An Interior Optimal Species Preservation Policy for the Two Types' Symbiotic Noah Ark

Shiri Shamir

Department of Economics

University of Haifa

Mount Carmel, Haifa 31905, Israel

and

The Eitan Berglas School of Economics

Tel Aviv University

P.O. Box 39040, Tel Aviv 69978, Israel

E-mail: sshamir@econ.haifa.ac.il

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Abstract

Weitzman's (1998) seminal work applied the metaphor of Noah's ark and the related libraries model to the problem of species preservation under budget constraints. In this paper we consider the symbiotic Noah's Ark problem with two types of species: a keystone species and a keystone-dependent species, which relies on the keystone species for survival. The central planner maximizes the expected biodiversity value under budget constraint and obtains the optimal preservation policy. One of Weitzman's main conclusions was that under an appropriate independence assumption, an optimal policy yields an extreme outcome (almost all species either fully survive or die out). In contrast, we show that our symbiotic model with two types of species generates a *unique interior optimal policy*. Moreover, we find that under an interior optimal preservation policy, the expenditure on the keystone species' survival is greater than 50% of the given budget.

JEL classifications codes: D81, O21, Q57

Key words: optimal policy, biodiversity, symbiotic relationship, keystone species.

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1. Introduction

Biodiversity benefits human society by providing ecosystem services. In agriculture, biodiversity offers a reservoir of genetic traits from wild varieties, while in medicine, natural chemicals can be mined for potentially useful medicinal compounds. This is in addition to the aesthetic, cultural, or spiritual rewards that human beings derive from the natural world (Daily, 1999). The economic valuation of these ecosystem services faces many challenges. The understanding of interdependencies between: the species and their ecological functions, and the species and the ecosystems, is far from being complete (Kassar and Lasserre, 2004). Moreover, the society faces many unanswered questions related to biodiversity maintenance or preservation. Indeed, how can we manage and protect biodiversity? (Loreau, 2010).

One of the most important analyses of optimal preservation policies was pioneered by Weitzman's seminal paper "The Noah's Ark Problem" (1998), which applied the metaphor of Noah's ark (borrowed from his earlier work (1993)) and the related libraries model to the problem of species preservation under budget constraints. Weitzman shows how biodiversity theory can be used to determine optimal preservation policies, based on genetic distances between species. Specifically, Weitzman maximized the expected biodiversity under budget constraints, assuming the independence of the survival probabilities of different species. He also derived a general criterion to identify species that will enter Noah's ark, i.e. which species will survive.

Other papers extended the Weitzman framework. Mainwaring (2001) noted that Weitzman's model is unsuitable for studying global problems of biodiversity loss, and Weikard (2002) pointed out that applying Weitzman's model to genetic diversity is difficult. Heide et al. (2005) wrote that *"…Weitzman's general criterion only holds under a very strict condition: namely, in the absence of ecological relationships among species... This represents at best a very specific case, and can certainly not be regarded as providing general information about which species to protect through which protection projects…Weitzman's criterion is only suitable if ecological relationships are of little importance, notably if the loss of a species has little impact on an ecosystem".* (See also Montgomery et al. (1999), Haight (1995), and Marshall et al. (2000).)

In our paper we introduce a symbiotic model of biodiversity in order to relax the Weitzman's independence condition. We consider a symbiotic model with two types of species: a keystone species and a keystone-dependent species, which relies on the keystone species for survival, and identify the preservation methods that maximize the expected biodiversity value under budget constraints. Since the budget is usually not sufficient for full preservation, we can assume that only part of the preservation methods can be implemented under this given budget. Those methods, which maximize the expected biodiversity value, are called *optimal preservation policies*. Our ecological value function extends Weitzman's diversity index which was the number of different genes available to a set of species. We assume that the marginal contribution in ecological value terms of the keystone-dependent species is positive. We apply the Weitzman framework of the uncertain environment for the two types symbiotic model and consider the maximization problem of the expected biodiversity value under budget constraints to our symbiotic model.

The main result of our keystone species and keystone-dependent model is that there exists a unique optimal policy and this unique optimal policy is interior policy. Moreover, the optimal expenditure on the keystone species is greater than 50% of the given budget. These results may be applied to a policy recommended by us to a Nature and Parks Authority.

The paper is organized as follows: Section 2 describes a case study of the pines and pine mushrooms as a symbiotic model with positive marginal contributions in ecological value terms. In Section 3 we introduce the mathematical symbiotic model .In Section 4 we present the central planner problem. The results for the symbiotic model are given in Section 5. Section 6 compares the symbiotic model with Weitzman's model – algebraically and graphically – under linear costs. In Section 7 we present some possible extensions

2. The Symbiotic Case Study: the Pine and Pine Mushroom in National Parks

There is an interesting example of the symbiotic model of species preservation which clarifies the idea of the symbiotic model. The pine (*Pinus*) and pine mushroom found in National Parks (related species are found in forests throughout the world) are an example for a symbiotic relationship between species. The pine mushrooms in the natural park do not appear without the pines. While the main danger for pines is fires, the risks for the pine mushrooms come from fruit picking. The pines benefit humans by providing products for building and medicinal use as well as through their contribution to the landscape, while the mushrooms can be utilized both as edible fungi and for medical purposes. The relationship between these two types of species is defined as a symbiotic relationship – specifically, commensalism. The tree and fungus live in a mutually beneficial relationship in which mushrooms draw carbohydrates from the host tree's roots, and, in return, help the plant to absorb water and mineral nutrients. Because the mushrooms colonize the roots of the pine trees, they cannot survive without them. In addition, the pine mushroom's marginal contribution in ecological value terms is positive.

3. The Mathematical Symbiotic Model

We begin by denoting the keystone species by P and keystone-dependent species by M (P and M stand for the Pine species and the Pine Mushroom species respectively.)

The uncertainty (following Weitzman's approach) is based on survival probabilities' analysis. These probabilities are computed in Weitzman's independent probabilities model by the formula:

$\mathrm{Pr}ob(S) = \prod_{i \in S} P_i \cdot \prod_{i \notin S} (1 - P_i)$, where P_i is the survival probability of species i.

The probabilities in the symbiotic model that we are interested in are:

S1) $\mathrm{Pr}ob(\{M\}) = 0.$ Note that {M} stands for the case where the pine mushroom survives and the pine dies out. Thus, the probability of this event {M} is zero.

The other formulae ((S2)-(S3)) follow from the multiplication rule, where we use the conditional probability denoted by P_M^C (of the survival of the pine mushroom conditioned on the survival of the pine); where P_P is the pine survival probability.

S2) $\mathrm{Pr}ob(\{P, M\}) = P_P \cdot P_M^C$

S3) $\mathrm{Pr}ob(\{P\}) = P_P (1 - P_M^C).$

The expected biodiversity is defined to be the expectation of the biodiversity value function V(.) computed in our uncertainty environment. Thus, we have in the symbiotic case:

$$W^S = V(\{P\}) \cdot \Pr ob(\{P\}) + \cdot V(\{P, M\}) \Pr ob(\{P, M\}) \qquad (1)$$

Where, V(S) stands for the ecological value of a set S of species.

Inserting (S1)-(S3) in formula (1) and denoting $E_M \equiv V(\{P, M\}) - V(\{P\})$. E_M is the positive marginal contribution in the biodiversity value of the keystone-dependent species M to the keystone species P, denote $M_P \equiv V(\{P\})$. Then:

$$W^S = P_P M_P + P_P P_M^C E_M \qquad (2)$$

This can be compared to the Weitzman formula (equation 5, p. 1283) :

$$W^W = P_P M_P + P_M M_M - J P_P P_M \qquad (3)$$

Where $M_P \equiv V(\{P\})$, $M_M \equiv V(\{M\})$ and J are positive constants[5].

The cost function $C(P_P, P_M^C)$ in the symbiotic model embodies the idea that the central planner can increase the survival probabilities of both species by investing capital and using technology, so the probabilities are given by (S1-S3). Moreover, we assume that the cost function is separable $C(P_P, P_M^C) = C(P_P, 0) + C(0, P_M^C)$ so in the case of pine and pine mushroom this is carried out through programs to prevent (pine) forest fires and enforcement of prohibitions on mushroom harvesting.

4. The Central Planner's Problem in the Symbiotic Model

In order to identify the significance of the symbiotic model and its comparison to Weitzman's model, we assume, for simplicity, that (0,0) and (1,1) represent the lower and the upper bounds of the survival probabilities (and conditional probabilities) of both species. (I.e. the null and the full preservation policies.)

The central planner problem in the symbiotic model is to maximize the expected symbiotic biodiversity under budget constraints. We assume that the costs $C(\cdot)$ in our symbiotic model are bounded and are given by a continuous and strongly increasing function $C^S(P_P, P_M^C)$.We assume that the budget is not sufficient enough for full

[5] In Weitzman's diversity index the constant $J \equiv V(\{P\}) + V(\{M\}) - V(\{P, M\})$ is positive.

preservation policy, i.e. C(1,1)>B, and in the null preservation policy, the costs vanish, i.e. $C(0,0)=0$.

We get:

$$\text{MAX} \quad P_P M_P + P_P P_M^C E_M$$

$$\text{s.t.} \quad C^S\left(P_P, P_M^C\right) \leq B$$

$$(0,0) \leq \left(P_P, P_M^C\right) \leq (1,1)$$

5. The Results

5.1 Existence of Optimal Policies and Budget Equality

In the central planner's problem optimal policies exist and every optimal policy satisfies the budget equality.

Proof: See appendix 1.

5.2 Uniqueness of the Optimal Policy in the Two Types' Symbiotic Model

Theorem A: Assume a continuous convex costs function $C^S\left(P_P, P_M^C\right)$ and positive budget B>0, then the optimal policy in the two types' symbiotic model, $P^* = \left(P_P^*, P_M^{C^*}\right)$, is unique.

Proof: See appendix 2.

5.3 Interiority Property and Optimal Expenditure in the Symbiotic Model

In this section we consider three different cases of cost functions $C^S\left(P_P, P_M^C\right)$ and present the related results. The first case is when the costs are linear. The second case is the infinite marginal cost case. The third case is when we have a common value cost function.

Case I: The Linear Costs Case in the Symbiotic Model

Theorem B.1: Assume that the symbiotic cost function is linear. That is, $C^S\left(P_P, P_M^C\right) = C_P P_P + C_M P_M^C$, where by assumption $C(1,1) = C_P + C_M > B > 0$. Assume further that the unique optimal policy $\left(P_P^*, P_M^{C^*}\right)$ is interior. Then the optimal expenditure on the keystone species preservation is greater than 50% of the budget.

Proof: we have:

$$\text{Max} \quad P_P M_P + P_P P_M^C E_M$$

$$\text{s.t.} \quad C_P P_P + C_M P_M^C \leq B$$

$$(0,0) \leq \left(P_P, P_M^C\right) \leq (1,1)$$

By our assumption (of interiority) the F.O.C. yields $\dfrac{M_P + P_M^{C^*} E_M}{P_P^* E_M} = \dfrac{C_P}{C_M}$. By $\dfrac{P_M^{C^*}}{P_P^*} < \dfrac{M_P + P_M^{C^*} E_M}{P_P^* E_M}$ it follows that $C_M P_M^{C^*} < C_P P_P^*$. Thus, budget equality yeilds $C\left(P_P^*, 0\right) = C_P P_P^* > \dfrac{B}{2}$. Q.E.D.

Case II: The Infinite Marginal Cost in the Symbiotic Model

Assume the following assumptions:

1. The cost function is separable, i.e. $C\left(P_P, P_M^C\right) = C\left(P_P, 0\right) + C\left(0, P_M^C\right)$, strictly increasing in $\left(P_P, P_M^C\right)$ and convex.
2. The marginal cost of P_P at 1 is infinity, $MC_{P_P}(1,0) = \infty$
3. The marginal cost of P_M^C at 1 is infinity, $MC_{P_M^C}(0,1) = \infty$

Remark: Assumptions (2) and (3) are natural intuitive assumptions. Specifically, they imply that at full preservation of a species, the marginal cost is infinite. Note that in the Mt. Carmel National Park, obtaining full preservation of the pine (i.e. with no fire risk at all) can be done by investing huge, but finite, amounts of money, however the marginal cost of pine's preservation increases asymptotically to infinity.

Theorem B.2: Under the above assumptions the unique optimal policy is an interior optimal policy.

Proof:

I. Let $P^* = \left(P_P^*, P_M^{C*}\right)$ be the unique optimal policy.

1. Assume first that $P^* = (1,1)$. By budget equality $B = C\left(P_P^*, P_M^{C^*}\right) = C(1,1) > B$, a contradiction.

2. Assume now $P_P^* = 1 \text{and} P_M^{C^*} < 1$. Thus, P^* is a corner optimal preservation policy. The First Order Condition is: $\frac{MW_{P_P}}{MW_{P_M^C}} \geq \frac{MC_{P_P}}{MC_{P_M^C}}$ at P^*, where $MW_{P_P}, MW_{P_M^C}$, MC_{P_P} and $MC_{P_M^C}$ and denote the partial derivatives of $W^S\left(P_P, P_M^C\right)$and $C^S\left(P_P, P_M^C\right)$ with respect to their arguments. It follows that:

$$\infty > \frac{MW_{P_P}}{MW_{P_M^C}}\Bigg|P^* = \frac{M_P + P_M^{C^*} E_M}{P_P^* E_M} \geq \frac{MC_{P_P}}{MC_{P_M^C}}\Bigg|P^* = \frac{C_{P_P}(1,0)}{C_{P_M^C}\left(0, P_M^{C^*}\right)} = \infty \qquad (4)$$

A contradiction.

3. Obviously, since B>0 therefore $W^S\left(P_P^*, P_M^{C*}\right) > 0$, it follows that $P_P^* > 0$. We have proved that $0 < P_P^* < 1$, i.e. $P^*$is an interior optimal policy in the first argument.

II. Similarly, we can prove that under assumption (3) that $P_M^{C^*} < 1$. Q.E.D.

Case III: The Common Value Costs in the Symbiotic Model

We assume now the following conditions:

There exists a common function F for both species that is $F:[0,1] \rightarrow \Re_+$ with F(0)=0, $F'(1) = \infty$, $F' > 0$ and $F'' \geq 0$that satisfies $C^S\left(P_P, P_M^C\right) = F\left(P_P\right) + F\left(P_M^C\right)$.

(An example of such a function is $F(x) = K \cdot \left(1 - \sqrt{1 - x}\right)$.)

Theorem B.3: Under the above assumptions, the unique optimal policy is an interior policy that satisfies $P_M^{C^*} < P_P^*$, and $C\left(P_P^*, 0\right) > \frac{B}{2}$.

That is, the optimal expenditure on the keystone species in the unique interior optimal policy $P^* = \left(P_P^*, P_M^{C^*}\right)$ is greater than 50% of the given budget.

Proof: Obviously, by Theorem B.2 we have an interior optimal policy. For interior optimal policy we have the F.O.C.

$$\frac{P_M^{C^*}}{P_P^*} < \frac{M_P + P_M^{C^*} E_M}{P_P^* E_M} = \frac{F'\left(P_P^*\right)}{F'\left(P_M^{C^*}\right)} \qquad (5)$$

Assume by negation that $P_M^{C^*} \geq P_P^*$. Then by convexity $F'' \geq 0$ we have that F' is increasing, therefore $F'\left(P_P^*\right) \leq F'\left(P_M^{C^*}\right)$. Thus, $1 \leq \frac{P_M^{C^*}}{P_P^*} < \frac{M_P + P_M^{C^*} E_M}{P_P^* E_M} = \frac{F'\left(P_P^*\right)}{F'\left(P_M^{C^*}\right)} \leq 1$. A contradiction. Therefore: $P_M^{C^*} < P_P^*$ and by $F' > 0$ and therefore F is strictly increasing. Thus $F\left(P_P^*\right) > F\left(P_M^{C^*}\right)$, therefore $B = C\left(P_P^*, P_M^C\right) = F\left(P_P^*\right) + F\left(P_M^{C^*}\right) < 2F\left(P_P^*\right)$. Which yields that $C\left(P_P^*, 0\right) > \frac{B}{2}$. Q.E.D.

6. Comparison with Weitzman

There is a dichotomous between the symbiotic model and the Weitzman's model.

Indeed we have that when we have independence, i.e., when $P_M^C = P_M$ then we have either $P_P = 1$ and/or $P_M = 0$.

Proof: See Appendix 3.

6.1 The Symbiotic Model vs. Weitzman's Model

The central planner problem in both models is to maximize the expectation of the biodiversity value under budget constraints. By inserting W^S and W^W from equations (9) and (10) and combining with the costs functions $C\left(P_P, P_M^C\right)$, which embody the idea that the central planner can increase the survival probabilities of both species by investing capital and using technology, we obtain the following:

The Symbiotic Model		Weitzman's Model	
MAX	$P_P M_P + P_P P_M^C E_M$	MAX	$P_P M_P + P_M M_M - J P_P P_M$
s.t.	$C^S\left(P_P, P_M^C\right) \le B$	s.t.	$C_P P_P + C_M P_M \le B$
	$(0,0) \le \left(P_P, P_M^C\right) \le (1,1)$		$(0,0) \le \left(P_P, P_M\right) \le (1,1)$

6.2 Extreme Optimal Policy Theorem in Weitzman's Model with Two Species

The extreme optimal policy in Weitzman's model yields that one species either fully preserved or dies out, while the other species survival probability determined by the budget equality.

Weitzman proved that any optimal policy, under linear costs and independent survival probabilities, is an extreme policy.

6.3 Algebraic and Graphical Solutions of Weitzman's Model vs. the Symbiotic Model under Linear Costs

6.3.1 An Algebraic and Graphical Example of Weitzman's Model

Let us define

$$W^W\left(P_P, P_M\right) = P_P M_P + P_M M_M - J P_P P_M$$

$$C_P P_P + C_M P_M = B$$

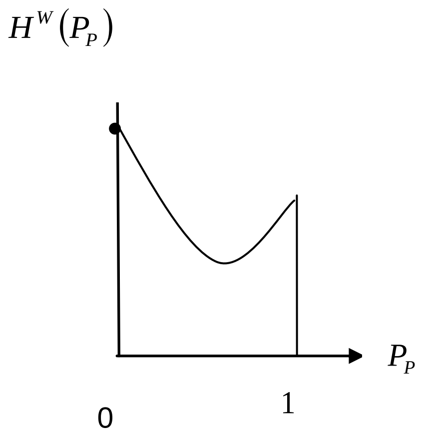
$$P_M\left(P_P\right) = \frac{B}{C_M} - \frac{C_P P_P}{C_M}$$

We assume that $C_P + C_M > B > 0$ and we denote Weitzman's expected diversity by:

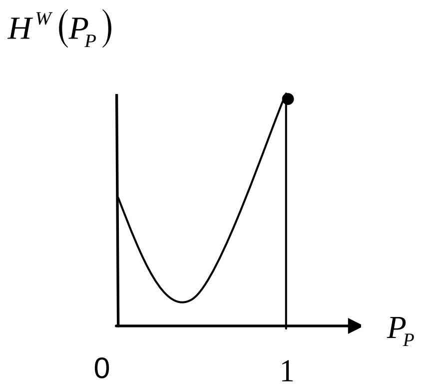
$$H^W(P_P) = W^W(P_P, P_M(P_P)) =$$
$$= P_P M_M + M_M\left(\frac{B}{C_M} - \frac{C_P P_P}{C_M}\right) - J P_P\left(\frac{B}{C_M} - \frac{C_P P_P}{C_M}\right) = \alpha + \beta P_P + \gamma P_P^2$$

Clearly $\alpha > 0$ and $\gamma > 0$. Thus, the function $H^W(P_P)$ is a strictly convex function over the interval [0,1] and hence the maximum of the strictly convex function $H^W(P_P)$ is obtained on the boundary (that is, either 0 or 1 or both).

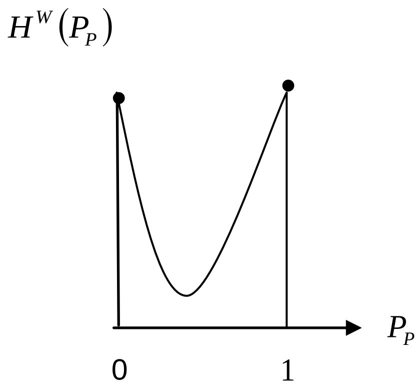
I. Optimal policy is in the Left Edge



II. Optimal policy is in the Right Edge



III. Two extreme optimal policies – one is in the Left Edge and the other is in the Right Edge



6.3.2 A Graphical Example of the Symbiotic Model

Let :

$$W^S(P_P, P_M) = P_P M_P + P_P P_M^C E_M$$

$$C_P P_P + C_M P_M^C = B$$

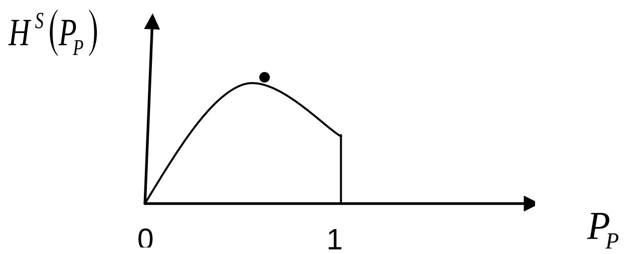
$$P_M^C(P_P) = \frac{B}{C_M} - \frac{C_P P_P}{C_M}$$

$$H^S(P_P) = W^S\left(P_P, P_M^C(P_P)\right) = P_P M_P + P_P E_M\left(\frac{B}{C_M} - \frac{C_P P_P}{C_M}\right) = \tilde{\beta} P_P - \tilde{\gamma} P_P^2$$

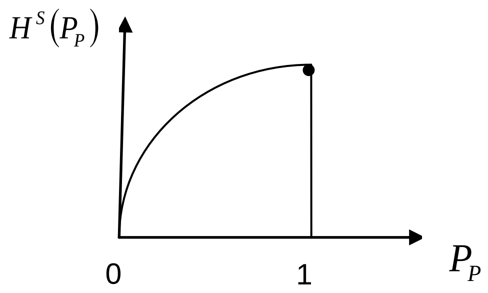
Clearly $\tilde{\gamma} > 0$ and thus, in the symbiotic case, $H^S(P_P)$ is a strictly concave function over the interval [0,1], and vanishing where $P_P = 0$. This strict concave function has a unique maximum.

This unique maximum might be interior or at the Right Edge 1 of the interval [0,1]

I. An interior optimal policy in (0,1)



II. An extreme optimal policy in the Right Edge



7. Extensions

Our symbiotic model in this paper can be useful to different applications. A possible application of our symbiotic model might be a central library with a single branch library, where each loan of a book is made via the central library. Our model implies that the optimal expenditure on the central library is greater than 50% of the given budget.

Furthermore, the two-species model can be extended to the case with (K+1) species, where one of the species is a keystone species and the other K species are keystone-dependent species (following Weitzman's uncertainty environments and our symbiotic approach). We plan to investigate this (K+1) species' symbiotic model and compare it to Weitzman general independent case in a subsequent paper.

Next, a different extension of our symbiotic model with pine and pine mushroom can be treated when one of the species is a predatory keystone-dependent species. Here, we can also obtain a unique optimal policy, where the objective of the central planner is to minimize the reduction of the expected biodiversity by the predatory species. In this model the predatory species' marginal contribution is negative. We found that in this case the central planner problem can be transformed into a standard central planner problem where we have a symbiotic structure.

Finally, we conclude that the results of this paper might guide conservation organizations or natural parks authorities how to allocate given budget on preservation of species. In the Carmel National park in Israel, the Israel Nature and Parks Authority (NPA) or the Jewish National Fund (KKL) are responsible for natural reserves. They have low budget and need to allocate it optimally. Our present paper gives them preliminary tools for optimal preservation policies, which are in contrast to Weitzman's criteria.

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<u>Appendix 1 – Existence of Optimal Policies and Budget Equality</u>

<u>Proof</u>:

A continuous function over a non-empty, closed and bounded set in $\Re^2_+$ has a maximum. Thus, optimal policies exist.

Let $P^* = \left(P_P^*, P_M^{C*}\right)$ be an optimal policy and assume by negation that there is not budget equality. That is, $C\left(P_P^*, P_M^{C*}\right) < B$. Then by $C(1,1) > B$, we have either $P_P^* < 1$ or $P_M^{C*} < 1$. Obviously, by the strict inequality we can obtain higher expected biodiversity, while keeping the budget constraint. A contradiction .Q.E.D

<u>Appendix 2 - Uniqueness of the Optimal Policy in the Noah's Ark Symbiotic Model</u>

<u>Proof</u>:

Let $P^* = \left(P_P^*, P_M^{C*}\right)$ be an optimal policy. We have $W\left(P^*\right) = P_P^* M_P + P_P^* P_M^{C^*} E_M = P_P^*\left(M_P + P_M^{C^*} E_M\right) = P_P^* Y^*$, where $Y^* \equiv M_P + P_M^{C^*} E_M$. Obviously B>0, $P_P^* > 0$ and $W\left(P^*\right) > 0$.

Assume by negation that $P^{**} = \left(P_P^{**}, P_M^{C**}\right)$ is another different optimal policy; therefore, its expected biodiversity value satisfies $W\left(P^{**}\right) = P_P^{**} M_P + P_P^{**} P_M^{C^{**}} E_M = P_P^{**}\left(M_P + P_M^{C^{**}} E_M\right) = P_P^{**} Y^{**}$, where $Y^{**} \equiv M_P + P_M^{C^{**}} E_M$.

Since P^* and P^{**} are both optimal policies we obtain $W\left(P^*\right) = W\left(P^{**}\right)$. In other words, $P_P^* Y^* = P_P^{**} Y^{**} > 0$, where $\left(P_P^*, Y^*\right)$ and $\left(P_P^{**}, Y^{**}\right)$ are strictly positive. The

Cobb-Douglas function $U(P_P, Y)$ in two variables P_P and Y given by $U(P_P, Y) = P_P \cdot Y$ is strictly quasi-concave in the pair of variables (P_P, Y), where P_P and Y are strictly positive. Since $P^* \neq P^{**}$, then necessarily (since $W(P^*) = W(P^{**})$) $P_P^* \neq P_P^{**}$ and $P_M^{C*} \neq P_M^{C**}$. Hence by strict quasi-concavity

$$W\left(\frac{P^* + P^{**}}{2}\right) = \left(\frac{P_P^* + P_P^{**}}{2}\right) \cdot \left(M_P + E_M \cdot \frac{P_M^{C*} + P_M^{C**}}{2}\right) =$$
$$= \left(\frac{P_P^* + P_P^{**}}{2}\right) \cdot \left(\frac{Y^* + Y^{**}}{2}\right) > Min\{P_P^* \cdot Y^*, P_P^{**} \cdot Y^{**}\} = W(P^*)$$

Also, by the convexity of $C(\cdot)$ in (P_P, P_M^C) and the fact that P^* and P^{**} are satisfying the budget constraint:

$$C\left(\frac{P^* + P^{**}}{2}\right) \leq \frac{1}{2}C(P^*) + \frac{1}{2}C(P^{**}) \leq \frac{1}{2}B + \frac{1}{2}B = B$$

We conclude that $\frac{P^* + P^{**}}{2}$ satisfies the budget constraint and the value of its expected biodiversity is strictly higher than $W(P^*)$. A contradiction. Q.E.D.

<u>Appendix 3 - Impossibility of the Representation of the Symbiotic Probability Model by an Independent Probability's Weitzman's Model</u>

<u>Proof</u>:

Assume, $P_M^C = P_M$. By (S1) and (S2) we get :

$$P_M = \Pr ob(\{\{M\}, \{P, M\}\}) = \Pr ob(\{M\}) + \Pr ob(\{P, M\}) = 0 + \Pr ob(\{P, M\}) = P_P \cdot P_M,$$

Thus we have either $P_P = 1$ and/or $P_M = 0$. Q.E.D.