, i.e.

$$\frac{d\hat{\tau}}{d\delta} \lesseqqgtr 0 \,, \quad (35)$$

if

$$-\frac{\hat{x}\,\phi'(\hat{x})}{\phi(\hat{x})}\;\frac{q_x(\hat{x},n\,\hat{x})}{n\,q_X(\hat{x},n\,\hat{x})}\;\gtreqless\;\frac{\hat{x}\,c''(\hat{x})}{c'(\hat{x})}\;, \tag{36}$$

where

$$\phi(x)\equiv\frac{q_x(x,n\,x)}{q_x(x,n\,x)+n\,q_X(x,n\,x)}\;. \tag{37}$$

Proof: see Appendix A.8.

As stated in Proposition 3, increased costs of financial insurance have an unambiguously positive effect on the individual level of management effort to improve ecosystem quality. Nevertheless, the effect on the optimal regulation can go either way, depending on the characteristics of the ecosystem under consideration, in particular the 'technology' and the costs of ecosystem management as specified by Condition (36). On the left hand side of Condition (36), the expression $\phi(x)$ is the share of marginal ecosystem quality improvement on the individual scale out of total marginal ecosystem quality improvement including the positive externalities on the aggregate scale. It is, in short, the individual share of marginal quality improvement. With this, the first factor on the left hand side of Condition (36) is the elasticity of the individual share of marginal quality improvement with respect to management effort. The second factor simply is the marginal rate of substitution between management efforts on the individual and on the aggregate scale. On the right hand side of Condition (36) is the elasticity of marginal costs. Hence, the Pigouvian subsidy decreases / is unchanged / increases with the costs of financial insurance, if the elasticity of the individual share of marginal quality improvement times the marginal rate of substitution between effort on the individual and aggregate scale is greater than / equal to / less than the elasticity of marginal costs. In particular, in the case of constant marginal costs, the Pigouvian subsidy decreases with the costs of financial insurance, or, equivalently, it increases with improved access to financial insurance, if and only if the elasticity of the individual share of marginal quality improvement is positive. This, however, does not need to be the case. Overall, whether the Pigouvian subsidy increases or decreases with improved access to financial insurance depends on how ecosystem processes operating at different scales influence ecosystem quality; it does not depend on how exactly ecosystem service provision is influenced by ecosystem quality.

Although the Pigouvian subsidy is an appropriate measure of the extent of regulation necessary to reach the efficient allocation in a decentralized economy, a different measure is required in order to determine how welfare and how the welfare loss due to the external effects is affected by the costs of financial insurance. The welfare loss is the difference in welfare between the efficient allocation and the laissez-faire allocation. Employing the welfare function (27), it is given by

$$\hat{W} - W^\star = n \left[\mu\left(q\left(\hat{x}, n\,\hat{x}\right)\right) - \left[\delta\, e(\hat{a}) + \frac{\rho}{2}\left(1-\hat{a}\right)^2\right] \sigma^2\left(q\left(\hat{x}, n\,\hat{x}\right)\right) - c(\hat{x})\right]$$
$$- n \left[\mu\left(q\left(x^\star, n\,x^\star\right)\right) - \left[\delta\, e(a^\star) + \frac{\rho}{2}\left(1-a^\star\right)^2\right] \sigma^2\left(q\left(x^\star, n\,x^\star\right)\right) - c(x^\star)\right] > 0\ . \quad (38)$$

Proposition 6
Define

$$\tau^\star = (n-1)\, q_X\left(x^\star, n\,x^\star\right) \left[\mu'\left(q\left(x^\star, n\,x^\star\right)\right) - \left[\delta\, e(a^\star) + \frac{\rho}{2}\left(1-a^\star\right)^2\right] \sigma^{2\prime}\left(q\left(x^\star, n\,x^\star\right)\right)\right] > 0\ . \quad (39)$$

1. *Welfare in the laissez-faire equilibrium decreases / is unchanged / increases with increasing costs of financial insurance, i.e.*

$$\frac{d}{d\delta}\, W^\star \lesseqqgtr 0 \quad (40)$$

if

$$\tau^\star\, \frac{dx^\star}{d\delta} \lesseqqgtr e(\hat{a})\, \sigma^2\left(q\left(x^\star, n\,x^\star\right)\right)\ . \quad (41)$$

2. *The welfare loss due to free-riding decreases / is unchanged / increases with increasing costs of financial insurance, i.e.*

$$\frac{d}{d\delta}\left(\hat{W} - W^\star\right) \lesseqqgtr 0 \quad (42)$$

if

$$e(\hat{a})\, \left[\sigma^2\left(q\left(x^\star, n\,x^\star\right)\right) - \sigma^2\left(q\left(\hat{x}, n\,\hat{x}\right)\right)\right] - \tau^\star\, \frac{dx^\star}{d\delta} \lesseqqgtr 0\ . \quad (43)$$

Proof: see Appendix A.9.

The intuition behind the result is as follows. In the optimum, welfare obviously decreases, if the costs of financial insurance increase. Or, the other way around, an improved access to financial insurance is unambiguously beneficial. This is different in the laissez-faire equilibrium without environmental policy. Whether welfare decreases or increases with increasing costs of financial insurance depends on the relative magnitude of two effects: on the one hand, as a direct effect, the expenditures for financial insurance increase, which decreases welfare. On the other hand, the individual effort to improve ecosystem quality increases (Proposition 3). This improves welfare by an amount equal to the positive externality $\tau^\star$, which the individual effort of ecosystem management has on the other ecosystem managers.

Whether an improved access to financial insurance is beneficial from a welfare point of view or not depends on the balance between these two effects.

In a similar way the balance of two effects determines how the welfare loss due to free-riding is affected by the costs of financial insurance. On the one hand, the difference between the expenditures for financial insurance in the laissez-faire equilibrium and in the optimum is spread with increasing costs per unit of financial insurance. This effect worsens the market failure. On the other hand, there is the effect that higher costs of financial insurance lead to increased individual management effort in equilibrium, which tends to decrease the extent of market failure. Again, the positive externality $\tau^\star$ determines how valuable it is, in welfare terms, that individual ecosystem management effort increases with the costs of financial insurance.

The net effect of increased costs of financial insurance on welfare in the laissez faire equilibrium and on the welfare loss due to the externality is ambiguous. In order to determine more clearly under which conditions the welfare loss decreases or increases, we consider an example which is simple enough to enable a closed-form solution. Let

$$q(x, X) = x^{1-\gamma}\, X^{\gamma} \quad \text{with} \quad 0 < \gamma < 1 \ . \tag{44}$$

This specification is analytically very convenient, and it corresponds to the well established species-area-relationship at each of the two hierarchical levels, i.e. q is a power function of both x and X. We further assume constant marginal costs of management effort, $c(x) = c \cdot x$, which corresponds to a constant price of land set aside for biodiversity protection. In order to focus on the insurance effect we disregard that improved ecosystem quality increases the mean level of ecosystem services, i.e. $\mu(q) = \mu =$ constant. Finally, the variance of ecosystem services depends on ecosystem quality as follows[10]

$$\sigma^2(q) = (\delta - \eta\, q)^{\frac{1}{\eta}} \quad \text{with} \quad \eta < 1 \quad \text{and} \quad \delta > 0 \ . \tag{45}$$

This specification includes (for different η) large variety of functions satisfying Conditions (8). For $\eta > 0$, it is possible to obtain the ecosystem service at zero variance, provided ecosystem quality is high enough. This is not possible for $\eta < 0$. Whether the welfare losses due to the public good problem decreases, is unchanged, or increases with the costs of financial insurance depends on the type of ecosystem, as specified by the parameter η.

[10]For $\eta > 0$, we define $\sigma^2(q)$ by (45) for all $q \leq \delta/\eta$ and by $\sigma^2(q) \equiv 0$ for all $q > \delta/\eta$. In the case $\eta = 0$, the specification (45) becomes $\sigma^2(q) = \exp(-q/\delta)$.

Proposition 7

Given the specifications of the example, with increasing costs δ per unit of financial insurance,

1. *welfare in the laissez-faire equilibrium decreases / is unchanged / increases if*

$$\gamma \frac{n-1}{n} \lesseqqgtr 1-\eta \ . \tag{46}$$

2. *the welfare loss due to free-riding decreases / is not affected / increases if*

$$\eta \gtreqqless 0 \ . \tag{47}$$

Proof: see Appendix A.10.

The intuition behind the result is as follows. The case $\eta < 0$ corresponds to an ecosystem which is poorly manageable: an increase of ecosystem quality reduces the uncertainty of ecosystem service provision only weakly and cannot remove the uncertainty completely. In this case the effect that the difference in total expenditures between the efficient allocation and the laissez-faire equilibrium increases with the costs of financial insurance outweighs the welfare-increasing effect of increased individual management effort. Hence, for $\eta < 0$ (and given the other specifications of the example), increasing costs of financial insurance increase the welfare loss due to free-riding. In the case $\eta = 0$, the costs of financial insurance play no role for the extent of welfare loss.

By contrast, an ecosystem which is characterized by $\eta > 0$ is effectively manageable: an increase of ecosystem quality strongly reduces the uncertainty of ecosystem service provision, and, eventually, uncertainty is completely removed. For such an ecosystem, the welfare loss decreases with increasing costs of financial insurance, i.e. an improved access to financial insurance increases the need for environmental regulation.

However, even if $\eta > 0$ increasing costs of financial insurance not necessarily increase welfare: in the efficient allocation, welfare is unambiguously reduced; in the laissez-faire equilibrium, welfare can, in principle, increase with costs of financial insurance. This is the case if not only $\eta > 0$, but also $\gamma \frac{n-1}{n} > 1-\eta$ (cf. Condition 46). Hence, if improved ecosystem quality reduces the uncertainty of ecosystem service provision very effectively, and, moreover, the aggregate effort's contribution to improved ecosystem quality is very important, i.e. that the positive externality of individual effort is high, welfare actually increases if the costs of financial insurance increase. For such a type of ecosystem, an improved access to financial insurance has an adverse effect on welfare in the laissez-faire.

5 Conclusions

We have analyzed how risk-averse ecosystem users manage an ecosystem for its services. The ecosystem model captures three stylized facts, as identified in the ecological literature: (i) the mean level of ecosystem services increases with biodiversity; (ii) the variance of ecosystem services decreases with biodiversity; (iii) biodiversity is influenced by ecosystem processes operating at different hierarchical scales. We have considered two such scales: individual management action affects processes at the lower scale, while aggregate action affects processes at the higher scale. Thus, an individual management action has not only a private benefit, but also a positive external effect on other ecosystem users.

We have demonstrated that conservative biodiversity management has an insurance value, which depends on the ecosystem managers' risk-aversion and on ecosystem properties. Because ecosystem managers, when choosing a management action under uncertainty, take into account the ecosystem's insurance value, the level of individual effort to improve ecosystem quality increases with increasing uncertainty and risk-aversion. As a consequence, higher higher costs of financial insurance and higher risk-aversion lead to a higher level of biodiversity. Thus, under uncertainty and without the possibility of financial insurance the ecosystem management is more conservative, and the resulting level of biodiversity is higher, than it would be in a world of certainty or if an actuarially fair financial insurance would be available.

Due to the external effect of individual management effort, the laissez-faire equilibrium is not efficient. In order to study how the public good-problem is affected by uncertainty, we have analyzed how (i) the extent of regulation necessary to implement the efficient allocation and (ii) the welfare loss due to free-riding depend on the availability of financial insurance.

How the Pigouvian subsidy, as a measure of the extent of efficient regulation, is affected by financial insurance depends on how management effort at the individual scale and at the aggregate scale contribute to ecosystem quality. For constant marginal costs of management effort, the Pigouvian subsidy decreases with higher costs per unit of financial insurance if the elasticity of the individual share of marginal quality improvement is positive, and increases otherwise. Hence, the extent of the regulatory intervention necessary to implement the efficient allocation depends on the hierarchical structure of how ecosystem management affects biodiversity, but not on how exactly biodiversity influences the provision of ecosystem services. In contrast, the latter crucially determines whether the welfare loss due to free-riding decreases or increases with improved access to financial insurance: If biodiversity reduces the variance of ecosystem services very strongly, the welfare loss increases, if financial insurance becomes more easily accessible. If additionally the external effect of individual management actions is large, welfare in the laissez-faire equilibrium decreases, if financial insurance is introduced. If, on the other hand, biodiversity hardly reduces the variance of ecosystem services, the welfare loss decreases, if the costs of financial insurance decline. In other words, for poorly manageable ecosystems, the free-rider problem decreases when financial insurance becomes available.

These results highlight that the ecosystem properties determine how efficient environmental policy and welfare change if financial insurance becomes available for ecosystem users and that it is not possible, in the absence of environmental regulation, to judge whether improved access to financial insurance is desirable or not in welfare terms.

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A Appendix

A.1 Proof of Proposition 1

The first order conditions for problem (4) read (with $i = 1, \ldots n$)

$$\frac{\partial}{\partial z_i} U(z^\star) = \frac{\partial}{\partial z_i} \mathcal{E}_{z^\star}[u(y)] = 0 \ . \tag{A.1}$$

Employing the definition of the risk premium, Equation (2), one obtains

$$\begin{aligned} \frac{\partial}{\partial z_i} \mathcal{E}_{z^\star}[u(y)] &= \frac{\partial}{\partial z_i} u\left(\mathcal{E}_{z^\star}[y] - R(z^\star)\right) \\ &= u'\left(\mathcal{E}_{z^\star}[y] - R(z^\star)\right) \cdot \left(\frac{\partial}{\partial z_i} \mathcal{E}_{z^\star}[y] - \frac{\partial}{\partial z_i} R(z^\star)\right) \end{aligned} \tag{A.2}$$

As $u' > 0$, this implies, with Equation (3),

$$\frac{\partial}{\partial z_i} \mathcal{E}_{z^\star}[y] - \frac{\partial}{\partial z_i} R(z^\star) = 0 \tag{A.3}$$

$$V_{z_i}(z^\star) = -\frac{\partial}{\partial z_i} \mathcal{E}_{z^\star}[y] \ . \tag{A.4}$$

A.2 Derivation of the fair insurance premium

Define the probability density function

$$\tilde{f}(y) = \left[\int_0^{\tilde{y}} f(\nu)\, d\nu \right]^{-1} \begin{cases} f(y) & \text{if} \quad y \leq \tilde{y} \\ 0 & \text{if} \quad y > \tilde{y} \end{cases} , \tag{A.5}$$

where

$$f(y) = \frac{1}{\sqrt{2\pi \text{var } y}}\, e^{-\frac{(y-\mathcal{E}y)^2}{2\text{var } y}} . \tag{A.6}$$

Clearly, $\lim_{\tilde{y}\to\infty} \tilde{f}(y) = f(y)$. Consider the insurance contract under the probability density function (A.5) that losses compared to $\tilde{y}$ are insured against, i.e. if $y_i < \tilde{y}$, the insurance company pays an amount $a_i\,(\tilde{y} - y_i)$ of money to the insurant. The contract considered in this paper is the particular case $\tilde{y} \to \infty$. The fair premium π_i, i.e. the premium, which equals the expected payoff, of such a contract is

$$\pi_i = \mathcal{E}_{z_i}\, \left[a_i\,(\tilde{y} - y_i)\right] . \tag{A.7}$$

This premium has to be paid in any event. If the actual income y_i is below $\tilde{y}$, however, the insurant additionally receives the indemnification benefit. The net payment to (or from) the insurance company amounts to

$$\mathcal{E}_{z_i}\, \left[a_i\,(\tilde{y} - y_i)\right] - a_i\,(\tilde{y} - y_i) = -\mathcal{E}_{z_i}\, \left[a_i\, y_i\right] + a_i\, y_i = a_i\,(y_i - \mathcal{E}y_i) . \tag{A.8}$$

This expression does not depend on $\tilde{y}$, but only on the mean $\mathcal{E}y_i$ of the probability distribution of incomes. In particular, we obtain the same expression in the limit $\tilde{y} \to \infty$.

A.3 Expected utility function (17)

With $f(x)$, as defined by Equation (A.6), being the probability density function of the normal distribution of income y with mean $\mathcal{E}y$ and variance var y, the von Neumann-Morgenstern expected utility from the Bernoulli utility function (16) is

$$\tilde{U} = \mathcal{E}[u(y)] = -\int e^{-\rho\, y} f(y) dy = -e^{-\rho\left[\mathcal{E}y - \frac{\rho}{2}\text{ var } y\right]} . \tag{A.9}$$

Using a simple monotonic transformation of $\tilde{U}$, one obtains the expected utility function U (Equation 17).

A.4 Derivation of the risk premium (18)

The risk premium R has been defined in Equation (2) as

$$u\left(\mathcal{E}[y] - R\right) = \mathcal{E}\left[u(y)\right] . \tag{A.10}$$

With the Bernoulli utility function (16) the left hand side of this equation is given by

$$u\left(\mathcal{E}[y]-R\right)=-e^{-\rho\,[\mathcal{E}y-R]}\ , \tag{A.11}$$

and the right hand side is given by Equation (A.9). Hence, we have

$$-e^{-\rho\,[\mathcal{E}y-R]}=-e^{-\rho\,\left[\mathcal{E}y-\frac{\rho}{2}\,\mathrm{var}\;y\right]}\ . \tag{A.12}$$

Rearranging yields the result stated in Equation (18).

A.5 Proof of Proposition 2

Ad 1. This is confirmed by plugging $a_i = 1$ into Equation (22).
Ad 2. We show that

$$\frac{\partial V^\star_{x_i}(x_i,a_i)}{\partial a_i}=\frac{\partial V^\star_{a_i}(x_i,a_i)}{\partial x_i}<0\ . \tag{A.13}$$

Differentiating the private insurance value of ecosystem management (Equation 22) with respect to a_i yields

$$\frac{\partial V^\star_{x_i}(x_i,a_i)}{\partial a_i}=\rho\,(1-a_i)\,\sigma^{2\prime}(q(x_i,X))\ [q_{x_i}(x_i,X)+q_X(x_i,X)] \tag{A.14}$$

$$\frac{\partial V^\star_{a_i}(x_i,a_i)}{\partial x_i}=(1-a_i)\,\rho\,\sigma^{2\prime}(q(x_i,X))\ [q_{x_i}(x_i,X)+q_X(x_i,X)]\ . \tag{A.15}$$

Both expressions on the right hand sides are equal and negative by Assumption (8).

A.6 Proof of Proposition 3

The objective function (17) with the Constraints (14) and (15) plugged in is strictly concave, which is assured by Conditions (8) and (11).

According to Proposition 1, the individual ecosystem manager's optimal decision is characterized by the conditions (omitting the arguments of the functions)

$$\mu'\,[q_x+q_X]-c'-\delta\,e(a^\star)\,\sigma^{2\prime}\,[q_x+q_X]+V^\star_{x_i}=0 \tag{A.16}$$

$$-\delta\,e'\,\sigma^2+V^\star_{a_i}=0 \tag{A.17}$$

Ad 1. Differentiating these conditions implicitly with respect to δ, we obtain

$$\frac{dx^\star}{d\delta}=\frac{e(a^\star)\,\sigma^{2\prime}\,[q_x+q_X]}{A^\star}>0 \tag{A.18}$$

$$\frac{da^\star}{d\delta}=-\ \frac{e'}{\delta\,e''+\rho}<0\ , \tag{A.19}$$

where

$$\begin{aligned}A^\star=&\left[\mu''-\left[\delta\,e(a^\star)+\frac{\rho\,(1-a^\star)^2}{2}\right]\sigma^{2\prime\prime}\right][q_x+q_X]\,[q_x+n\,q_X]\\&+\left[\mu'-\left[\delta\,e(a^\star)+\frac{\rho\,(1-a^\star)^2}{2}\right]\sigma^{2\prime}\right][q_{xx}+(n+1)\,q_{xX}+n\,q_{XX}]-c''<0\ .\end{aligned} \tag{A.20}$$

Ad 2. Differentiating Conditions (A.16) and (A.17) implicitly with respect to ρ, we obtain

$$\frac{dx^\star}{d\rho} = -\frac{\frac{1}{2}(1-a^\star)^2\,\sigma^{2\prime}\,[q_x + q_X]}{A^\star} > 0 \tag{A.21}$$

$$\frac{da^\star}{d\rho} = \frac{1-a^\star}{\delta\, e'' + \rho} > 0\ , \tag{A.22}$$

where $A^\star$ is defined as above.

A.7 Proof of Proposition 4

The objective function given in (28) is strictly concave, which is assured by Conditions (8) and (11).

According to Proposition 1, the solution to the optimization problem (28) is characterized by the conditions (omitting the arguments of the functions)

$$\mu'\,[q_x + n\,q_X] - c' - \delta\, e(a^\star)\,\sigma^{2\prime}\,[q_x + n\,q_X] + \hat{V}_{x_i} = 0 \tag{A.23}$$

$$-\delta\, e'\,\sigma^2 + \hat{V}_{a_i} = 0 \tag{A.24}$$

Ad 1. Differentiating these conditions implicitly with respect to δ, we obtain

$$\frac{d\hat{x}}{d\delta} = \frac{e(\hat{a})\,\sigma^{2\prime}\,[q_x + n\,q_X]}{\hat{A}} > 0 \tag{A.25}$$

$$\frac{d\hat{a}}{d\delta} = -\frac{e'}{\delta\, e'' + \rho} < 0\ , \tag{A.26}$$

where

$$\begin{aligned}\hat{A} = \left[\mu'' - \left[\delta\, e(\hat{a}) + \frac{\rho\,(1-\hat{a})^2}{2}\right]\sigma^{2\prime\prime}\right]\,[q_x + n\,q_X]^2 \\ + \left[\mu' - \left[\delta\, e(\hat{a}) + \frac{\rho\,(1-\hat{a})^2}{2}\right]\sigma^{2\prime}\right]\,[q_{xx} + 2\,n\,q_{xX} + n^2\,q_{XX}] - c'' < 0\ .\end{aligned} \tag{A.27}$$

Ad 2. Differentiating Conditions (A.23) and (A.24) implicitly with respect to ρ, we obtain

$$\frac{d\hat{x}}{d\rho} = -\frac{\frac{1}{2}(1-\hat{a})^2\,\sigma^{2\prime}\,[q_x + n\,q_X]}{\hat{A}} > 0 \tag{A.28}$$

$$\frac{d\hat{a}}{d\rho} = \frac{1-\hat{a}}{\delta\, e'' + \rho} > 0\ , \tag{A.29}$$

where $\hat{A}$ is defined as above.

A.8 Proof of Proposition 5

Ad 1. In order to derive the comparative statics of $\hat{\tau}$ with respect to δ, we differentiate (34) with respect to δ. This yields (omitting arguments)[11]

$$\frac{d\hat{\tau}}{d\delta} = (n-1)\Bigg[\Bigg[\left[q_{xX} + n\,q_{XX}\right]\left[\mu' - \frac{\rho\,(1-\hat{a})^2}{2}\,\sigma^{2\prime} - \delta\,e(\hat{a})\,\sigma^{2\prime}\right] \qquad \text{(A.30)}$$
$$+\, q_X\left[\mu'' - \frac{\rho\,(1-\hat{a})^2}{2}\,\sigma^{2\prime\prime} - \delta\,e(\hat{a})\,\sigma^{2\prime\prime}\right]\left[q_x + n\,q_X\right]\Bigg]\frac{d\hat{x}}{d\delta} - q_X\,e(\hat{a})\,\sigma^{2\prime}\Bigg]$$

From Equation (A.25), we have (with $\hat{A}$ given by Equation A.27)

$$\frac{d\hat{x}}{d\delta} = \frac{e(\hat{a})\,\sigma^{2\prime}\left[q_x + n\,q_X\right]}{\hat{A}}\;. \qquad \text{(A.31)}$$

Using this in (A.30) and simplifying yields

$$\frac{d\hat{\tau}}{d\delta} = \frac{(n-1)\,e(\hat{a})\,\sigma^{2\prime}}{\hat{A}}\Bigg[q_X\,c''+ \qquad \text{(A.32)}$$
$$\left[\mu' - \left[\delta\,e(\hat{a}) + \frac{\rho\,(1-\hat{a})^2}{2}\right]\sigma^{2\prime}\right]\left[-q_X\,q_{xx} - n\,q_{xX}\,q_X + n\,q_x\,q_{XX} + q_x\,q_{xX}\right]\Bigg]\;.$$

Since $\hat{A}$ is negative and the $\sigma^{2\prime}$ is negative, too, the change of $\hat{\tau}$ following an increase in δ has the same sign as

$$\left[\mu' - \left[\delta\,e(\hat{a}) + \frac{\rho\,(1-\hat{a})^2}{2}\right]\sigma^{2\prime}\right]\left[-q_X\,q_{xx} - n\,q_{xX}\,q_X + n\,q_x\,q_{XX} + q_x\,q_{xX}\right] + q_X\,c''\;. \qquad \text{(A.33)}$$

Rearranging, this expression has the same sign as

$$-\left[\frac{\hat{x}\,q_{xx}}{q_x} + \frac{\hat{X}\,q_{xX}}{q_x} - \frac{\hat{x}\,q_{xX}}{q_X} - \frac{\hat{X}\,q_{XX}}{q_X}\right]\left[\mu' - \left[\delta\,e(\hat{a}) + \frac{\rho\,(1-\hat{a})^2}{2}\right]\sigma^{2\prime}\right]q_x + \hat{x}\,c''\;, \qquad \text{(A.34)}$$

which is equal to, using the efficiency condition (A.23),

$$-\left[\frac{-\hat{X}\,q_{XX} - \hat{x}\,q_{Xx}}{q_X} - \frac{-\hat{x}\,q_{xx} - \hat{X}\,q_{xX}}{q_x}\right]\frac{c'\,q_x}{q_x + n\,q_X} + \hat{x}\,c''\;. \qquad \text{(A.35)}$$

Using the abbreviation (37) and rearranging leads to Condition (36).

Ad 2. Differentiating the optimal subsidy (34) with respect to ρ leads to

$$\frac{d\hat{\tau}}{d\rho} = (n-1)\Bigg[\Bigg[\left[q_{xX} + n\,q_{XX}\right]\left[\mu' - \frac{\rho\,(1-\hat{a})^2}{2}\,\sigma^{2\prime} - \delta\,e(\hat{a})\,\sigma^{2\prime}\right] \qquad \text{(A.36)}$$
$$+\, q_X\left[\mu'' - \frac{\rho\,(1-\hat{a})^2}{2}\,\sigma^{2\prime\prime} - \delta\,e(\hat{a})\,\sigma^{2\prime\prime}\right]\left[q_x + n\,q_X\right]\Bigg]\frac{d\hat{x}}{d\delta} - q_X\,\frac{1}{2}\,\sigma^{2\prime}\Bigg]$$

[11] The term with $d\hat{a}/d\delta$ drops out by means of the first order condition (A.24).

Using (A.28) and rearranging yields

$$\frac{d\hat{\tau}}{d\rho} = \frac{(n-1)\,\frac{1}{2}\,\sigma^{2\prime}}{\hat{A}} \Big[q_X\, c''+ \tag{A.37}$$
$$\Big[\mu' - \Big[\delta\, e(\hat{a}) + \frac{\rho\,(1-\hat{a})^2}{2}\Big]\ \sigma^{2\prime}\Big]\ [-q_X\, q_{xx} - n\, q_{xX}\, q_X + n\, q_x\, q_{XX} + q_x\, q_{xX}]\Big]\ ,$$

which is negative, if and only if Condition (36) is fulfilled.

A.9 Proof of Proposition 6

Part 1. is proven by differentiating welfare in equilibrium, i.e.

$$W^\star = n\ \Big[\mu\,(q\,(x^\star, n\,x^\star)) - \Big[\delta\, e(a^\star) + \frac{\rho}{2}\,(1-a^\star)^2\Big]\ \sigma^2\,(q\,(x^\star, n\,x^\star)) - c(x^\star)\Big] \tag{A.38}$$

with respect to δ, and plugging in the equilibrium conditions (A.16) and (A.17).
Part 2. is proven by differentiating Equation (38) with respect to δ, using the envelope theorem, $d\hat{W}/d\delta = \partial\hat{W}/\partial\delta$, and the equilibrium conditions (A.16) and (A.17).

A.10 Proof of Proposition 7

With the specifications (44), $c(x) = c\cdot x$, and $\mu(q) = \mu$, we have (using A.18)

$$\frac{dx^\star}{d\delta} = \frac{e(a^\star)}{\delta\, e(a^\star) + \frac{\rho\,(1-a^\star)^2}{2}}\ \left[\frac{1-\eta}{\delta - \eta\, q\,(x^\star, n\,x^\star)}\ \frac{q\,(x^\star, n\,x^\star)}{x^\star}\right]^{-1}\ . \tag{A.39}$$

and

$$\tau^\star = (n-1)\,\frac{\gamma}{n}\,\frac{q\,(x^\star, n\,x^\star)}{x^\star}\ \left[\delta\, e(a^\star) + \frac{\rho\,(1-a^\star)^2}{2}\right]\ \frac{\sigma^2\,(q\,(x^\star, n\,x^\star))}{\delta - \eta\, q\,(x^\star, n\,x^\star)}\ . \tag{A.40}$$

Thus,

$$\tau^\star\,\frac{dx^\star}{d\delta} = e(a^\star)\,\sigma^2\,(q\,(x^\star, n\,x^\star))\ \gamma\,\frac{n-1}{n}\,\frac{1}{1-\eta}\ . \tag{A.41}$$

Ad 1. Using this in Equation (41), we find that welfare in the laissez-faire equilibrium decreased / is unchanged / increases with δ, if

$$e(a^\star)\,\sigma^2\,(q\,(x^\star, n\,x^\star))\ \gamma\,\frac{n-1}{n}\,\frac{1}{1-\eta} \lesseqqgtr e(a^\star)\,\sigma^2\,(q\,(x^\star, n\,x^\star))\ . \tag{A.42}$$

Rearranging leads to Condition (46).
Ad 2. Using (A.41) in Equation (43), we have (note that $\hat{a} = a^\star$)

$$\frac{d}{d\delta}\,\left(\hat{W} - W^\star\right) = n\, e(a^\star)\left[\left(1 - \gamma\,\frac{n-1}{n}\,\frac{1}{1-\eta}\right)\ (\delta - \eta\, n^\gamma\, x^\star)^{\frac{1}{\eta}} - (\delta - \eta\, n^\gamma\, \hat{x})^{\frac{1}{\eta}}\right]\ . \tag{A.43}$$

With the specifications of the example, the condition for the efficient allocation is

$$\left[\delta\, e(\hat{a}) + \frac{\rho\,(1-\hat{a})^2}{2}\right]\ (\delta - \eta\, n^\gamma\, \hat{x})^{\frac{1}{\eta}-1}\ n^\gamma = c\ , \tag{A.44}$$

i.e.,

$$(\delta - \eta\, n^{\gamma}\, \hat{x})^{\frac{1}{\eta}} = \left(\frac{c}{n^{\gamma}}\right)^{\frac{1}{1-\eta}} \left[\delta\, e(\hat{a}) + \frac{\rho\,(1-\hat{a})^2}{2}\right]^{-\frac{1}{1-\eta}} \tag{A.45}$$

The equilibrium condition is

$$\left[\delta\, e(\hat{a}) + \frac{\rho\,(1-\hat{a})^2}{2}\right] (\delta - \eta\, n^{\gamma}\, x^{\star})^{\frac{1}{\eta}-1} \left(1 - \gamma\,\frac{n-1}{n}\right) n^{\gamma} = c\ , \tag{A.46}$$

such that

$$(\delta - \eta\, n^{\gamma}\, x^{\star})^{\frac{1}{\eta}} = \left(\frac{c}{n^{\gamma}}\right)^{\frac{1}{1-\eta}} \left[\delta\, e(\hat{a}) + \frac{\rho\,(1-\hat{a})^2}{2}\right]^{-\frac{1}{1-\eta}} \left(1 - \gamma\,\frac{n-1}{n}\right)^{-\frac{1}{1-\eta}} \ . \tag{A.47}$$

Hence,

$$\frac{d}{d\delta}\left(\hat{W} - W^{\star}\right) = n\, e(a^{\star}) \left(\frac{c}{n^{\gamma}}\right)^{\frac{1}{1-\eta}} \left[\delta\, e(\hat{a}) + \frac{\rho\,(1-\hat{a})^2}{2}\right]^{-\frac{1}{1-\eta}} \left[\frac{1 - \gamma\,\frac{n-1}{n}\,\frac{1}{1-\eta}}{\left(1 - \gamma\,\frac{n-1}{n}\right)^{\frac{1}{1-\eta}}} - 1\right] . \tag{A.48}$$

A Taylor-series expansion-argument yields the result that the expression in brackets is negative for $\eta > 0$, zero for $\eta = 0$ and positive for $0 < \eta < 1$.