Uncertainty, Precaution and Biodiversity Management

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Abstract

We analyze ecosystem management under 'unmeasurable' Knightian uncertainty or ambiguity which, given the uncertainties characterizing ecosystems, might be a more appropriate framework relative to the classic risk case (measurable uncertainty). We consider this approach as a formal way of modelling the precautionary principle and provide policy rules for biodiversity management under precautionary approaches. We specify the ambiguity framework by the e-contamination and k-ignorance approaches which are associated with decision making in the context of least favorable priors and maxmin criteria.

The k-ignorance approach is applied to a descriptive dynamic model of interacting species and provides land allocation and harvesting rules for keeping species biomasses above some minimum safety level with a given probability. The e-contamination approach is applied to an optimal biodiversity management problem for which we provide similar land allocation and harvesting rules. Comparing solutions under risk and under ambiguity, we quantify the impact of the precautionary principle.

Keywords: Knightian uncertainty, ambiguity, risk, precautionary principle, biodiversity management, land allocation, harvesting.

JEL Classification: D81, Q20

1 Introduction

Biodiversity loss has emerged as a major issue on both academic and policy grounds. As stated in the recent Millennium Ecosystem Assessment report (MEA 2005a):

> Humans are fundamentally, and to a significant extent irreversibly, changing the diversity of life on earth, and most of these changes represent a loss of biodiversity.

It is estimated, in the same report, that during the past several hundred years, humans have increased the species extinction rate by as much as 1000 times over background rates over the planet's history. In the MEA report (2005b), it is acknowledged that ecosystem management practices that maintain diversity, functional groups, and trophic levels are more likely to decrease the risk of large losses of ecosystem services than practices that ignore these factors.

These statements suggest that the development of management rules that could help to prevent loss of biodiversity is a desirable goal. The attainment of this goal is hindered, however, both by the complexity of ecosystems and by important and interrelated uncertainties, a number of which include sources such as major gaps in global and national monitoring systems; the lack of a complete inventory of species and their actual distributions; limited modelling capacity and lack of theories to anticipate thresholds; emergence of surprises and unexpected consequences. These uncertainties may impede adequate scientific understanding of the underlying ecosystem mechanisms and the impacts of policies applied to ecosystems. For the purposes of our analysis we will refer to the overall uncertainty associated with these sources as scientific uncertainty.

A feature of the above structure of uncertainty is that it might be difficult or even impossible to associate probabilities with uncertain prospects affecting the ecosystem evolution. This is close to the concept of uncertainty as introduced by Frank Knight (1921) to represent a situation where there is ignorance, or not enough information, to assign probabilities to events. Knight argued that uncertainty in this sense of

unmeasurable uncertainty is more common in economic decision making. Knightian uncertainty is contrasted to risk (measurable or probabilistic uncertainty) where probabilities can be assigned to events and they are summarized by a subjective probability measure or a *single* Bayesian prior. This concept of Knightian uncertainty or 'ambiguity' has been associated formally with a concept of *multiple priors* (Gilboa and Schmeidler, 1989), as well as with a concept of uncertainty or ambiguity aversion which in general increases with an ignorance parameter (Chen and Epstein, 2002).

In economics, decision making under risk implies expected utility maximization. Under Knightian uncertainty as described above, it was Wald (1950) who suggested that a *maxmin* solution could be a reasonable solution to a decision problem, where an *a priori* probability distribution does not exist or is not well known to the researcher. One way to approach the maxmin solution is to use the idea of *least favorable prior* (LFP)[1] decision theory, as developed by Gilboa and Schmeidler (1989), which results in maxmin expected utility theory and represents an axiomatic foundation of Wald's criterion.

Decision theory based on the LFP can be associated with the concepts of *precautionary principle* (PP) and *safe minimum standards* (SMS). The precautionary principle is an approach where actions are taken to anticipate and avert serious or irreversible harm, such as for example extinction of species for the case of biodiversity preservation, in advance of, or without, a clear demonstration that such action is necessary. Marchant (2003) states that "the PP prescribes how to bring scientific uncertainty into the decision-making processes by explicitly formalizing precaution and bringing it to the forefront of deliberations". On the other hand the ideas of LFP or *worst case scenario* (WCS) and irreversible changes can be intuitively put together, since the emergence of a WCS could lead to an irreversible change. Therefore a direct link can be

[1] Given a set of prior probability distributions associated with the multiple priors framework, the LFP is the one that corresponds to the least favorable outcomes. It can be associated with the concept of the worst case scenario. Under Knightian uncertainty the researcher cannot choose *one* prior to define expected utility as is done under risk.

made between LFP ideas and the PP. Scientific uncertainty can be manifested in multiple priors. The decision maker cannot chose among them but one or more of these priors, the LFP, lead to irreversible change. To prevent the irreversible change, which is not clearly demonstrated since the decision maker does not know that the LFP will prevail, a precautionary approach should be taken, which implies that the decision rule should be based on LFP. Thus, the maxmin expected utility can be used as a conceptual framework for designing management rules which adhere to the PP.

Closely related to the these concepts is the idea of SMS for the preservation of biodiversity (e.g. Holt and Tisdell, 1993), where SMS could be defined in terms of minimum viable populations and minimum habitat requirements. Using the LFP and maxmin framework, SMS can be defined so that species extinction is prevented under the least favorable situation associated with the uncertainties obscuring the scientific understanding of the ecosystems' mechanisms. This policy can be regarded as management which embodies some type of PP.

The purpose of this paper is to combine these concepts and provide management rules for preserving biodiversity which follow a precautionary principle under ambiguity. The precautionary approach is formalized by using multiple priors and LFP ideas, and maxmin decision rules, which lead to SMS and optimal management rules that embody the PP. Furthermore, by comparing SMS and optimal management rules under conditions of scientific uncertainty and risk it is possible to obtain some quantification of the implications of the PP in terms of decision variables of interest such as harvesting and land allocation rules.

In the rest of the paper we present two approaches to biodiversity management under ambiguity in models of multiple species. In the first we apply the k-ignorance approach for specifying the multiple priors model and we derive, in terms of a descriptive non-optimizing model of species interactions, harvesting and land allocation rules for species to keep species populations above some minimum safety standard with a given probability. In the second we apply the e-contamination approach for specifying the multiple priors model to an optimal biodiversity man-

agement problem and we provide similar land allocation and harvesting rules. By comparing solutions under risk and under ambiguity we provide a measure of the impact of adopting precautionary approaches in ecosystem management.

2 Modelling Uncertainty Using Multiple Priors

Let the set of states of the world be Ω, and consider an individual observing some realization $\omega_t \in \Omega$. The basic idea underlying the multiple priors approach is that beliefs about the evolution of the process $\{\omega_t\}$ cannot be represented by a probability measure. Instead, beliefs conditional on ω_t are too vague to be represented by such a single probability measure and are represented by a *set* of probability measures (Epstein and Wang, 1994). Thus for each $\omega \in \Omega$, we consider $\mathcal{P}(\omega)$ as a set of probability measures about the next period's state. Formally $\mathcal{P}$ is a correspondence $\mathcal{P} : \Omega \to \mathcal{M}(\Omega)$ assumed to be continuous, compact-valued and convex-valued and $\mathcal{M}(\Omega)$ is the space of all Borel probability measures.

The individual ranks uncertain prospects or acts α. Let u be a standard utility function. The utility of any act α in an atemporal model is defined as (Gilboa and Schmedler, 1989; Chen and Epstein, 2002)

$$U(\alpha) = \min_{Q \in \mathcal{P}} \int u(\alpha)\, dQ \tag{1}$$

while in continuous time framework, recursive multiple prior utility is defined as:

$$V_t = \min_{Q \in \mathcal{P}} E_Q \left[\int_t^T e^{-\beta(s-t)} ds \right] \tag{2}$$

These definitions of utility in the context of multiple-priors corresponds to an intuitive idea of the 'worst case'. Utility is associated with the utility corresponding to the least favorable prior. With utility defined in this way, decision making by using the maxmin rule follows neutrally, since maximizing utility in the multiple-prior case implies maximizing the utility which corresponds to the LFP.

The individual's set of priors can be further specified for the purposes

of the analysis by the following formulations

- e-contamination

  The individual set of priors representing beliefs is defined as

$$\mathcal{P}_\epsilon = \{(1-\epsilon)\, P + \epsilon Q : Q \in \mathcal{M}\,(\Omega)\ \ ,\ \epsilon \in [0,1]\} \tag{3}$$

  where P is a reference probability measure. If $\epsilon = 0$, then the set of probability measures is a singleton $\mathcal{P} = \{\pi\}$. When $\epsilon \in (0,1)$, $\mathcal{P}$ is multi-valued and this multi-valuedness reflects both the presence of uncertainty and the individual's aversion to uncertainty (Epstein and Wang, 1994). The case of $\epsilon = 1$ can be regarded as the case of *ignorance.* Using the concepts of multiple-prior utility defined above, the definition of utility in the non-singleton $\mathcal{P}_\epsilon$ implies Q will be the probability measure associated with the least favorable outcome. Then $\mathcal{P}_\epsilon$ embodies the LFP idea.

- k-ignorance

  The individual considers the reference probability measure P and another measure $Q \in \mathcal{M}\,(\Omega)$. The discrepancy between the two measures is defined by the relative entropy

$$R(Q//P) = \int_0^{+\infty} e^{-\delta t} E_Q[\frac{1}{2}\varepsilon_t^2]dt \tag{4}$$

  where ε is a measurable function associated with the distortion of the probability measure P to the probability measure Q. According to the k-ignorance approach the individual incorporates into her/his decision-making problem the instantaneous relative entropy constraint $Q(\tau) = \{Q : E_Q[\frac{1}{2}\varepsilon_t^2] \leq \tau$, for all $t\}$, which means that probability measures differing from the reference measure P by at least as much as τ should be taken into account. If Q is a probability measure associated with the least favorable outcome, then k-ignorance also embodies an LFP or worst case scenario idea.

In relation to biodiversity management these approaches allow us to model the uncertainties or ambiguities underlying our scientific knowledge about the ecosystems in a way that, as will be shown later, leads to well defined policy rules.

3 Safe Minimum Standards

Economists try to manage ecosystems and biodiversity in an optimal way most of the times, despite the fact that the complexity of ecosystems might make the optimization exercises difficult even at a theoretical level. On the other hand, if we are interested in preserving diversity it might be useful to think about managing ecosystems using safety rules, which when applied prevent, in a probabilistic way, species or a set of species from becoming extinct.[2] Safety rules in biodiversity preservation could acquire greater importance when the ecosystem manager faces Knightian uncertainty, or ambiguity which, as discussed above, is a case potentially very relevant in ecosystem management. In this situation worst case events might cause surprises and extinction of species. Since these irreversible changes have occurred in reality, dealing with worst case scenarios means that ecosystem management and biodiversity preservation are associated with a PP, which implies that the management rules are such that species will not become extinct under worst case scenarios.

3.1 A Deterministic Model

We examine first the determination of safe minimum standards for preventing biodiversity loss, in terms of minimum population levels in the context of a deterministic model. The deterministic model developed here is used as a vehicle for the introduction of uncertainty in analyzing biodiversity management which is the main target of this paper. Population levels are directly controlled by harvesting, and available habitat which is determined by land allocations rules.

[2] Safety regulation is a more general issue in economics. For a general discussion of the role of economic analysis in the development of environmental health and safety regulation, see Arrow et al. (1996). For a discussion of safety standards in species protection, see for example Holt and Tisdell (1993).

We start by considering an ecosystem manager who manages a landscape, normalized to unity, where two species coexist. Let $B_i(t)$ for $i = 1, 2$ be the initial biomasses of the two species at time t. It is assumed that the evolution of the initial biomasses $(B_{10,}B_{20})$ through time can be described by the linear system of deterministic differential equations

$$\begin{aligned} \dot{B}_1 &= B_1[f_1(\mathbf{w}) - d_1] - a_{12}B_2 - h_1 \\ \dot{B}_2 &= B_2[f_2(\mathbf{w}) - d_2] - a_{21}B_1 - h_2 \end{aligned} \tag{5}$$

which can be rewritten as:

$$\dot{\mathbf{B}} = A\mathbf{B} - \mathbf{h}, \tag{6}$$

$$A = \begin{bmatrix} f_1(\mathbf{w}) - d_1 & -a_{12} \\ -a_{21} & f_2(\mathbf{w}) - d_2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

In equation (6), $f_i(\mathbf{w}) - d_i$, $i = 1, 2$ are the instantaneous, net of death, growth rates, where $f_i(\mathbf{w})$ are positive and bounded functions of $\mathbf{w}$, and $\mathbf{w} = (w_1, w_2)$ denotes the land allocation rule $w_1 + w_2 = 1$. Furthermore, $\mathbf{h} = (h_1, h_2)$, where h_i denotes the harvesting of each species, and finally a_{12} and a_{21} are interaction coefficients. We assume that harvesting and land allocation are held fixed. The land in this formulation is considered as a fixed resource used by each species and, in a sense, can be regarded as implying an underlying competition between species for this resource. So species interact both through the interaction coefficients and land allocation. For example if the two species are complementary then $f_i(\mathbf{w}) = 0$ if $w_j = 0$, $i, j = 1, 2$, $i \neq j$. In the control problem we assume that harvesting and land allocation are held fixed.[3]

The idea behind safe standards is to use harvesting and land allocation as controls in order to stir the dynamic system (5), which is linear in the state variables (B_1, B_2) to a desired steady state $(\overline{B}_1, \overline{B}_2)$, which is no less than a desirable lower bound $\left(B_1^S, B_2^S\right)$. That is, $(\overline{B}_1, \overline{B}_2) \geq$

[3] This is the simplest possible case but it provides insights into the solution of the problem. More complex feedback and/or time flexible mechanisms are areas for further research.

$\left(B_1^S, B_2^S\right) > (B_{10}, B_{20}) > 0$. The control of the system can be obtained in the following way.

By considering the characteristic equation of (6)

$$|A - \lambda I| = 0 \Leftrightarrow [f_1(\mathbf{w}) - d_1 - \lambda][f_2(\mathbf{w}) - d_2 - \lambda] - a_{12}a_{21} = 0 \Leftrightarrow$$
$$\lambda_{12} = \frac{+[(f_1(\mathbf{w}) - d_1) + (f_2(\mathbf{w}) - d_2)] \pm \sqrt{\Delta}}{2}$$
$$\Delta = [(f_1(\mathbf{w}) - d_1) + (f_2(\mathbf{w}) - d_2)]^2 + 4(f_1(\mathbf{w}) - d_1)(f_2(\mathbf{w}) - d_2)a_{12}a_{21}$$

we can calculate the characteristic roots $\lambda_{1,}\lambda_2$ of matrix A. Then by a suitable choice of the values for the controls $(\mathbf{w},\ \mathbf{h})$, we seek:[4]

- negative characteristic roots λ_i, $i = 1, 2$
- a solution of the above two-dimensional system of population dynamics which satisfies:

  $$B_1(t) \geq \overline{B}_{1,}\ B_2(t) \geq \overline{B}_2\ ,\ as\ \ t \to \infty$$

  where $(\overline{B}_1, \overline{B}_2)$ is the pre-specified desirable steady state. This solution takes the form $(B_1(t), B_2(t)) = \Omega e^{\Lambda t}\mathbf{c} - A^{-1}\mathbf{h}$, with Ω being the matrix of characteristic vectors $\mathbf{x}$ $(A\mathbf{x} = \lambda\mathbf{x})$, and $e^{\Lambda t}$ being a diagonal matrix with diagonal elements $e^{\lambda_i t}$, and $\mathbf{c} = \Omega^{-1}\mathbf{B}_0 + \Omega^{-1}A^{-1}\mathbf{h}$ being a vector of constants which we calculate using the initial conditions, (B_{01}, B_{02}) on biomasses.

As t increases, there is global convergence of the systems to the limit values $(\overline{B}_1, \overline{B}_2)$ which are higher than the desired lower bounds $\left(B_1^S, B_2^S\right)$.

Provided that a solution to the control of the system exists, then in the simple deterministic case we can attain population paths and

[4] This is a special case of the more general problem of choosing controls to stabilize a dynamic system at the desired steady state where feeback control laws of the form $w_i = \gamma_{wi}(\mathbf{B})$, $h_i = \gamma_{hi}(\mathbf{B})$ are to be determined. This is, however, beyond the scope of the present paper. (For the general problem, see for example Sastry (1999) or Khalil (2002).)

steady state populations which are above minimum bounds.[5] Although at this stage our model is deterministic, we can think of large random shocks to the population levels, which are not explicitly modelled here, but which might move the biomasses to extinction if they occurred. By choosing $\left(B_1^S, B_2^S\right)$ sufficiently high, we can protect against such events. Therefore a suitable choice of the controls of our system (w_1, w_2, h_1, h_2) can be interpreted as a safety rule for the preservation of an SMS of biodiversity in the landscape.

Model (5) is a simplification because of its linear structure, however it provides a basis for dealing with more realistic non-linear models of population dynamics with harvesting. Consider the general non-linear model with harvesting,

$$\dot{B}_i = B_i F_i \left(B_1, B_2, ..., B_n, \mathbf{w}\right) - h_i \ , i = 1, ..., n \ , \ \sum_{i=1}^{n} w_i = 1$$

Choose a vector $\mathbf{B}^S = \left(B_1^S, B_2^S, ..., B_n^S\right)$ of lower bounds and linearize the above system to obtain

$$\dot{B}_i = \sum_{j=1}^{n} \left[F_i \left(\mathbf{B}^S, \mathbf{w}\right) + B_i^S \sum_{j=1}^{n} \left. \frac{\partial F_i \left(\mathbf{B}, \mathbf{w}\right)}{\partial B_j} \right|_{\mathbf{B}=\mathbf{B}^S} \right] B_j - h_i, \ i = 1, ..., n$$

This system is of the same structure as (5) and the same approach can be applied. We move now to a stochastic environment.

3.2 Safety Standards in a Stochastic Environment

3.2.1 SMS under risk

In this section we consider the more realistic case where the evolution of biomasses is stochastic. We assume at this stage that the manager of the ecosystem has a single subjective prior distribution. A single prior is the main characteristic of the vast majority of continuous time dynamic models which assume probabilistic sophistication, implying that we analyze the problem under risk (measurable uncertainty). We fol-

[5] This is because we control the system from initial levels which are above the lower bounds.

low this approach because it is an intuitive way to proceed to the case of Knightian (unmeasurable) uncertainty, but also because it allows us, by comparing solutions under risk and solutions under uncertainty, to obtain a quantification of the precautionary principle, since PP can be associated with the Knightian uncertainty framework.

We assume that the evolution of the initial biomasses $B_{10,}B_{20}$ through time is given by the system of stochastic differential equations

$$\begin{aligned} dB_1 &= B_1[f_1(\mathbf{w}) - d_1]dt - a_{12}B_2dt - h_1dt + \sigma_1(t)dz_1 \\ dB_2 &= B_2[f_2(\mathbf{w}) - d_2]dt - a_{21}B_1dt - h_2dt + \sigma_2(t)dz_2 \end{aligned} \tag{7}$$

where the parameters are defined in system XX, and dz_1, dz_2 denote two correlated Brownian motions, with ρ being the correlation coefficient between them.[6]

Using matrix notation, equation (7) can be written as:

[6] In the system of equations (7), the quantities $\sigma_i(t)$ can be defined as $\frac{\phi_i(\mathbf{B}_0)}{\varphi_i(t)}$, with $\phi_i(\mathbf{B}_0)$ being suitable functions of the two initial biomasses, and $\varphi_i(t)$ functions of time t. In system (7), $(dz_{1t}, dz_{2t}) \rightsquigarrow N(0,t)$, that is they follow normal distributions with zero mean and variance t, or standard deviation $s = \sqrt{t}$. It has been observed that when we have normal or almost normal distributions, 99.7% of the observations are in the interval $(m-3s, m+3s)$. For an appropriate specification of the dynamics of the uncontrolled system, the two biomasses $B_i(t)$ should not take negative values. Therefore, we could choose as $\varphi_i(t)$, the function $3\sqrt{t}$ or the function t. We use the above normalization to specify the dynamics of our system, since in the case where σ_i is independent of time t, the term $\sigma_i dz_i$, even with small probability, could take very large, negative values. The way in which we choose the quantities $\sigma_i(t)$, as will be shown further on, does not affect the final results, due to the fact that the Brownian motions dz_i have zero mean. In the rest of this paper, for purposes of notational simplicity, we write σ_i instead of $\sigma_i(t)$.

$$d\mathbf{B} = ABdt - \mathbf{h}dt + \Sigma d\mathbf{Z} \quad \text{where} \tag{8}$$

$$d\mathbf{B} = \begin{bmatrix} dB_1 \\ dB_2 \end{bmatrix}$$

$$A = \begin{bmatrix} f_1(\mathbf{w}) - d_1 & -a_{12} \\ -a_{21} & f_2(\mathbf{w}) - d_2 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}, \mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

$$d\mathbf{Z} = \begin{bmatrix} dz_1 \\ dz_2 \end{bmatrix}$$

Equation (8), multiplied from the left by a suitable matrix, becomes (see Oksendal (2000)): [7]

$$d(e^{-At}B_t) = e^{-At}dB - e^{-At}ABdt = -e^{-At}hdt + e^{-At}\Sigma dZ \tag{9}$$

$$\text{where } e^F = \sum_{n=1}^{\infty} \frac{1}{n!}F^n = F + \frac{1}{2!}F^2 + \frac{1}{3!}F^3 + ... \tag{10}$$

$$\text{where } F = -At \tag{11}$$

Equivalently:

$$e^{-At}B_t - B_0 = -\int_0^t e^{-As}h_s ds + \int_0^t e^{-As}\Sigma dZ_s$$

$$B_t = e^{At}B_0 - \int_0^t e^{A(t-s)}h_s ds + \int_0^t e^{A(t-s)}\Sigma dZ_s \tag{12}$$

$$\text{with } \mathbf{B}_0 = \begin{bmatrix} B_{10} \\ B_{20} \end{bmatrix}$$

[7] In our case F is the matrix $-At$. The elements of this matrix converge to a real number. This holds because each element of this matrix is upper bounded by the sum $a_k = \sum_{k=1}^{\infty} \frac{2^{k-1}}{k!}(-tx)^k$, with x being the maximum of the four elements of matrix A in equation (8). For the above general term a known convergence criterion holds: $\limsup |\frac{a_{k+1}}{a_k}| < 1$ and therefore the series converge.

where

$$e^{At} = \begin{bmatrix} A_1 \, A_2 \\ A_3 \, A_4 \end{bmatrix} \tag{13}$$

with A_i for $i = 1, ..., 4$ depending on the values of the interaction coefficients a_{ij} and on f_i, h_i and which we can calculate using relationship (10). Using relationships (12) and (13), we can derive:

$$B_{1t} = A_1 B_{10} + A_2 B_{20} + g_1(h_1, h_2) + \int_0^t G_1 dZ_1 + \int_0^t G_2 dZ_2 \tag{14}$$

$$B_{2t} = A_3 B_{10} + A_4 B_{20} + g_2(h_1, h_2) + \int_0^t G_3 dZ_1 + \int_0^t G_4 dZ_2 \tag{15}$$

with G_i being functions of f_i , h_i, and σ_i, with the property that they belong to the class $V = V(0, T)$.[8] ,[9] The four integrals in equations (14) and (15) are stochastic integrals with the property that for all the possible combinations of i, j,

$$E \int_0^t G_i dZ_j = 0 \tag{16}$$

Suppose that the ecosystem manager is interested in the preservation of biomasses at levels higher than $\frac{1}{n}$ of their initial values. Using equations (14) and (15), because of (16) and the fact that the biomasses are bounded by their initial values, and using standard operations from the probability theory, we obtain upper and lower bounds for the probabilities of biomasses to be higher than the desirable levels. Then we have that:

[8] The quantities $g_1(h_1, h_2)$, $g_2(h_1, h_2)$ are due to the integral $\int_0^t e^{A(t-s)} h_s ds$.

[9] V is the set of measurable and adapted functions f with the property $E \int_0^T f(t, \omega)^2 dt < \infty$. Then for the corresponding stochastic integral it holds that $E \int_0^T f(t, \omega) dZ_t = 0$.

$$(A_1 + \frac{A_2}{l} + \frac{g_1}{B_{10}}) - \frac{1}{n} \leq \Pr(B_{1t} > \frac{1}{n}B_{10}) \leq \frac{n}{B_{10}}(A_1B_{10} + A_2B_{20} + g_1)$$
$$= n(A_1 + \frac{A_2}{l} + \frac{g_1}{B_{10}}) \tag{17}$$
$$(lA_3 + A_4 + \frac{g_2}{B_{20}}) - \frac{1}{n} \leq \Pr(B_{2t} > \frac{1}{n}B_{20}) \leq \frac{n}{B_{20}}(A_3B_{10} + A_4B_{20} + g_2)$$
$$= n(lA_3 + A_4 + \frac{g_2}{B_{20}}) \tag{18}$$
$$\text{where } l = \frac{B_{10}}{B_{20}}$$

In expressions (17) and (18), A_i is defined as $A_i = A_i(w_{1,}w_2, h_1, h_2)$ and thus the associated probability bounds depend on the land allocation weights (w_1, w_2) and on the harvesting rules (h_1, h_2). Therefore the landscape manager can suitably specify some invariant land allocation and harvesting rule $(w_{1,}w_2, h_1, h_2)$, so that the probability that the biomass of species i at any instant of time exceeds a prespecified level, which is proportional to the initial species biomass, does not fall below a lower bound and does not exceed an upper bound.[10] So with the parameters defined as above we have proven that:

Proposition 1 *Given a land allocation rule and harvesting rule* $(w_{1,}w_2, h_1, h_2)$, *the upper and lower bounds for the probabilities that the biomasses of species* $i = 1, 2$ *are higher than* $\frac{1}{n}$ *of the initial biomasses, are given by equations (17) and (18) respectively.*

The land allocation and harvesting rule $(w_{1,}w_2, h_1, h_2)$ that satisfies proposition 1 therefore provides a *safety rule*, since it bounds the probability of having the biomasses at any point in time above a certain level, which is $\frac{1}{n}B_{i0}$, $i = 1, 2$. By choosing this level, that is by choosing $1/n$, relations (17) and (18) can be used to determine a land allocation and a harvesting rule $(w_{1,}w_2, h_1, h_2)$ for desired probability bounds. For example, a rule $(w_{1,}w_2, h_1, h_2)|_p^x$ could be specified such that the biomasses at the period's end exceed by $x\%$ the initial biomasses, with a probability

[10] The elements A_i can be calculated by using numerical methods, but this goes beyond the scope of the present work.

that is between p and $p + \Delta p$. Thus $x\%$ can be regarded as an SMS for keeping species from extinction and the rule $(w_{1,}w_2, h_1, h_2)|_p^x$ can be regarded as a safety rule which may prevent the loss of biodiversity or the irreversible extinction of a species. There is, of course, the issue of the existence of such a rule for plausible parameter values, but due to the complexity of the problem, this could be handled by numerical methods.

The multi-species case Our model can be extended to the multi-species case. In this case, the evolution of the biomass of the kth species is given by:

$$dB_k = B_k[f_k(\mathbf{w}) - d_k]dt - h_k dt - \sum_{j \neq k} a_{kj} B_j dt + \sigma_k dz_k \quad j, k = 1, ..., n \tag{19}$$

with $\mathbf{w} = (w_1, ..., w_n)$ being the land allocation rule.

Using matrix notation, the system of equations (8) now takes the form:

$$d\mathbf{B} = ABdt - \mathbf{h}dt + \Sigma d\mathbf{Z} \text{ where} \tag{20}$$

$$d\mathbf{B} = \begin{bmatrix} dB_1 \\ ..... \\ dB_n \end{bmatrix}$$

$$A = \begin{bmatrix} f_1(\mathbf{w}) - d_1 & -a_{12} & ... \; ... & -a_{1n} \\ -a_{21} & f_2(\mathbf{w}) - d_2 & ... \; ... & -a_{2n} \\ ..... & ... & .... \; .... & .... \\ -a_{n1} & -a_{n2} & ... \; .. & f_n(\mathbf{w}) - d_n \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & \sigma_n \end{bmatrix}, \; \mathbf{h} = \begin{bmatrix} h_1 \\ ..... \\ h_n \end{bmatrix}$$

$$d\mathbf{Z} = \begin{bmatrix} dZ_1 \\ .... \\ dZ_n \end{bmatrix}$$

Applying the same methodology as above, we again obtain that:

$$B(t) = e^{At}B_0 - \int_0^t e^{A(t-s)}h_s ds + \int_0^t e^{A(t-s)}\Sigma dZ_s$$

$$\text{where now } \mathbf{B}_0 = \begin{bmatrix} B_{10} \\ .... \\ B_{n0} \end{bmatrix}$$

with the exponential matrix now being the nxn:

$$e^{At} = \begin{bmatrix} A_{11} & ... & A_{1n} \\ .. & A_{kj} & .. \\ A_{n1} & ... & A_{nn} \end{bmatrix}$$

Therefore the kth biomass is given by:

$$B_{kt} = \sum_{i=1}^{n} A_{ki}B_{i0} + g_k(h_{1,}.., h_n) + \sum_{i=1}^{n} \int_0^t G_i dZ_i \quad k = 1, ..n$$

Following the same approach as above the upper and lower bounds are given by

$$(\frac{A_1}{l_{1k}} + A_k + \frac{A_n}{l_{nk}} + \frac{g_k}{B_{k0}}) - \frac{1}{\gamma} \leq \Pr(B_{kt} > \frac{1}{\gamma}B_{k0}) \leq n(\frac{A_1}{l_{1k}} + A_k + \frac{A_n}{l_{nk}} + \frac{g_k}{B_{k0}})$$
$$\text{with } l_{jk} = \frac{B_{k0}}{B_{j0}} \; k = 1, ...n \quad j \neq k \tag{21}$$

Therefore it has been shown in the multi-species case that:

Proposition 2 *Given land allocation and harvesting rules* $(w_1, ..., w_n; h_1, ..., h_n)$, *upper and lower bounds can be determined for the probabilities that the biomasses of species* $i = 1, 2, ..., n$ *are higher than* $\frac{1}{\gamma}$ *of the initial biomasses values. The safety rules and the corresponding bounds are characterized by (21).*

3.2.2 SMS under Knightian Uncertainty

Suppose now that the ecosystem manager operates under conditions of ambiguity or Knightian uncertainty, which could be a realistic approximation of the actual ecosystem conditions. Along the lines of our previous discussion, this type of uncertainty can be modelled in terms of the multiple priors approach. In particular, we assume that the manager has multiple priors regarding the evolution of the species biomasses. We further specify the set of priors by following the k-ignorance approach.

Specifically, in the two biomasses case the system of equations given by (8) can now be written as:

$$
\begin{aligned}
d\mathbf{B} &= ABdt - \mathbf{h}dt + \Sigma R d\mathbf{Z} \text{ where} \qquad (22)\\
d\mathbf{B} &= \begin{bmatrix} dB_1 \\ dB_2 \end{bmatrix} \\
A &= \begin{bmatrix} f_1(\mathbf{w}) - d_1 & -a_{12} \\ -a_{21} & f_2(\mathbf{w}) - d_2 \end{bmatrix} \\
\Sigma &= \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}, \; R = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{bmatrix} \\
\mathbf{h} &= \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}, \; d\mathbf{Z} = \begin{bmatrix} dZ_1 \\ dZ_2 \end{bmatrix}
\end{aligned}
$$

where ρ is the correlation coefficient between the two Brownian motions in the initial system (8), and dZ_1, dZ_2 are two independent Brownian motions.

In the k-ignorance approach, the manager has a reference prior about the biomasses' evolution, those expressed by dZ_i, and considers, because of ambiguity, a decision-making problem with multiple priors such that the prior which according to her/his beliefs is farther away from the reference prior, does not differ from this reference prior, in terms of relative entropy, more than a positive number. To obtain the set of priors which reflect ambiguity, using as a benchmark model the model of

the reference prior (7) or (19), we consider measurable drift distortions.[11] More specifically the initial Brownian motions, dZ_i, $i = 1, 2$, are replaced by

$$Z_i(t) = \widehat{Z}_i(t) + \int_0^t \varepsilon_i(s)ds \quad , \; i = 1, 2 \tag{23}$$

where $\widehat{Z}_i$ are Brownian motions and ε_i are measurable functions. By doing this, system (22) takes the form:

$$d\mathbf{B} = ABdt - \mathbf{h}dt + \Sigma R\mathbf{E}dt + \Sigma Rd\widehat{\mathbf{Z}} \text{ where}$$
$$\mathbf{E} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \tag{24}$$

Applying the same methodology as before, system (24) becomes:

$$d(e^{-At}B_t) = e^{-At}dB - e^{-At}ABdt = -e^{-At}hdt + e^{-At}\Sigma Rd\widehat{Z} \tag{25}$$
$$e^{-At}B_t - B_0 = -\int_0^t e^{-As}h_s ds + \int_0^t e^{-As}\Sigma REds + \int_0^t e^{-As}\Sigma Rd\widehat{Z}_s$$
$$B_t = e^{At}B_0 - \int_0^t e^{A(t-s)}h_s ds + \int_0^t e^{A(t-s)}\Sigma REds + \int_0^t e^{A(t-s)}\Sigma Rd\widehat{Z}_s \tag{26}$$

If we compare equation (26) with (12), we can see that there is an extra term, $\int_0^t e^{A(t-s)}\Sigma REdt$, which acts as a measure of precaution and reflects the impact of Knightian uncertainty or ambiguity, relative to the case of risk. This has as a result the introduction of two extra terms in equations (14) and (15). Therefore the upper and lower bounds change, depending on the structure of the problem's parameters.

Specifically, when considering distortions in the benchmark model, the initial measure P is replaced by another probability measure Q. The discrepancy between the two measures is measured by the relative entropy, $R(Q//P) = \int_0^{+\infty} e^{-\delta t}E_Q[\frac{1}{2}\varepsilon_t^2]dt$. According to the k-ignorance

[11] In the terminology of robust control , we introduce model uncertainty into the decision-making problem. (See, for example, Holt and Sargent (2001).)

framework, we consider the instantaneous relative entropy constraint[12] $Q(\tau) = \{Q : E_Q[\frac{1}{2}\varepsilon_t^2] \leq \tau$, for all $t\}$, which restricts the set of models the decision maker considers at each instant of time. In this case the worst case perturbation is:

$$\varepsilon_{it}^* = -\sqrt{2\tau_i} \tag{27}$$

It should be clear that (27) reflects the idea of the LFP with the multiple priors model. By adopting this approach the distortions are now constant negative numbers and therefore we can calculate the new integral and afterwards the adjusted bounds. Particularly, examining the possible cases of the signs of the matrix $e^{A(t-s)}\Sigma RE$ in the integral $\int_0^t e^{A(t-s)}\Sigma RE dt$, we derive for these matrices that:

$$e^{A(t-s)} = \begin{bmatrix} + & - \\ - & + \end{bmatrix}, \Sigma = \begin{bmatrix} + & 0 \\ 0 & + \end{bmatrix}, \; e^{A(t-s)}\Sigma = \begin{bmatrix} + & - \\ - & + \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{bmatrix}, E^* = \begin{bmatrix} -\sqrt{2\tau_1} \\ -\sqrt{2\tau_2} \end{bmatrix}$$

$$RE^* = \begin{bmatrix} -\sqrt{2\tau_i} \\ -\sqrt{2\tau_1}\rho - \sqrt{2\tau_2}\sqrt{1-\rho^2} \end{bmatrix}$$

$$e^{A(t-s)}\Sigma RE^* = \begin{bmatrix} \Theta^- - (\sqrt{2\tau_1}\rho + \sqrt{2\tau_2}\sqrt{1-\rho^2})\Theta^- \; , \Theta^- < 0 \\ \Theta^+ - (\sqrt{2\tau_1}\rho + \sqrt{2\tau_2}\sqrt{1-\rho^2})\Theta^+ \; , \Theta^+ > 0 \end{bmatrix}$$

Therefore from equation XX we obtain that if the term $(\sqrt{2\tau_1}\rho + \sqrt{2\tau_2}\sqrt{1-\rho^2})$ is negative, then the first element of the matrix is negative and the second is positive. When $\tau_1 = \tau_2$ the above condition is satisfied if $\rho + \sqrt{1-\rho^2} < 0 \iff \rho < -\frac{\sqrt{2}}{2}$. [13] Then (17) and (18) become:

[12] This is in contrast to the robust control approach where we consider a lifetime constraint.

[13] When this condition is not satisfied we are not able to derive a similar result.

$$\Theta^- + (A_1 + \frac{A_2}{l} + \frac{g_1}{B_{10}}) - \frac{1}{n} \leq \Pr(B_{1t} > \frac{1}{n}B_{10}) \leq n(A_1 + \frac{A_2}{l} + \frac{g_1}{B_{10}}) + \tag{28}$$

$$\Theta^+ + (lA_3 + A_4 + \frac{g_2}{B_{20}}) - \frac{1}{n} \leq \Pr(B_{2t} > \frac{1}{n}B_{20}) \leq n(lA_3 + A_4 + \frac{g_2}{B_{20}}) + \tag{29}$$

$$\text{where } l = \frac{B_{10}}{B_{20}}$$

It can be seen from (28) and (29) that under ambiguity and a given land allocation and harvesting rules, the probability bounds corresponding to sustaining the species biomasses above a certain level change relative to the risk case. In particular when $\sqrt{2\tau_1}\rho + \sqrt{2\tau_2}\sqrt{1-\rho^2} < 0$, the bounds that correspond to the first species are lower, and the bounds corresponding to the second species are higher as compared to the bounds obtained under the risk aversion case. In general, however, the final outcome depends on the type of correlation between the two biomasses and the regulator's beliefs about the worst case scenario. This is interesting since it implies that our approach of dealing with ambiguity does not lead simply to wider bounds, but takes into account the structure of the ecosystem. Further research could involve simulations to determine the land allocation and harvesting rules which correspond to the same safety standard for biomasses and the same probability bounds. That is, a rule $(w_{1,}^r w_2^r, h_1^r, h_2^r)\big|_p^x$ under risk and a rule $(w_{1,}^a w_2^a, h_1^a, h_2^a)\big|_p^x$ under ambiguity. Comparison of the rules could provide some quantitative measure associated with precaution.

4 Optimal Land Allocation Rules under Uncertainty

In this section we consider biodiversity preservation rules in an optimizing framework. We develop our conceptual framework in a non-dynamic environment, as a first step before extending to a fully dynamic framework. We consider again a landscape normalized to unity with two species (Brock and Xepapadeas, 2002). Let B_{0i}, for $i = 1, 2$ be the initial biomasses and let $\widetilde{g}_i$ be the one period random growth rate of each species. If a proportion of $w_i \epsilon [0, 1]$ of the available land, with $w_1 + w_2 = 1$, was devoted to the growth of species i, then at the end of the period the

biomasses would be $w_i \widetilde{g}_i B_{0i}$ for $i = 1, 2$.

Assume that at the end of the period, the biomasses left in the landscape have a positive per unit 'existence' value $p_i \geq 0$, while the harvested biomasses have a value determined by a known exogenous market price $p_i^m \geq 0$, for $i = 1, 2$.

The landscape manager harvests, according to a harvesting rule, a proportion $h_i \epsilon [0, 1]$ of the available biomasses at the end of the period. Then the total value of the ecosystem's biomasses at the end of the period is defined as:

$$tv = \sum_{i=1}^{2} \{p_i^m h_i w_i \widetilde{g}_i B_{0i} + p_i (1 - h_i) w_i \widetilde{g}_i B_{0i}\} \tag{30}$$

Assume, in turn, that the ecosystem manager faces ambiguity about the growth rates, that is the manager has multiple priors. According to the e-contamination approach, the agent's beliefs about the probability distributions of the growth rates could be described as:

$$P_1 = P_{\epsilon_1}^{g_1} = \{(1 - \epsilon_1) g_1 + \epsilon_1 m_1, \; m_1 \in M_1 (\Omega)\} \tag{31}$$

$$P_2 = P_{\epsilon_2}^{g_2} = \{(1 - \epsilon_2) g_2 + \epsilon_2 m_2, \; m_2 \in M_2 (\Omega)\} \tag{32}$$

$$\epsilon_i \in [0, 1] \; i = 1, 2 \tag{33}$$

where g_i are the reference or benchmark estimates of the species growth rates and can be thought of as representing a point mass of unity at g_i, and $M_i (\Omega)$ represent the entire set of probability measures, with support $[g_i - b_i, g_i + b_i]$.[14] Thus g_i can be considered a consensus model, where least favorable or worst scenario cases, or precautionary approaches, could be represented by measures with most mass concentrated on $g_i - b_i$, while less precautionary approaches could be represented by measures with most mass concentrated on $g_i + b_i$. The manager's ambiguity is an increasing function of ϵ_i, where a value of $\epsilon_i = 0$ corresponds to the no ambiguity case.

[14] For an application of this approach to a global environmental problem, see Brock and Xepapadeas (2003).

Let u be a standard utility function with $u'(\cdot) > 0$, $u''(\cdot) < 0$. The aim of the social planner is to optimally choose w_1, w_2 in order to maximize the utility of tv given by (30), under the assumption of ambiguity expressed by the e-contamination structure (31)-(32). Following Gilboa and Schmeidler (1989), utility corresponding to the sets of probability measures $M_i(\Omega)$ can be defined as:

$$Eu(tv) \equiv \int \int u(tv) dP_{\epsilon_1}^{g_1} dP_{\epsilon_2}^{g_2} = \min_{\mu_{2 \in P_2}} \int \min_{\mu_{1 \in P_1}} \int u(tv) d\mu_1 d\mu_2 \qquad (34)$$

Assume that the agent wants to preserve biodiversity in the landscape, which means that $w_i > 0$, $h_i < 1$ $i = 1, 2$. Then the utility function is increasing in w_1 and w_2, and there are unique probability measures with the property to concentrate the mass with probability 1, at the left of the believable range, $g_i - b_i$. Therefore, utility (34) becomes:

$$u\left(\sum_{i=1,2} \left((1-\epsilon_i)\left[p_i + (p_i^m - p_i)h_i\right]B_{0i}g_i + \epsilon_i[p_i + (p_i^m - p_i)h_i]B_{0i}\left(g_i - b_i\right)\right)\right) =$$
$$(w_1[p_1 + (p_1^m - p_1)h_1]B_{01}(g_1 - b_1\epsilon_1) + w_2[p_2 + (p_2^m - p_2)h_2]B_{02}(g_2 - b_2\epsilon_2))$$

Because u is an increasing function, the agent's utility maximization problem is equivalent to the maximization of the quantity:

$$w_1[p_1 + (p_1^m - p_1)h_1]B_{01}(g_1 - b_1\epsilon_1) + w_2[p_2 + (p_2^m - p_2)h_2]B_{02}(g_2 - b_2\epsilon_2) \quad (35)$$

4.1 Risk Aversion

If the social planner was not ambiguity averse, but risk averse in the usual sense ($\epsilon_i = 0$), then the manager's problem would be the problem of the optimal choice of w_i, in order to maximize

$$w_1[p_1 + (p_1^m - p_1)h_1]B_{01}g_1 + w_2[p_2 + (p_2^m - p_2)h_2]B_{02}g_2 \qquad (36)$$

For a given harvesting rule, $(h_1, h_2) < 1$, we can write (36) as:

$$a_1 w_1 + a_2 w_2 \qquad (37)$$

where a_1, a_2 take values depending on known parameters, with:

$$a_1 = [p_1 + (p_1^m - p_1)h_1]B_{01}g_1$$
$$a_2 = [p_2 + (p_2^m - p_2)h_2]B_{02}g_2$$

Assume that we were not interested in preserving biodiversity in the landscape, then using (37), we obtain that:

$$\text{if} \quad \begin{array}{l} a_1 > a_2, \text{ then } w_1 = 0, \ w_2 = 1 \\ a_1 < a_2, \text{ then } w_1 = 1, \ w_2 = 0 \\ a_1 = a_2, \text{ then } w_1 = 0.5, \ w_2 = 0.5 \end{array}$$

In this specific case, the biomasses of the two species, at the end of the period and before harvesting, will be:[15]

$$\text{if} \quad \begin{array}{l} a_1 > a_2, \text{ then } B_1 = 0, B_2 = B_{02}g_2 \\ a_1 < a_2, \text{ then } B_1 = B_{01}g_1, B_2 = 0 \\ a_1 = a_2, \text{ then } B_1 = 0.5B_{01}g_1, B_2 = 0.5B_{02}g_2 \end{array}$$

Assume now that we were interested in biodiversity preservation, in the sense that we want positive period's end biomasses, before harvesting. This means that we would like w_i to be greater than zero, and to be at least $0 < \gamma < 0.5$ for $i = 1, 2$, which imposes a restriction on the available biomass B_i at the end of the period, since $B_i = w_i g_i B_{0i} = \gamma g_i B$. Then we obtain that:

$$\text{if} \quad \begin{array}{l} a_1 > a_2, \text{ then } w_1 = 1 - \gamma, \ w_2 = \gamma \\ a_1 < a_2, \text{ then } w_1 = \gamma, \ w_2 = 1 - \gamma \\ a_1 = a_2, \text{ then } w_1 = 0.5, \ w_2 = 0.5 \end{array} \tag{38}$$

In this case, the biomasses of the two species at the end of the period will be:

$$\text{if} \quad \begin{array}{l} a_1 > a_2, \text{ then } B_1 = (1-\gamma)B_{01}g_1, B_2 = \gamma B_{02}g_2 \\ a_1 < a_2, \text{ then } B_1 = \gamma B_{01}g_1, B_2 = (1-\gamma)B_{02}g_2 \\ a_1 = a_2, \text{ then } B_1 = 0.5B_{01}g_1, \ B_2 = 0.5B_{02}g_2 \end{array} \tag{39}$$

[15] It is clear that in this case all biomass will be harvested at the period's end.

In conditions (39), the third case of $a_1 = a_2$ is a special case and shows that, since through (37) (a_1, a_2) depends on the harvesting rules, a harvesting rule (h_1^r, h_2^r) could be chosen such that $a_1 = a_2$, or

$$(h_1^r, h_2^r) < 1 : [p_1 + (p_1^m - p_1)h_1^r]B_{01}g_1 = [p_2 + (p_2^m - p_2)h_2^r]B_{02}g_2$$

This implies that the combinations of these harvesting rules are on a straight line, shown in figure 1 as the RR line, and defined as:

$$h_1^r = \frac{Z_2 - Z_1}{Y_1} + \frac{Y_2}{Y_1}h_2^r, \tag{40}$$

$$Z_i = p_i B_{0i} g_i, \;\; Y_i = (p_i^m - p_i)B_{0i}g_i, \;\; i = 1, 2 \tag{41}$$

4.2 Ambiguity Aversion

We consider now the ambiguity framework with the e-contamination structure. Using the same harvesting rule as in the previous case, we obtain after manipulations, that equation (37) can be written as:

$$a_1(1 - \frac{b_1\epsilon_1}{g_1})w_1 + a_2(1 - \frac{b_2\epsilon_2}{g_2})w_2 \tag{42}$$

Using relationship (42), we derive that if the structure of the model is suitable, which means that the values of the parameters g_i, b_i, ϵ_i which reflect the confidence to the estimated model are such that:

$$\begin{array}{c} a_1(1 - \frac{b_1\epsilon_1}{g_1}) < a_2(1 - \frac{b_2\epsilon_2}{g_2}), \text{ then } w_1 = \gamma, \; w_2 = 1 - \gamma \\ a_1(1 - \frac{b_1\epsilon_1}{g_1}) > a_2(1 - \frac{b_2\epsilon_2}{g_2}), \text{ then } w_1 = 1 - \gamma, \; w_2 = 1 - \gamma \\ a_1(1 - \frac{b_1\epsilon_1}{g_1}) = a_2(1 - \frac{b_2\epsilon_2}{g_2}), \text{ then } w_1 = 0.5, \; w_2 = 0.5 \end{array} \tag{43}$$

By comparing (38) and (43) it can easily be seen that depending on the magnitude of the term $\theta_i = b_i\epsilon_i/g_i$, which embodies uncertainty, the land allocation policy under aversion, described by (38), can be reversed under ambiguity aversion. Furthermore, by using the last line of (43), it can be seen that for any harvesting rule h_1^a there is a corresponding value h_2^a such that $w_1 = 0.5$, $w_2 = 0.5$. In a way similar to the risk case, the combinations of these harvesting rules are on a straight line defined, shown in figure 1 as the AA line, and defined as:

$$h_1^a = \frac{(Z_1 - Z_2) - (\theta_2 Z_2 - \theta_1 Z_1)}{Y_1 (1 - \theta_1)} + \frac{Y_2 (1 - \theta_2)}{Y_1 (1 - \theta_1)} h_2^r \ , \ i = 1, 2 \qquad (44)$$

The positions of the two harvesting lines are hypothetical, but the shaded area between them can be regarded as a measure of the impact of ambiguity on the harvesting rules, which can be further interpreted as a quantitative measure of precaution. The relative position of the two lines depends on the problem's parameters. The relative positions of AA and RR suggest that ambiguity leads to a more conservative harvesting. This is not, however, always true, since a possible position ZZ for harvesting line under ambiguity suggests a possible range of more aggressive harvesting.

The model can also be extended to a more realistic set up by allowing for interactions among species growth rates which could be of either a 'competitive' or a 'facilitative' type.

5 Concluding Remarks and Areas for Further Research

We introduce the conceptual framework of multiple priors in order to analyze unmeasurable Knightian uncertainty (or ambiguity) which, given the multiple types of uncertainty characterizing ecosystems, might be regarded as a more appropriate framework relative to the classic risk case (measurable uncertainty). We believe that this approach can be regarded as a formal way of modelling the precautionary principle and provide policy rules for biodiversity management under precautionary approaches. We specify the multiple priors framework by the e-contamination and k-ignorance approaches which are associated with decision making in the context of least favorable priors and maxmin criteria.

We apply the k-ignorance approach to a descriptive dynamic model of interacting species and we provide land allocation and harvesting rules for keeping species biomasses above some minimum safety level with a given probability. We solve the problem under risk and ambiguity and, by comparing solutions, we provide a measure of the impact of adopt-

ing a precautionary approach. We apply the e-contamination approach to an optimal biodiversity management problem and we provide similar land allocation and harvesting rules, as well as the impact of the precautionary principle. It is interesting to note that the adoption of an ambiguity aversion framework does not imply 'uniform' conservatism with respect to all species, relative to the classic risk aversion case, as might have been expected intuitively. Rules could indicate, depending on the type of species interactions, conservative behavior towards one group and aggressive towards another.[16]

Our conceptual framework can be extended along three possible lines. In addition to the e-contamination and k-ignorance approaches, robust control methods (e.g. Hansen and Sargent, 2001) can also be used to specify multiple priors approaches and maxmin decision making. The conceptual framework of Knightian uncertainty or ambiguity can be extended to formal prey-predator or mechanistic resource-based models of species competition, along with simulation to obtain a true sense of the quantitative results. Finally, it might be worth exploiting the possibility of combined presence of measurable (risk) and unmeasurable (ambiguity) uncertainty in models described by two qualitatively different but interrelated dynamic systems. These could be, for example, coevolutionary models where population dynamics which evolve in a fast time scale are characterized by measurable uncertainty and a single prior, while trait dynamics which evolve in slow time are characterized by unmeasurable uncertainly and multiple priors.

[16] This is a result that has been noticed in monetary policy models and portfolio section models under ambiguity aversion (e.g. Onatski and Williams, 2003; Vardas and Xepapadeas, 2005).

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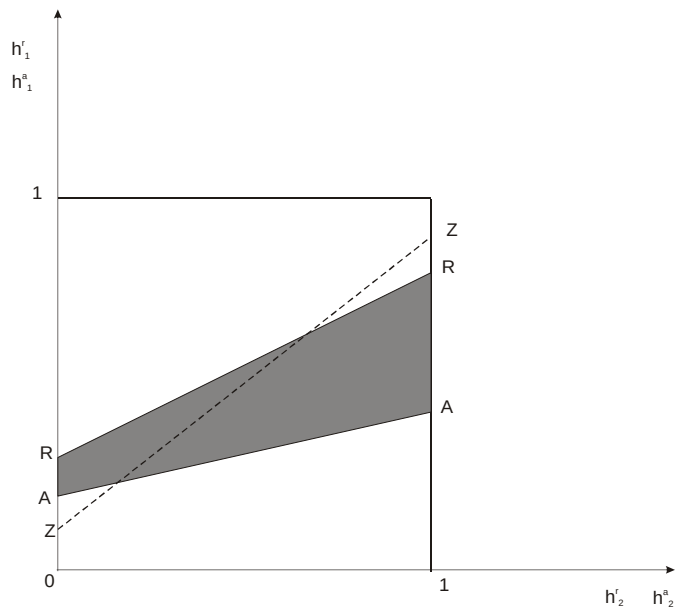

Figure 1: Harvesting lines