# The Biodiversity Optimization Problem with a Single Parasite and a Single Species*^

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## ABSTRACT

A given preservation budget should be divided between two species: Palm and the parasite Ficus Religious, which needs the palm to survive. In this parasite framework we obtain an interior solution. This is in contrast to the well-known extreme solution result of Weitzman's (Econometrica 1998) libraries model in the Noah's Ark problem. Moreover, in the parasite model, at least half of the budget is invested in the Palm preservation.

Keywords: Biological Diversity Function, Libraries, Parasites, Linear Budget Constraint.

## 1. Introduction:

Humans utilize the goods and services nature provides in order to survive. Ecosystem goods (such as seafood, forage, timber, and many pharmaceutical and industrial products) and ecosystem services (processes of natural ecosystems that support human activity and sustain human life) yield benefits humans populations derive, directly or indirectly, from the functions of biodiversity (de Groot, 1992; Daily, 1997). Due especially to increase human activities many of the original natural areas have been destroyed, leading to the extinction of species at rates never observed before. The more is known about biodiversity and the loss of species, the stronger the calls are for protecting what is left of the natural environment. Well-considered arguments for nature protection should not only be based on ecological grounds but also on economic grounds.

However, financial resources for biodiversity conservation programs are not sufficient to protect all habitats and species. Mankind simply cannot protect everything. This requires choosing conservation priorities in order to support the most species at the least cost. Moran *et al.* (1996; 1997) point out that there is no single correct method for establishing biodiversity conservation priorities at any level of organization. Nevertheless, the question of how to determine priorities for maintaining or increasing biodiversity under a limited budget constraint has occupied the minds of environmental economists since the seminal work by Martin L. Weitzman (1992; 1993). He developed "… a more-or-less consistent conceptual framework and a more-or-less usable measure on the value of diversity that can tell us how to trade off one form of diversity against the other." (Weitzman, 1992, p. 363; see also Weitzman, 1995; 1998). Weitzman's 'Noah's Ark Problem' introduced a framework for ranking conservation priorities under budget constraints. A main assumption was that there are no symbiotic relations between the species. Criticisms of Weitzman's framework were that it was unsuitable for studying global problems of biodiveristy loss, and that it was difficult to apply to genetic diversity (Weikard 2002, Mainwaring 2001). V.D.Heide (2005) wrote that *"…Weitzman's general criterion only holds under a very strict condition: namely, in the absence of ecological relationships among species... This represents at best a very specific case, and can certainly not be regarded as providing general information about which species to protect through* *which protection projects…Weitzman’s criterion is only suitable if ecological relationships are of little importance, notably if the loss of a species has little impact on an ecosystem.”.*

These criticisms were part of the motivation for this study, which examines possible solutions to Noah’s ark problem with interactions among species in ecological habitats. Specifically, in this paper we extend Weitzman’s model to a model of a symbiotic relationship between a parasite and a host species, where the parasite needs the host species in order to survive. We posit a linear costs variety of prevention rules, whereby a "central planner" optimizes over a biodiversity function under a linear budget constraint. The main result is that - contrary to Weitzman's – one may obtain an interior solution; moreover, at least half of the budget is invested in the host species. The paper begins with a description of Weitzman model of two species and then we proceed with the two-species symbiotic model.

In general, the Noah’s ark problem is a metaphor to the conservation dilemma under budget constraint and development stresses. Weitzman used the term “library” as a model to “species”.  This is demonstrated by the metaphor of a library that can be burnt with possible loss of its collection of books. Weitzman celebrated result for independent species yields an extreme solution. Extreme solution is defined by Weitzman as:” *”almost all” (n-1 out of n) species/libraries are wither fully boarded* $P_i = \overline{P_i}$ *, or not boarded at all* $P_i = \underline{P_i}$ *.at most one species/library j is “fractionally boarded” with interior probability* $P_j$ *, where* $\underline{P_j} < P_j < \overline{P_j}$ **“**

In the present extension, we assume the following policy problem: A given preservation budget is to be divided optimally between two species: Palm and a parasite, Ficus Religious, which needs the palm to survive. For this particular case one may obtain an interior solution, contrary to the extreme solution result of Weitzman's (Econometrica 1998) Noah's ark problem.

## 2. The Diversity Function of "Species/Library" for Two Species

### 2.1. Weitzman's two libraries model:

The book collection of any given library i consists of $M_i$ different books[1]. An overlap of book collections may occur between two libraries. The definition of the diversity of libraries is the number of different books in the library's collections.

The number of the same books held simultaneously by the two libraries is denoted by J.

Weitzman specified independent probabilities of survival of the different libraries.

For n=2 in Weitzman (1998) basic model, and when both probabilities are independent, Weitzman obtained the ***diversity function*** $W(P_0, P_1)$ to be:

$$(4w)\ W(P_0,P_1) = P_0P_1(M_0 + M_1 - J) + P_0(1-P_1)M_0 + (1-P_0)P_1M_1 + (1-P_0)(1-P_1)\cdot 0$$

Which yields:

$$(5w)\quad W(P_0,P_1) = M_0P_0 + M_1P_1 - JP_0P_1$$

From this specification he obtained the result that under linear budget constraint there exists an extreme solution.

### 2.2. The Parasite framework:

Consider, for example, an "indifferent" species (Palm) and a "beneficiary" species (Ficus Religious), where the probability of the latter's survival depends on the event that the Indifferent species will survive. Denote the conditional probability of the beneficiary species' survival, conditional on the event that species exists, by $P_1$. Using

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[1] See list of symbols in the appendix.

the multiplicative formula for the probability of the intersection[2], we obtain an expression which we would like to compare with Weitzamn's results (4*w*) and (5*w*):

(1) $$W(P_0,P_1)=P_0P_1(M_0+M_1-J)+P_0(1-P_1)M_0=P_0M_0+P_0P_1(M_1-J)$$

In addition, there is a value $P_0U_0+P_0P_1U_1$ to the physical structures (using the library metaphor) or biomass of the Palm and of the Ficus Religious. Moreover, as noted above, we assume that costs are linear in the probabilities (the cost of the parasite conservation is the linear in the conditional probability).

Given a budget B and parameters $M_0,M_1,J,C_0,C_1,U_0,U_1$[3], we can specify the biodiversity optimization problem as:

(2) $$\begin{aligned}\text{MAX}\quad & Z(P_0,P_1)=P_0U_0+P_0P_1U_1+P_0M_0+P_0P_1(M_1-J)\\ \text{s.t.}\quad & P_0C_0+P_1C_1\le B\\ & 0\le P_0\le 1\\ & 0\le P_1\le 1\end{aligned}$$

Obviously, whenever $C_0+C_1\ge B$, there is budget equality in the optimal solution $(P_0^*,P_1^*)$, i.e.,

(3) $$P_0C_0+P_1C_1=B$$

Inserting in (2), we obtain,

(4) $$\begin{aligned}&F(P_0)=P_0U_0+P_0M_0+P_0\left(\frac{B-P_0C_0}{C_1}\right)(U_1+M_1-J)=\\ &=P_0(U_0+M_0)+P_0\cdot\frac{B(U_1+M_1-J)}{C_1}-P_0^2\cdot\frac{C_0(U_1+M_1-J)}{C_1}\end{aligned}$$

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[2] The multiplicative formula: $P(A\cap B)=P(B)\cdot P(A|B)$

[3] See list of symbols in the appendix

In this case, contrary to Weitzman's result, the quadratic function is concave, and may have an interior maximum. The first order condition for interior solution $0 < P_0^* < 1$ is:

$F'(P_0^*) = 0$, that is:

(5) $$U_0 + M_0 + \frac{B(U_1 + M_1 - J)}{C_1} - 2P_0^* \frac{C_0(U_1 + M_1 - J)}{C_1} = 0$$

(6) $$P_0^* = \frac{(U_0 + M_0)C_1}{2C_0(U_1 + M_2 - J)} + \frac{B}{2C_0}$$

(7) $$P_1^* = \frac{B - P_0^* C_0}{C_1}$$

(8) $$P_1^* = \frac{B}{2C_1} - \frac{U_0 + M_0}{2(U_1 + M_1 - J)}.$$

Sufficient conditions for interior solution are:

(5) $$2 - \frac{C_1}{C_0} \cdot \frac{(U_0 + M_0)}{(U_1 + M_1 - J)} > \frac{B}{C_0} > \frac{C_1}{C_0} \cdot \frac{(U_0 + M_0)}{(U_1 + M_1 - J)}$$

While a necessary condition for an interior solution is given by:

(6) $$\frac{U_0 + M_0}{C_0} < \frac{U_1 + M_1 - J}{C_1}$$

(7) $$\frac{U_0 + M_0}{U_1 + M_1 - J} < \frac{C_0}{C_1}$$

That is, for interior solution we need to assume that the value of the beneficiary species / the value of the indifferent species is greater than the proportion of the expenditures coefficients.

Hence:

***Lemma 1***: whenever $P_0^*$ is an interior solution, $P_0 C_0 > \frac{B}{2}$.

That is, the part of the budget spent on the Palm must be greater than the part of the budget spent on the Ficus Religious.

Proof: The expenditure on the indifferent species is:

$$(7) \qquad C_0 P_0^* = \frac{(U_0 + M_0)C_1}{2(U_1 + M_1 - J)} + \frac{B}{2} > \frac{B}{2}$$

which is bigger that the expenditure on the beneficiary species, which is:

$$(8) \qquad C_1 P_1^* = \frac{B}{2} - \frac{(U_0 + M_0)C_1}{2(U_1 + M_1 - J)} < \frac{B}{2}$$

Q.E.D.

We also notice that the expenditure on the beneficiary species is positive, although it depends on the indifferent species. Note also, whenever $P_1^* > 0$, it affects the size of $P_0^*$, by reducing the survival probability compared to the case when no parasite exists.

## 3. Further research:

We intend to develop a variety of biodiversity functions similar that of Weitzman's biodiversity functions for the case of n>2, and their inter-correlations. Next, we intend to generalize our model to the central library problem with branches.

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## Appendix 1: List of Symbols

**General symbols:**

$M_0$ - Different books at library 0 or gene pool of the indifferent species

$M_1$ - Different books at library 1 or gene pool of the parasite species

$J$ - Number of books held jointly in common by library 0 and library 1

$U_0$ - Value of the building structure of library 0 or the direct utility or how much we value the existence of the indifferent species

$U_1$ - Value of the building structure of library 1 or the direct utility or how much we value the existence of the parasite species

$C_0$ - Cost of preserving species 0

$C_1$ - Cost of preserving species 1

B - the total amount of the budget

$W(P_0, P_1)$ - The expected diversity function

**Weitzman's model:**

$P_0$ - Probability of survival of species 0

$P_1$ - Probability of survival of species 1

**Parasite model:**

$P_0$ - Probability of survival of the indifferent species

$P_1$ - Conditional probability of the parasite's survival

$C_1$ - The cost of preserving parasite under the assumption that the conditional probability is $P_1$